A mathematical programming tool for an efficient decision-making on teaching assignment under non-regular time schedules

Abstract In this paper, an optimization tool based on a MILP model to support the teaching assignment process is proposed. It considers not only hierarchical issues among lecturers but also their preferences to teach a particular subject, the non-regular time schedules throughout the academic year, different type of credits, number of groups and other specific characteristics. Besides, it adds restrictions based on the time compatibility among the different subjects, the lecturers' availability, the maximum number of subjects per lecturer, the maximum number of lecturers per subject as well as the maximum and minimum saturation level for each lecturer, all of them in order to increase the teaching quality. Schedules heterogeneity and other features regarding the operation of some universities justify the usefulness of this model since no study that deals with all of them has been found in the literature review. Model validation has been performed with two real data sets collected from one academic year schedule at the Spanish University Universitat Politècnica de València (UPV).

Keywords: Teaching assignment problem, non-regular schedules, time compatibility, type of credits, mixed integer linear programming

1. Introduction

According to Schaerf (1999), the creation of an academic schedule consists of determining who will teach a subject and in which time slot it will be taught, given a set of constraints. The large number of subjects/courses and lecturers and their multiple assignment combinations makes it complicated even to simply finding a feasible solution. This task automation, commonly known as automated timetabling, was due to the low performance of manually developed schedules.

It began with Gotlieb (1963), cited in Schaerf (1999), and since then there have been numerous researchers having an increasing interest in automated university timetabling, with many works published on the topic, giving a view of the different approaches to solve this type of problems with optimal, heuristic and meta-heuristic methods (Burke and Petrovic 2002; McCollum and Ireland 2006; Landa-Silva and Obit 2008,), or more recently (Kingston 2013; Kristiansen and Stidsen 2013; Sorensen and Dahms 2014; Salem et al. 2015; Babaei et al. 2015; Bettinelli et al. 2015, Kristiansen et al. 2015; Pillay 2016; Dorneles et al. 2017; Tan et al. 2021). Not only that, but also some important groups and events have been held since the beginning of this century encouraging researchers to get deeper on this topic, such as the Unitime organization (Unitime 2020), a comprehensive educational scheduling system that supports developing course and exam timetables or the recent fourth International Timetabling Competition (ITC 2019), devoted to university course timetabling.

However, institutions peculiarities influence in the academic schedule, which prevents either creating a universal resolution model or comparing techniques in different scenarios (Al-Yakoov and Sherali 2006). Among the various tools to approach, it stands out mathematical programming, which has been widely used in the literature to solve problems of assigning/allocating limited resources to activities.

In this paper, a mixed integer linear programming (MILP) model to solve a real problem of university timetabling is proposed as well as checking its behavior within different scenarios by measuring the quality of the obtained results and its computational efficiency.

This real problem concerns to Universitat Politècnica de València (UPV), which is characterized by a matrix organization structured in faculties and departments. Each faculty offers different academic degrees in which multiple departments teaching different subjects are involved. Each subject has a number of credits of different nature: classroom theory, classroom practice, laboratory practice and field practice. In addition to that, each subject may differ in the types of credits and the number of groups, not only between different degrees but also in the same one. Moreover, the faculties establish the different schedules.

Regarding the departments, they are structured in knowledge areas. Each lecturer only belongs to a department and knowledge area. There are different ranks and types of contracts for lecturers that define the minimal number of teaching hours, which in turn may be reduced by management issues, research performance, work tutoring recognition, etc. This causes each lecturer to have a different "effective" teaching load, that is, a real number of hours to teach. Finally, each department calculates its saturation level as the ratio of the total credits requested by the different university faculties and the sum of all its lecturers' teaching loads. This department saturation index will become the target of individuals in order to level the teaching load of any of them.

The teaching assignment is a complicated task, especially in those large departments where there are also a huge number of part-time lecturers with very limited time availability, as they must handle their work inside and outside the university. This difficulty is even greater since the faculties determine its academic schedules that are characterized by being non-regular both the weekly schedule and the start time and duration of each subject.

In many cases, the process of teaching assignment among one-department lecturers is hierarchical, that is, senior lecturers have the highest priority to choose. It is clear that it may cause certain disadvantages and inefficiencies, being the most important that some teachers are assigned to subjects in which they are not expert or the fact that some subjects are assigned a high number of lecturers, both factors causing a decrease in the overall teaching quality.

The proposed MILP model for optimizing the teaching assignment handles with specific characteristics non addressed in previous works such as: non-regular times either in the weekly schedules or the duration of the classes, subjects with different types of credits and groups for every credit, different ranks for lecturers with different teaching loads and availability time intervals, inclusion of lecturer preferences (due to time availability or subjects contents) to teach each group of each type of credit for each subject, as well as lecturers' maximum and minimum saturation restrictions and limitations of the number of lecturers per subject and subjects per teacher.

Finally, a resolution approach is proposed to solve and validate the model. The model is solved by a two-steps procedure, that does preserve optimality in a reasonable computational time, solving each step with a general MILP solver. Two cases are considered from real data collected from one academic year schedule in two of the departments of the Universitat Politècnica de València (UPV). The first one is a real case corresponding to the teaching assignment of the ETSII faculty to the Applied Linguistics department. The aim is to validate and analyze the quality of the obtained solutions and the computational efficiency under different scenarios. The second one is also a real case corresponding to the teaching assignment of the Industrial Management Engineering Degree to the Business Organization department with the aim of giving some insights on the scalability and speed characteristics of the resolution methodology in a larger problem.

The rest of the paper is structured as follows: in section 2 a structured revision of the relevant research to our problem is conducted in order to show how the paper fits in the current literature as well as highlight its contributions. Section 3 includes a detailed description of the problem in order to facilitate subsequent mathematical programming modeling in section 4. In section 5, the previous model is solved with the objective of validating, analyzing the amount of solutions obtained and the computational efficiency of different scenarios, using the developed tool for its application to a real problem. Besides, the model is also solved for a larger real problem in order to measure the computational efficiency. Finally, in section 6, some conclusions and future research lines are outlined, performing a subjective evaluation of results as well as a proposal of possible improvements to model similar problems.

2. Literature review

In a first stage, the literature search was focused on mathematical programming models for the teaching assignment process. However, due to the lack of papers addressing that issue, it was decided to extend the literature review to some related ones such as the setting of university schedules, and particularly academic schedules, whose content may be transferred to some of the characteristics of the studied problem.

The references have been categorized in order to accelerate the process of identifying the type of problem addressed by each reviewed work and which features of the studied problem have previously been modeled or not. The classification has been made using the following dimensions: scheduled event, problem type, model, resolution method, resolution procedure, objectives, distinction of credits types, timetable, lecturer availability, lecturer preferences, rank and saturation.

Regarding the "**schedule event**", Schaerf (1999) distinguishes the following cases, depending on the institution and/or the events to be scheduled (classes or exams):

- Setting of school schedules: the focus of this type is to prevent a lecturer to have two classes at a time.
- Setting of university schedules: the main difference with the previous one is that university students may be enrolled in subjects belonging to different academic years. Therefore, it has to be ensured as much as possible that in no case a student would have to attend different classes at the same time. This also includes those school schedules in which electives subjects are also offered.
- Setting of exam schedules: the focus is to ensure that no student will have two exams on the same day and have all of them uniformly spread over the exam period.

Regarding the "**problem type**", a classification based on which elements among lecturers (L), subjects (S), time periods (T) and classrooms (C)) are scheduled. This proposal classifies problems into three types:

- Lecturer-subject (L-S): these problems consist of assigning lecturers once all the subjects are scheduled.
- Subject-period (S-T): these problems consist of determining in which period of time each subject is taught, being just an input its lecturer assignment or simply not addressed.
- Lecturer-subject-period (L-S-T): these problems consist of combining the above two types as in this case must be determined which lecturers are assigned to each subject as well as the period of time each subject is taught.

In the last two types it is possible the location (classroom) to be addressed as an additional aspect, leading to S-T-C and L-S-T-C, respectively. In these cases, classrooms capacity and availability for the assignment of subjects to the different time slots are also taken into account.

Another dimension refers to the "**model**" used to address the problem, which distinguishes between linear programming (LP), integer programming (IP), binary programming (BP), mixed integer linear programming (MILP) and nonlinear programming (NLP).

In the dimension regarding the "**resolution method**" of the former mathematical programming model it is distinguished between exact (search for the best/optimal solution) or heuristic (finding a feasible/good solution) methods.

Regarding the "**resolution procedure**", when the problem is too large it is often divided into two subproblems where the solution obtained in the first one becomes the input of the second one. Two steps resolution reduces the computational time while keeping a good solution quality, as corroborated by Daskalaki and Birbas (2005) comparing with the study of Daskalaki et al. (2004). Therefore, this dimension leads to distinguish between one or two steps resolution.

Table 1 shows the reviewed literature according to the above dimensions.

[Table 1 near here]

The dimension "**objectives**" shows which ones are aimed to be optimized in the search for a solution (**Table 2**).

[Table 2 near here]

The dimension "**timetable**" refers to whether it is regular or non-regular in either the start times or the duration of the subjects.

The dimension "**credit type**" refers to whether different credits types within a subject (e.g. theory and practice credits) are distinguished as well as if various groups for each type of credit exist (e.g. one single group of theory and three of practice).

The dimension "**availability**" refers to the fact that each lecturer has a limited time availability due to their teaching duties in other university faculties or professional works outside the academic field. Therefore, a subject might be assigned to a lecturer only if it fits into the lecturer availability schedule.

Each lecturer may also have a "**preference**", which can be taken into account during the teaching assignment.

Besides, each lecturer may have a "**rank**" and therefore certain priorities. For example, higher priority could be given to meet the preferences of lecturers with "Professor" ranks rather than those who are just "Associate Professor".

Finally, the dimension "**saturation**" refers to the ratio between the workload assigned to each lecturer and their available time capacity. In order to make a fairer teaching assignment, it may be aimed the final lecturer workload to be between a minimum and a maximum percentage (saturation margins) of their total assigned workload so that the teaching quality and lecturers' satisfaction can be improved.

These dimensions are also collected in **Table 3**, as in the two previous Tables.

[Table 3 near here]

With the aim of fitting our work in the current research and clearly present its novelties, in the following paragraphs a structured analysis of the literature reviewed is presented. As shown in Table 1, the problem approached in this paper is classified as lecturer-subject (L-S) since the subjects' timetable as well as the different credit types and groups are previously scheduled by faculties. It is aimed to make and efficient teaching assignment taking into account, among others, lecturers' preferences as well as their rank and saturation margins. Some revised works deal either with the same problem type (L-S) such as Hultberg and Cardoso (1997) or include it in a broader one: L-S-T-C such as Badri (1996), Badri *et al.* (1998), Al-Yakoob and Sherali (2006), Skoullis *et al.* (2017) or L-S-T, such as Ismayilova *et al.* (2007), Tassopoulos and Beligiannis (2012), Katsaragakis *et al.* (2015), Fonseca *et al.* (2017) and Tassopoulos *et al.* (2020). However, as justified below, these works do not address some of the peculiarities of the problem under study.

It can be observed in Table 1 that many mathematical programming-based approaches for university timetabling have been developed in the last decade, most of them using predefined sets of diverse instances such as XHSTT (Fonseca *et al* 2017), Brazilian high school, Lectio or Greek high school datasets (Katsaragakis *et al.* 2015, Tassopoulos *et al.* 2012, 2020), with the aim to improve on the solutions previously found in quality and/or computational efficiency. The Integer Programming and Binary Programming based methods are the current state-of-the-art for many of those instances. The majority of them used a 1-step resolution procedure consisting of an exact resolution method (Badri et al.1998, Dimopoulou and Miliotis 2001, Daskalaki et al. 2004, Ismayilova et al. 2007, Santos et al. 2012, Fonseca et al. 2017, Savienic et al. 2020). Among them, some authors propose reformulations for already existing integer programming models including new cuts (e.g. Santos *et al.* 2012, Fonseca *et al.* 2017) or defining a column generation procedure (e.g. Santos *et al.* 2012, Saviniec *et al.* 2020) for improving dual bounds on hard combinatorial optimization problems. There are also an important number of approaches using heuristic and meta-heuristics procedures alone or jointly with exact methods that have obtained promising results in the last years, although to reach the optimal solution is not ensured (Tassopoulos and Beligiannis 2012, Katsaragakis et al. 2015, Skoullis et al. 2017). Other works exist that design resolution procedures composed by 2-steps mainly adopting a decomposition approach to reduce the computational effort dividing the original problem into linked smaller ones (Badri 1996, Al-Yakoob and Sherali 2006, Birbas et al. 2009, Ceschia et al. 2014, Fonseca et al. 2017, Tassopoulos et al. 2020).

The proposed model in this paper does not use any of the above data instances, but collects a real-world problem presenting some unusual features that have not been covered in the predefined data sets and, to our knowledge, not simultaneously addressed by any work. This can be checked in Tables 1-3 where the most relevant characteristics of our paper are compared with those addressed by the revised literature. For this reason, the results obtained in this paper cannot be compared with other algorithms already published in the literature. More specifically, we adopt a two-step resolution procedure with an exact resolution method whose efficiency has been proven by its application to two real data sets from the UPV. Indeed, in recent years, advances in the computational efficiency of general-purpose MIP solvers have motivated researchers to investigate the potential of exact algorithms for these type of problems (Tassopoulos et al. 2020).

On the other hand, the main objective of our paper is to find a feasible teaching assignment so that the lecturer satisfaction level is maximized. It should be noted that in the **Table 2** and regarding the reviewed works the objective of satisfying the preferences lecturer-subject is described as "secondary" since, as corroborated by **Tables 1-3**, the few studies that do take it into consideration are multi-objective and aim to balance it with other objectives.

Other relevant characteristics of the papers revised can be consulted in **Table 3.** As it can be observed, just a few works (Daskalaki et al. 2004, Daskalaki and Birbas 2005, Al-Yakoob and Sherali 2006, Fontseca et al. 2017) consider different credit types within a subject, as they usually take them into account as different subjects. Unlike these works, different credit types within a subject, as well as different groups of each credit are considered in this paper.

Besides, all the analyzed papers deal with regular timetables (Table 3) that simplifies the problem under study since only one planning period (e.g. week) should be scheduled. Indeed, one of the main contributions of the proposed model in this paper is the non-regular nature of the academic schedule that makes it necessary to distinguish between weeks. However, most of the analyzed works split the planning horizon into periods and the time slots of each subject have the same duration. This means that if the duration of a class wants to be extended, then two consecutive slots have to be assigned. In our case, the schedule do not follow any pattern and daily change either in the number of classes or its duration. This is due to the fact that the teaching assignment of each individual department comes from different faculties, each of them with different schedules philosophies.

Several papers take into account the limited lecturer availability such as Daskalaki *et al.* (2004), Daskalaki and Birbas (2005), Birbas *et al.* (2009), Santos *et al.* (2012), Tassopoulos and Beligiannis (2012), Katsaragakis *et al.* (2015), Fonseca *et al.* (2017), Skoullis *et al.* (2017), Saviniec *et al.* (2020), Tassopoulos *et al.* (2020). Nevertheless, the consideration of regular timetables by all the revised papers implicitly forces them to assume a regular lecturers` availability. Therefore, the most general case in which the lecturer time availability is non-regular is also a contribution. It leads to different daily availability time slots that may also vary during each of the weeks of the horizon. This aspect is addressed by defining the start and end of each lecturer availability period. The non-regular nature of the schedule makes it relevant the inclusion of this feature since not all the subjects, type of credits and groups must be taught with the same timetable and/or duration during the planning period: even more, some of them must not be taught every week due to either scheduled activities at the University or calendar events (holidays).

On the other hand, the majority of the reviewed works consider that the number of credits (teaching hours) is equivalent to the number of classes as they are considered with the same duration. However, such assumptions may not be transferred to our problem since the schedule does not follow any pattern so that the duration of the classes can vary not only between subjects but also among the credit types. Consequently, it is not possible to count the number of credits from the number of classes. This fact leads us to calculate each class credits by the difference between its end and start time. Moreover, setting the schedule from the subjects start and end times allows us to tackle the teaching assignment in a variable schedule environment.

Lecturer preferences have been included in the literature related to time periods (Daskalaki *et al.* 2004, Daskalaki and Birbas 2005, Dimopoulou and Miliotis 2001), working shifts (Birbas *et al.* 2009), preference resources (Fonseca *et al* 2017), assignment and time periods (Badri 1996, 1998, Al-Yakoob and Sherali 2006, Ceschia *et al.* 2014) and assignment lecturer-subject-time slot (Ismayilova *et al.* 2007). However, unlike the previous papers, our model includes coefficients in the objective function to express the lecturers' preference degree as regards not only to teach certain subjects in certain time periods but more specifically to teach each of their groups and credit types.

It can be also observed from the analysis of the literature, that very few papers address some features regarding the rank or organizational hierarchy during the assignment: Al-Yakoob and Sherali (2006) restrict certain subjects to expert lecturers and Birbas et al. (2009) distinguish between full-time and part-time lecturers. However, in this paper, when maximizing the different lecturers´ preferences more priority is given to those lecturers in higher positions of the organizational hierarchy. For that, a rank not limited to a predefined number of hierarchy levels, such as the two ones addressed in the previous works, is considered.

Finally, different papers include some features affecting in greater or lesser extent to the lecturers' saturation: lecturers not wanting consecutive subjects (Al-Yakoob and Sherali, 2006), limits for lecturers' teaching workloads (Ismayilova *et al.*, 2007), (Birbas *et al.* 2009, Katsaragakis *et al.* 2015, Fonseca *et al.* 2017, Skoullis *et al.* 2017, Tassopoulos *et al.* 2020), upper limit regarding the number of daily lessons of a subject taught by a lecturer (Santos *et al.* 2012), similar daily workload (Tassopoulos and Beligiannis 2012) and minimization of extra days of the teacher (Saviniec *et al.* 2020). As it can be seen, the most extended way to approach lecturers´ saturation is to force their teaching workload to be inside a range composed by a lower and an upper limit. Our approach is similar to this last one, but due to the fact that each lecturer can present different maximum number of hours potentially assigned depending on several factors, the saturation is restricted by a maximum and minimum percentage of the load.

Summarizing, in **Table 4** the main differences between the model proposed in this paper and the reviewed works are presented to clearly show the novelty of our proposal. The focus on satisfying the lecturer-subject preferences by means a multi-objective approach, the different priority of lecturers when satisfying their preference based on a pre-defined hierarchy with any number of levels (rank), the inclusion of non-regular times in the weekly schedules, the duration of the classes and the lecturers´ availability, the definition of subjects with different types of credits and groups for every credit can be considered as the most relevant contributions of this paper.

This set of differences from the revised papers represents a proof of the original and innovative model created to respond to the described problem. Besides these contributions, as it can be observed in Tables 1-3, other characteristics of our problem have been previously considered in other works, although to the best of our knowledge, there is no paper dealing with all of them simultaneously. For a better understanding of these aspects, a detailed description of our problem features is presented in the next section.

[Table 4 near here]

3. Problem description

This paper aims to develop a mathematical programming model (MILP) for an efficient teaching assignment. This leads to the setting of an academic schedule in which all the subjects are assigned to certain lecturers, which in turn implies the definition of the individual lecturers' schedules for each week of each semester within the academic year. The main elements of the problem are two, lecturers and subjects, each of them with specific characteristics.

Regarding the first element, lecturers, the following characteristics are highlighted (**Fig.1**):

- They initially have a teaching load, depending on their rank and type of contract, which may be reduced by research and teaching merits, leading to the so-called teaching effective load (measured in hours).
- Their saturation level is given by the ratio between their assigned credits (hours) and their teaching effective load.
- They express their preferences to teach a particular credit type (theory, laboratory, etc.) and group (english group, evening group, etc.) of each of the subjects based on a parameter called preference which depends on how eager is the lecturer to teach it. The preference parameter is defined on a scale from 1 to 10, in which 10 corresponds to the highest preference value. Normally, their preferred subjects match those found in their expertise knowledge area and are likely to be assigned to them.
- They hold different ranks which will be taken into consideration. The model allows to weight the total satisfaction of the teaching assignment according to their rank.
- They have a limited time availability prior to the teaching assignment. Each lecturer may have several availability time slots on the same day and it could be different for each day and each week of the horizon. Therefore, the start and end times for each of them for each day of each week of each semester are known beforehand.

[Figure 1 near here]

Regarding the second element, subjects, the following characteristics are highlighted (**Fig.2**):

- They are made up of a number of credits, equivalent to the number of taught hours, which in turn, may be of different nature: classroom theory, classroom practice, laboratory practice, computers practice and field practice.
- Each type of credit can be given in different groups. For a given subject it might be just a single classroom theory credit but its laboratory practice ones being splitted into two or more groups.
- The duration of each group of each credit type of each subject as well as its start and end times may be different; that is, it is a non-regular schedule.
- The time slot in which each group of each credit type of each subject is taught is defined and known prior to the teaching assignment.

[Figure 2 near here]

The problem to be addressed consists of finding the most efficient teaching assignment so that all the credits are assigned and the overall preference is maximized. As aforementioned, the overall preference is calculated as the sum of the individual lecturer preferences regarding the teaching of a specific group of a credit type of a subject.

The ideal outcome would be one in which each lecturer was assigned that group/credit type/subject with their highest preference. However, it is impossible to assign their preferred choice since a large number of elements must be considered. In this case, a balanced assignment must be achieved, so that the overall preference is maximized, while respecting the different restrictions. Besides, certain assignments will be prioritized depending on the lecturer rank.

On the other hand, the proposed model reduces the search space for each lecturer to those subjects that could be assigned to them because their schedule is compatible with their time availability.

Additionally, the assignment gets more complicated due to the fact that no overlap may exist between subjects taught by the same lecturer. Therefore, the start and end times of different groups of different credit types of different subjects must be compared to ensure that no overlap exists. It must also be ensured that the teaching load assigned to each lecturer does not exceed their capacity. Finally, to achieve a feasible solution all the subjects must be taught by the set of lecturers from the department.

The above requirements represent the so-called hard constraints that must be included in the model in order to find a feasible solution. However, it is possible to include other aspects such as the consideration of different objectives and policies of each department. Such policies are the so-called soft constraints and are intended to ensure a minimum teaching quality either from the lecturers or the students' points of view. They are as follows:

- Limit the maximum and/or minimum saturation level of each lecturer. The teaching load assigned to each lecturer may not exceed a percentage α of their teaching effective load availability and not being below a minimum threshold.
- Do not allow a lecturer to teach in more than a certain number of different groups in the same semester. This undoubtedly will benefit the students and lecturers.
- Do not allow a group of a certain credit type of a subject to be taught by more than a certain number of lecturers.

4. Problem modeling

To solve the above problem a mixed integer linear programming (MILP) model to maximize a multi-objective function related to the overall lecturers' satisfaction and the teaching quality is proposed, subject to constraints such as lecturers' availability, schedule compatibility and some departmental policies, as aforementioned.

Regarding the nomenclature, the indexes, sets, parameters and decision variables are shown in **Tables 5-11.**

[Tables 5-11 near here]

Two objectives are initially approached in this model: maximize the overall lecturer´s satisfaction and the teaching quality.

Equation **(1)** refers to the first one, being $pref_{lscg}$ the weight (preference-related) assigned by a lecturer l to the fact of teaching a class in the group g of the credit type c of the subject s, and $rank_r$ the parameter which prioritizes the lecturers' requirements satisfaction depending on their rank. Since the aim is to maximize the objective function, the greater the lecturers' preference to teach a certain subject is and the higher their rank is, the higher are the value of the parameters $pref_{lscg}$ and $rank_r$.

Max $[O.F.1]$ =

$$\sum_{r} \sum_{l \in C(l,r)} \sum_{s \in S_{possible}(l,s)} \sum_{c \in C(s,c)} \sum_{g \in G(s,c,g)} \sum_{z \in Z(s,z)} \sum_{w \in W(z,w)} \sum_{d \in D(s,c,g,z,w,d)} (rank_r * pref_{lscg} * X_{lscgzwd}) \quad \textbf{(1)}$$

Equation **(2)** refers to the second one and aims to minimize the number of different lecturers teaching in the same group. This new objective aims to improve the teaching quality by allowing lecturers to focus on a limited set of specific skills. Depending on the case, this objective could be formulated as a soft constraint of the problem.

Min $[O.F.2]$ =

$$\sum_{l} \sum_{s \in S_{possible}(l,s)} \sum_{c \in C(s,c)} \sum_{g \in G(s,c,g)} Y_{lscg} \quad \textbf{(2)}$$

The objectives expressed in equations **(1)** and **(2)** may be combined from the weighted sum of scaled values method. This will be achieved by dividing each of the objectives between the maximum values that can reach so that the ratio is within the range [0-1]. Typically, this maximum value may be obtained in a simple manner ignoring the considered constraints.

The parameters *weight1* and *weight2* are also included in the objective function in order to modify both parameters in the experimental phase to test the effect of prioritizing one or another objective and validate the model behavior. The following equation **(3)** represents the objective function obtained from the combination of the previous ones.

Max $[O.F.3]$ =

$$weight_1 * (\sum_{r} \sum_{l \in R(l,r)} \sum_{s \in S_{possible}(l,s)} \sum_{c \in C(s,c)} \sum_{g \in G(s,c,g)} \sum_{z \in Z(s,z)} \sum_{w \in W(z,w)} \sum_{d \in D(s,c,g,z,w,d)}$$

$$\frac{rank_r * pref_{lscg} * X_{lscgzwd}}{OF1_{max}}) - \quad weight_2 * (\frac{\sum_l \sum_s \sum_c \sum_g Y_{lscg}}{OF2_{max}}) \quad (3)$$

Equation **(4)** prevents a group *g* of the credit type *c* of the subject *s* from being assigned more than a single lecturer in the same time slot of the day *d* of the week *w* of the semester *z*. That is, a class can not be taught by greater than one lecturer.

$$\sum_l X_{lscgzwd} \le 1 \quad (4)$$

$\forall s,\ \forall c \in C(s,c),\ \forall g \in G(s,c,g),\ \forall z \in Z(s,z),\ \forall w \in W(z,w),\ \forall d \in D(s,c,g,z,w,d)$

Equation **(5)** prevents a lecturer *l* from being assigned greater than λ different groups in the same semester *z*.

$$\sum\nolimits_{s \in Z(s,z)} \sum\nolimits_{c \in C(s,c)} \sum\nolimits_{g \in G(s,c,g)} Y_{lscg} \le \lambda \quad (5)$$

$$\forall p,\ \forall z$$

Equation **(6)** limits to μ the number of lecturers teaching the classes of a group *g* of the credit type *c* of the subject *s*. It is aimed to improve the teaching quality.

$$\sum\nolimits_l Y_{lscg} \le \mu \quad (6)$$

$$\forall s, c \in C(s,c),\ g \in G(s,c,g)$$

By means of equations **(7)** and **(8)**, the decision variable representing the teaching assignment of subjects to lecturers are obtained (**Y_{lscg}**). Through **(7)** it is assured that in case that group *g* of the credit type *c* of the subject *s* is assigned to lecturer *l*, then the value of Y_{lscg} is 1, otherwise, the value is 0 (8).

$$\sum\nolimits_{z \in Z(s,z)} \sum\nolimits_{w \in W(z,w)} \sum\nolimits_{d \in D(s,c,g,z,w,d)} X_{lscgzwd} \le n * Y_{lscg} \quad (7)$$

$$\forall l,\ \forall s \in S_{possible}(l)\ ,\ \forall\ c \in C(s,c),\ g \in G(s,c,g)$$

$$\sum\nolimits_{z \in Z(s,z)} \sum\nolimits_{w \in W(z,w)} \sum\nolimits_{d \in D(s,c,g,z,w,d)} X_{lscgzwd} \ge Y_{lscg} \quad (8)$$

$$\forall l,\ \forall s \in S_{possible}(l)\ ,\ \forall\ c \in C(s,c),\ g \in G(s,c,g)$$

Equations **(9)** - **(11)** prevent lecturers from being assigned a group *g* of a credit type *c* of a subject *s* if they are not available. This problem occurs if such a class begins earlier than the lecturer earliest availability time or ends later than their latest availability time.

In equation **(9)**, the value of the decision variable $DEVST_{lscgzwdt}$ is 1 if the class begins once the lecturer availability slot has already started, that is: $stassig_{scgzwd} \ge stavail_{zwdt}$.

$$[(stassig_{scgzwd} - stavail_{lzwdt}) + \varphi] * (-1 + 2 * DEVST_{lscgzwdt}) \ge 0 \quad (9)$$

$\forall l,\ \forall s \in S_{possible}(l),\ \forall c \in C(s, c),\ \forall g \in G(s, c, g),\ \forall z \in Z(s, z),\ \forall w \in W(z, w),\ \forall d \in D(s, c, g, z, w, d),\ \forall t \in T(l, z, w, d, t)$

In equation **(10)**, $DEVEN_{lscgzwdt}$ is 1 if the difference between $enavail_{scgzwd} - enassig_{lzwdt}$ is greater than 0, that is, the class ends earlier than the lecturer latest availability time. It must be noted that limits are also included, so that if such a difference is equal to 0 (the class ends at the same time than the lecturer latest availability time) then a feasible assignment exists and thus $DEVST_{lscgzwdt}$ and $DEVEN_{lscgzwdt}$ are equal to 1. For this to happen and avoid ambiguity, a small positive increase has been included in the equation **(10)**. This increase will be used in numerous equations presented here and always seeks to resolve the uncertainty associated with the fact that the difference is zero.

$$\left[\left(enavail_{scgzwd} - enassig_{lzwdt}\right) + \varphi\right] * \left(-1 + 2 * DEVEN_{lscgzwdt}\right) \geq 0 \quad \textbf{(10)}$$

$\forall l,\ \forall s \in S_{possible}\,(l),\ \forall c \in C\,(s, c),\ \forall g \in G\,(s, c, g),\ \forall z \in Z\,(s, z),\ \forall w \in W\,(z, w),\ \forall d \in D\,(s, c, g, z, w, d),\ \forall t \in T(l, z, w, d, t)$

Equation **(11)** stablishes that the assignment will be possible ($X_{lscgzwd}$ = 1) only if both $DEVST_{lscgzwdt}$ *and* $DEVEN_{lscgzwdt}$ are equal to 1; that is, if the class of the group *g* of the credit type *c* of the subject *s* taught the day *d* of the week *w* of the semester *z*, takes place between the earliest and latest availability (time) slots *t* of the lecturer *l*.

$$DEVST_{lscgzwdt} + DEVEN_{lscgzwdt} \geq 2 * X_{lscgzwd} \quad \textbf{(11)}$$

$\forall l,\ \forall s \in S_{possible}\,(l),\ \forall c \in C\,(s, c),\ \forall g \in G\,(s, c, g),\ \forall z \in Z\,(s, z),\ \forall w \in W\,(z, w),\ \forall d \in D\,(s, c, g, z, w, d),\ \forall t \in T(l, z, w, d, t)$

Equation **(12)** determines the number of hours (teaching load) assigned to lecturer *l*. They are calculated from the sum of the different classes duration taught by the lecturer (period between the start $stassig_{scgzwd}$ and the end $enassig_{scgzwd}$ of the subject) during the model horizon.

$$\sum_{s \in S_{possible}(l)} \sum_{c \in C(s,c)} \sum_{g \in G(s,c,g)} \sum_{z \in Z(s,z)} \sum_{w \in W(z,w)} \sum_{d \in D(s,c,g,z,w,d)} \left(enassig_{scgzwd} - stassig_{scgzwd}\right) * X_{lscgzwd} = CASSIG_l \quad \textbf{(12)}$$

$\forall l$

Equation **(13)** expresses through the parameter α the maximum allowed saturation level. For example, if $\alpha = 0.8$ then the maximum allowed load to be assigned to lecturer *p* is the 80% of its effective teaching load ($load_l$).

$$\mathrm{CASSIG_l} \leq \alpha * load_l \quad \textbf{(13)}$$

$\forall l$

On the contrary, equation **(14)** expresses through the parameter β the minimum allowed saturation level (also as a percentage of $load_l$).

$$\mathrm{CASSIG_l} \geq \beta * load_l \quad \textbf{(14)}$$

$\forall l$

Equations **(15)** - **(45)** avoid overlaps in lecturers' schedules, that is, a lecturer is assigned more than one subject in the same time slot. There are three types of overlapping cases:

- If the subject *s* starts earlier than the subject *s'* and ends later than the start of subject *s´* (**Fig.3**).
- If the subject *s* starts earlier than the subject *s´* and ends later than the end of subject *s´* (**Fig.4**).
- If the end of subject *s* and the start of the subject *s´* are at the same time. That´s a manner to consider the transition time between classes (**Fig.5**).

[Figures 3-5 over here]

In order to facilitate the reading and understanding of the above constraints, some of the equations (15)-(45) are grouped when defined for the identical indices and subsets (i.e. for the same ∀) in such a way that these common indices and subsets are written only for the last constraint.

Equations **(15)** and **(16)** force $OENSENS_{ss'cc'gg'zz'ww'dd'}$ to be equal to 1 only if *s´* starts later or at the same time than *s*. The first requires the decision variable to be equal to 1 if the difference is positive or zero while the second ensures the opposite case, that is, that the decision variable is 0 if the reverse difference ($stassig_{scgzwd} - {}'stassig_{s'c'g'z'w'd'}$) gives a positive value, where M is a positive number equal to the difference between the latest and the earliest schedule time. Equations **(17)** and **(18)** force $OSTS'ENS_{ss'cc'gg'zz'ww'dd'}$ to be equal to 1 only if *s* ends later or at the same time than the start of *s´*. As in the previous case, equation **(18)** ensures that the variable is equal to 0 if the difference $enassig_{scgzwd} - stassig_{s'c'g'z'w'd'}$ is negative. So, equations **(15)**, **(16)**, **(17)** and **(18)** determine whether the first case of overlap (the start of *s´* takes place between the start and end of *s*) or the third (end of *s* and start of *s´* are the same) exist.

$$'stassig_{s'c'g'z'w'd'} - stassig_{scgzwd} \leq M * OSTSSTS'_{ss'cc'gg'zz'ww'dd'} \quad \textbf{(15)}$$

$$stassig_{scgzwd} - {}'stassig_{s'c'g'z'w'd'} \leq M * (1 - OSTSSTS''_{ss'cc'gg'zz'ww'dd'}) \quad \textbf{(16)}$$

$$enassig_{scgzwd} - {}'stassig_{s'c'g'z'w'd'} + \varphi \leq M * OSTS'ENS_{ss'cc'gg'zz'ww'dd'} \quad \textbf{(17)}$$

$$'stassig_{s'c'g'z'w'd'} - \mathrm{enassig}_{\mathrm{scgzwd}} \leq \mathrm{M} * (1 - OSTS'ENS_{ss'cc'gg'zz'ww'dd'}) \quad \textbf{(18)}$$

$\forall\ s,\ \forall\ s',\ \forall c \in C(s,c),\ \forall\ c' \in C'(s',c'),\ \forall g \in G(s,c,g),\ \forall g' \in G'(s',c',g'),\ \forall z \in Z(s,z),\ \forall\ z' \in Z'(s',z'),\ \forall w \in W(z,w),\ \forall w' \in W'(z',w'),\ \forall\ d \in D(s,c,g,z,w,d),\ \forall\ d' \in D'(s',c',g',z',w',d')\ /\ s \neq s' \cap z = z' \cap w = w' \cap d = d'$

Equation **(19)** implies that, if both $OSTSSTS'_{ss'cc'gg'zz'ww'dd'}$ and $OSTS'ENS_{ss'cc'gg'zz'ww'dd'}$ are equal to 1, then only one of the subjects, *s* or *s´*, may be assigned to lecturer *l* the day *d* of the week *w* of the semester *z* in the time slot *t*. In other words, the first and third type of overlap occurs when both decision variables are equal to 1.

$$X_{lscgzwd} + X_{ls'c'g'z'w'd'} +$$

$$+ OSTSSTS'_{ss'cc'gg'zz'ww'dd'} + OSTS'ENS_{ss'cc'gg'zz'ww'dd'} \leq 3 \quad \textbf{(19)}$$

$\forall l, \forall s, \forall s', \forall c \in C(s,c), \forall c' \in C'(s',c'), \forall g \in G(s,c,g), \forall g' \in G'(s',c',g'), \forall z \in Z(s,z), \forall z' \in Z'(s',z'), \forall w \in W(z,w), \forall w' \in W'(z',w'), \forall d \in D(s,c,g,z,w,d), \forall d' \in D'(s',c',g',z',w',d') \,/\, s \neq s' \cap z = z' \cap w = w' \cap d = d'$

Equations **(20)** and **(21)** force $OSTS'ENS_{ss'cc'gg'zz'ww'dd'}$ to be equal to 1 only when *s´* ends after *s* starts, or when the end time of *s´* and the start time of *s* are the same. Equations **(22)** and **(23)** force $OENS'ENS_{ss'cc'gg'zz'ww'dd'}$ to be equal to 1 when *s* ends after or at the same time than *s´*. Equations **(20)** – **(23)** represent the second case of overlap (*s´* occurs between the start and end of *s*).

$$'enassig_{s'c'g'z'w'd'} - stassig_{scgzwd} + \varphi \leq M * OSTSENS'_{ss'cc'gg'zz'ww'dd'} \qquad \textbf{(20)}$$

$$stassig_{scgzwd} - 'enassig_{s'c'g'z'w'd'} \leq M * (1 - OSTSENS'_{ss'cc'gg'zz'ww'dd'}) \qquad \textbf{(21)}$$

$$enassig_{scgzwd} - 'enassig_{s'c'g'z'w'd'} + \varphi \leq M * OENS'ENS_{ss'cc'gg'zz'ww'dd'} \qquad \textbf{(22)}$$

$$'enassig_{s'c'g'z'w'd'} - enassig_{scgzwd} \leq M * (1 - OENS'ENS_{ss'cc'gg'zz'ws'dd'}) \qquad \textbf{(23)}$$

$\forall s, \forall s', \forall c \in C(s,c), \forall c' \in C'(s',c'), \forall g \in G(s,c,g), \forall g' \in G'(s',c',g'), \forall z \in Z(s,z), \forall z' \in Z'(s',z'), \forall w \in W(z,w), \forall w' \in W'(z',w'), \forall d \in D(s,c,g,z,w,d), \forall d' \in D'(s',c',g',z',w',d') \,/\, s \neq s' \cap z = z' \cap w = w' \cap d = d'$

Equation **(24)** forces that only one of the two subjects, *s* or *s´*, may be assigned to lecturer *l* the day *d* of the week *w* of the semester *z* in the time slot *t*, if decision variables $OSTSENS'_{ss'cc'gg'zz'ww'dd'}$ and $OENS'ENS_{ss'cc'gg'zz'ww'dd'}$ are equal to 1.

$$X_{lscgzwd} + X_{ls'c'g'z'w'd'} + $$

$$+ OSTSENS'_{ss'cc'gg'zz'ww'dd'} + OENS'ENS_{ss'cc'gg'zz'ww'dd'} \leq 3 \qquad \textbf{(24)}$$

$\forall l, \forall s \in S_{possible}(l), \forall s' \in S'_{possible}(l), \forall c \in C(s,c), \forall c' \in C'(s',c'), \forall g \in G(s,c,g), \forall g' \in G'(s',c',g'), \forall z \in Z(s,z), \forall z' \in Z'(s',z'), \forall w \in W(z,w), \forall w' \in W'(z',w'), \forall d \in D(s,c,g,z,w,d), \forall d' \in D'(s',c',g',z',w',d') \,/\, s \neq s' \cap z = z' \cap w = w' \cap d = d'$

It may be noted that an overlap in just one subject *j* may also occur: either between different credit types *c* or between different groups *g* corresponding to the same credit type *c*. The methodology used to solve this problem is identical to that used in the case of different subjects. The explanation of each constraint is similar to the equations **(15)** - **(24)**.

Equations **(25)** – **(34)** represent the overlap between different credit types *c* of a subject *s*. As the number of groups *g* may vary between the credit types *c* and *c´*, the groups g corresponding to type credit *c* and the groups *g´* corresponding to credit type *c´* must be distinguished.

It is not imposed that *g ≠ g´*, since in case *g* and *g´* coincided, that would imply an overlap in the students schedule, which is not possible since the teaching timetable is already an input where this problem has already been solved.

Equations **(25)** - **(29)** prevents the first and third type of overlap (previously described) from taking place.

$$'stassig_{s'c'g'z'w'd'} - stassig_{scgzwd} \leq M * OSTCSTC'_{ss'cc'gg'zz'ww'dd'} \quad \textbf{(25)}$$

$$stassig_{scgzwd} - 'stassig_{s'c'g'z'w'd'} \leq M * (1 - OSTCSTC'_{ss'cc'gg'zz'ww'dd'}) \quad \textbf{(26)}$$

$$enassig_{scgzwd} - 'stassig_{s'c'g'z'w'd'} + \varphi \leq M * OSTC'STC_{ss'cc'gg'zz'ww'dd'} \quad \textbf{(27)}$$

$$'stassig_{s'c'g'z'w'd'} - enassig_{scgzwd} \leq \mathrm{M} * (1 - OSTC'ENC_{ss'cc'gg'zz'ww'dd'} \quad \textbf{(28)}$$

$\forall\ s,\ \forall\ s',\ \forall c \in C(s,c),\ \forall\ c' \in C'(s',c'),\ \forall g \in G(s,c,g),\ \forall g' \in G'(s',c',g'),\ \forall z \in Z(s,z), \forall z' \in Z'(s',z'), \forall w \in W(z,w), \forall w' \in W'(z',w'), \forall d \in D(s,c,g,z,w,d),$
$\forall d' \in D'(s',c',g',z',w',d') / s \neq s' \cap c \neq c' \cap z = z' \cap w = w' \cap d = d'$

$$X_{lscgzwd} + X_{l's'c'g'z'w'd'} +$$

$$+ OSTC'STC_{ss'cc'gg'zz'ww'dd'} + OSTC'ENC_{ss'cc'gg'zz'ww'dd'} \leq 3 \quad \textbf{(29)}$$

$\forall\ l,\ \forall s \in S_{possible}(l),\ \forall\ s' \in S'_{possible}(l),\ \forall c \in C(s,c),\ \forall\ c' \in C'(s',c'),\ \forall g \in G(s,c,g),$
$\forall\ g' \in G'(s',c',g'),\ \forall z \in Z(s,z),\ \forall z' \in Z'(s',z'),\ \forall w \in W(z,w),\ \forall w' \in W'(z',w'),$
$\forall d \in D(s,c,g,z,w,d),\ \forall\ d' \in D'(s',c',g',z',w',d') / s \neq s' \cap c \neq c' \cap z = z' \cap w = w' \cap d = d'$

Equations **(30)** – **(34)** prevent the second type of overlap from occurring.

$$'enassig_{s'c'g'z'w'd'} - stassig_{scgzwd} + \varphi \leq M * OSTCENC'_{ss'cc'gg'zz'ww'dd'} \quad \textbf{(30)}$$

$$stassig_{scgzwd} - 'enassig_{s'c'g'z'w'd'} \leq M * (1 - OSTCENC'_{ss'cc'gg'zz'ww'dd'}) \quad \textbf{(31)}$$

$$enassig_{scgzwd} - 'enassig_{s'c'g'z'w'd'} + \varphi \leq M * OENC'ENC_{ss'cc'gg'zz'ww'dd'} \quad \textbf{(32)}$$

$$'enassig_{s'c'g'z'w'd'} - enassig_{scgzwd} \leq M * (1 - OENC'ENC_{ss'cc'gg'zz'ww'dd'}) \quad \textbf{(33)}$$

$\forall\ s,\ \forall\ s',\ \forall c \in C(s,c),\ \forall\ c' \in C'(s',c'),\ \forall g \in G(s,c,g),\ \forall g' \in G'(s',c',g'),\ \forall z \in Z(s,z), \forall z' \in Z'(s',z'), \forall w \in W(z,w), \forall w' \in W'(z',w'), \forall d \in D(s,c,g,z,w,d),$
$\forall d' \in D'(s',c',g',z',w',d') / s \neq s' \cap c \neq c' \cap z = z' \cap w = w' \cap d = d'$

$$X_{lscgzwd} + X_{ls'c'g'z'w'd'} +$$

$$+ OSTCENC'_{ss'cc'gg'zz'ww'dd'} + OENC'ENC_{ss'cc'gg'zz'ww'dd'} \leq 3 \quad \textbf{(34)}$$

$\forall\ l,\ \forall s \in S_{possible}(l),\ \forall\ s' \in S'_{possible}(l),\ \forall c \in C(s,c),\ \forall\ c' \in C'(s',c'),\ \forall g \in G(s,c,g),$
$\forall\ g' \in G'(s',c',g'),\ \forall z \in Z(s,z),\ \forall z' \in Z'(s',z'),\ \forall w \in W(z,w),\ \forall w' \in W'(z',w'),$

$\forall d \in D(s,c,g,z,w,d),\ \forall d' \in D'(s',c',g',z',w',d')\ /s \neq s' \cap c \neq c' \cap z = z' \cap w = w' \cap d = d'$

Equations **(35)** – **(44)** are similar to the aforementioned ones, although in this case the different groups schedules (g and g') corresponding to the same credit type c of the subject s are compared. The first five equations **(35)** – **(39)** prevent the first and third type of overlap from taking place, while the equations **(40)** – **(44)** prevent the second one.

$$'stassig_{s'c'g'z'w'd'} - stassig_{scgzwd} \leq M * OSTGSTG'_{ss'cc'gg'zz'ww'dd'} \quad \textbf{(35)}$$

$$stassig_{scgzwd} - 'stassig_{s'c'g'z'w'd'} \leq M * (1 - OSTGSTG'_{ss'cc'gg'zz'ww'dd'}) \quad \textbf{(36)}$$

$$enassig_{scgzwd} - 'stassig_{s'c'g'z'w'd'} + \varphi \leq M * OSTG'ENG_{ss'cc'gg'zz'ww'dd'} \quad \textbf{(37)}$$

$$'stassig_{s'c'g'z'w'd'} - enassig_{scgzwd} \leq M * (1 - OSTG'ENG_{ss'cc'gg'zz'ww'dd'}) \quad \textbf{(38)}$$

$\forall s,\ \forall s',\ \forall c \in C(s,c),\ \forall c' \in C'(s',c'),\ \forall g \in G(s,c,g),\ \forall g' \in G'(s',c',g'),\ \forall z \in Z(s,z),$
$\forall z' \in Z'(s',z'), \forall w \in W(z,w), \forall w' \in W'(z',w'), \forall d \in D(s,c,g,z,w,d),$
$\forall d' \in D'(s',c',g',z',w',d')\ /\ s = s' \cap c = c' \cap g \neq g' \cap z = z' \cap w = w' \cap d = d'$

$$X_{lscgzwd} + X_{ls'c'g'z'w'd'} +$$

$$+OSTGSTG'_{ss'cc'gg'zz'ww'dd'} + OSTG'ENG_{ss'cc'gg'zz'ww'dd'} \leq 3 \quad \textbf{(39)}$$

$\forall l,\ \forall s \in S_{possible}(l),\ \forall s' \in S'_{possible}(l),\ \forall c \in C(s,c),\ \forall c' \in C'(s',c'),\ \forall g \in G(s,c,g),$
$\forall g' \in G'(s',c',g'),\ \forall z \in Z(s,z),\ \forall z' \in Z'(s',z'),\ \forall w \in W(z,w),\ \forall w' \in W'(z',w'),$
$\forall d \in D(s,c,g,z,w,d),\ \forall d' \in D'(s',c',g',z',w',d')\ /s \neq s' \cap c = c' \cap g \neq g' \cap z = z' \cap w = w' \cap d = d'$

$$'enassig_{s'c'g'z'w'd'} - stassig_{scgzwd} + \varphi \leq M * OSTGENG'_{ss'cc'gg'zz'ww'dd'} \quad \textbf{(40)}$$

$$stassig_{scgzwd} - 'enassig_{s'c'g'z'w'd'} \leq M * (1 - OSTGENG'_{ss'cc'gg'zz'ww'dd'}) \quad \textbf{(41)}$$

$$enassig_{scgzwd} - 'enassig_{s'c'g'z'w'd'} + \varphi \leq M * OENG'ENG_{ss'cc'gg'zz'ww'dd'} \quad \textbf{(42)}$$

$$'enassig_{s'c'g'z'w'd'} - enassig_{scgzwd} \leq M * (1 - OENG'ENG_{ss'cc'gg'zz'ww'dd'}) \quad \textbf{(43)}$$

$\forall s,\ \forall s',\ \forall c \in C(s,c),\ \forall c' \in C'(s',c'),\ \forall g \in G(s,c,g),\ \forall g' \in G'(s',c',g'),\ \forall z \in Z(s,z),$
$\forall z' \in Z'(s',z'), \forall w \in W(z,w), \forall w' \in W'(z',w'), \forall d \in D(s,c,g,z,w,d),$
$\forall d' \in D'(s',c',g',z',w',d')\ /\ s = s' \cap c = c' \cap g \neq g' \cap z = z' \cap w = w' \cap d = d'$

$$X_{lscgzwd} + X_{ls'c'g'z'w'd'} +$$

$$+OSTGENG'_{ss'cc'gg'zz'ww'dd'} + OENG'ENG_{ss'cc'gg'zz'ww'dd'} \leq 3 \quad \textbf{(44)}$$

$\forall\ l,\ \forall s \in S_{possible}(l)\ ,\ \forall\ s^{'} \in S^{'}_{possible}(l),\ \forall c \in\ C(s,c),\ \forall\ c^{'} \in\ C^{'}(s^{'},c^{'}),\ \forall g \in\ G(s,c,g),$
$\forall\ g^{'} \in\ G^{'}(s^{'},c^{'},g^{'}),\ \forall z \in\ Z(s,z),\ \forall z^{'} \in\ Z^{'}(s^{'},z^{'}),\ \forall w \in\ W(z,w),\ \forall w^{'} \in\ W^{'}(z^{'},w^{'}),$
$\forall\ d \in\ D(s,c,g,z,w,d),\ \ \forall\ d' \in D'(s',c',g',z',w',d')\ /s \neq s^{'} \cap c =\ c' \cap g \neq g^{'} \cap z = z^{'} \cap w = w^{'} \cap d =\ d^{'}$

In order to avoid ambiguity caused by the handling of two decision variables to refer to the same element, it must be ensured that they are both identical when all indices coincide as it is established in the following equation **(45)**:

$$X_{lscgzwd} = X_{ls'c'g'z'w'd'} \quad \textbf{(45)}$$

$\forall\ l,\ \forall s \in S_{possible}(l)\ ,\ \forall\ s^{'} \in S^{'}_{possible}(l),\ \forall c \in\ C(s,c),\ \forall\ c^{'} \in\ C^{'}(s^{'},c^{'}),\ \forall g \in\ G(s,c,g),$
$\forall\ g^{'} \in\ G^{'}(s^{'},c^{'},g^{'}),\ \forall z \in\ Z(s,z),\ \forall z^{'} \in\ Z^{'}(s^{'},z^{'}),\ \forall w \in\ W(z,w),\ \forall w^{'} \in\ W^{'}(z^{'},w^{'}),$
$\forall\ d \in\ D(s,c,g,z,w,d),\ \ \forall\ d' \in D'(s',c',g',z',w',d')\ /s = s^{'} \cap c =\ c' \cap g = g^{'} \cap z = z^{'} \cap w = w^{'} \cap d =\ d^{'}$

Finally, it must be ensured that each group g of the credit type c of the subject s is assigned at least one lecturer. Equation **(46)** reflects that the summation of the credits (in hours) of the group g of the credit type c of the subject s has to be equal to the value of the parameter $credits_{scg}$.

$$\sum_{l} \sum_{z \in Z(s,z)} \sum_{w \in W(z,w)} \sum_{d \in D(s,c,g,z,w,d)} \left(enassig_{scgzwd} - stassig_{scgzwd}\right) * X_{lscgzwd} = credits_{scg} \quad \textbf{(46)}$$

$$\forall s,\ \forall c \in C(c,s),\ \forall g \in G\,(s,c,g)$$

All the described equations make the search space of the lecturer-subject assignment process narrower since it will only be possible to establish such a relationship in case that those restrictions allow $X_{lscgzwd}$ to be equal to 1.

5. Model validation: application to a real case

The proposed model has been applied to a real case in order to validate it and analyze the quality of the obtained solutions and the computational efficiency under different scenarios.

The input data comes from a real case corresponding to the teaching assignment of the ETSII faculty to the Applied Linguistics department for the German subjects for the academic year 2015-16 (ETSII 2015).

The main reason to choose such an assignment is due to the numerous groups that make up those German subjects, which in turn lead to multiple schedule overlaps. This fact makes the teaching assignment much more complicated. This will help to check whether the proposed model is managing those overlaps properly. In addition, different scenarios have been defined, in order to verify its proper execution.

The resolution methodology jointly with the obtained results are described in the following section

5.1. Resolution methodology

The application of the model to the above real case had a too high “sparse time" (previous to the model resolution), approximately 4,000% minutes higher than the optimization one which could potentially leave the computer out of memory with larger problems. The “sparse time" is the time employed by the modelling language software to combine the parametric MILP model in an algebraically form with the input data generating all the decision variables, macros and constraints that integrate the specific model to be solved. The MPL modelling language used in this paper is able to handle very large matrices with millions of variables and constraints through its own memory manager. However, the only limitation the model developer faces is how much machine memory is available (Maximal Software, 2021)

Therefore, in order to avoid the possibility of leaving the computer out of memory during the model generation with larger problems, a methodology resolution is proposed. This methodology is based on the fact that certain decision variables were identified as dependent, because once the start and end times of the subjects (parameters) are known, it is possible to calculate the value of the binary variables related to the overlap of the following subjects:

$OSTSSTS'_{ss'cc'gg'zz'ww'dd'}$, $OSTS'ENS_{ss'cc'gg'zz'ww'dd'}$, $OSTSENS'_{ss'cc'gg'zz'ww'dd'}$, $OENS'ENS_{ss'cc'gg'zz'ww'dd'}$, $OSTCSTC'_{ss'cc'gg'zz'ww'dd'}$, $OSTC'ENC_{ss'cc'gg'zz'ww'dd'}$, $OSTCENC'_{ss'cc'gg'zz'ww'dd'}$, $OENC'ENC_{ss'cc'gg'zz'ww'dd'}$, $OSTGSTG'_{ss'cc'gg'zz'ww'dd'}$, $OSTG'ENG_{ss'cc'gg'zz'ww'dd'}$, $OSTGENG'_{ss'cc'gg'zz'ww'dd'}$, $OENG'ENG_{ss'cc'gg'zz'ww'dd'}$.

Based on the above property, a resolution methodology in two stages is proposed, that reduces the “sparse time” in each step and, therefore, the memory requirements on the computer. This methodology is depicted in **Fig.6.**

[Figure 6 near here]

In the first step, the values of the above decision variables are calculated. For that, it is necessary to define an auxiliary model in order to obtain the value of the overlapping decision variables, taking as parameters the start and end time of each of the groups for each credit type for each subject in each week. In a second step, and taking the value of the above decision variables as parameters, the main model is solved and thus providing the solution for the teaching assignment problem. It has to be highlighted that splitting the model into two steps improves the reading time (sparse time) required for the optimization of each step, but it does not have any impact on the final optimal solution. The reason is that the solution obtained in the first step, i.e. the value of the decision variables related to the overlaps that passes as input data for the second step, will always be the same as long as the input data related to the start and end times of subjects are not changed.

Finally, it may be noted that the computer used is a HP Pavilion DM4 Notebook PC model, Intel ® Core ™ i5, 4.00 GB of RAM, 64-bit Windows Operating System. Modeling software has been MPL Modeling System 4.2p (4.2.14.107). Access 2002-2003 has been used as input and exit data storage. The solver used for optimization has been Gurobi 6.0.3. The runtime for each scenario has been limited to 15 minutes.

5.2. The case of the Teaching Assignment of German subjects from ETSII Faculty to the Department of Applied Linguistics

5.2.1. Input data

This real case concerns to the teaching assignment from the ETSII faculty to the department of applied linguistics. This assignment encompasses different subjects in German and are taught in different courses of different degrees of ETSII faculty. Data about these subjects (credit types, number of groups and timetable) has been obtained from the website of the ETSII faculty (ETSII 2015).

This example fits this paper purpose since there is a large number of overlaps among the groups of the different subjects, which will help to check whether the model properly works for the teaching assignment. It will be a good manner to check how the overlapping subjects are assigned to lecturers.

The number of lecturers, their teaching load, as well as their time availability and preferences has been estimated. This aspect is not relevant since the objective is to validate the model and analyze its behavior in different scenarios.

The data used to validate the proposed mathematical programming model are as follows:

- The academic course is made up of 2 semesters A y B; z = (A, B).
- Each semester is made up of 17 weeks; w = (w1, w2, …, w17).
- Each week is made up of 5 working days; d = (MO, TU, WE, TH, FR).
- 9 lecturers (l) must be assigned.
- 3 lecturers ranks are considered; r = (r1, r2, r3). Such ranks will be more or less weighted.
- Lecturers' time availability is continuous, that is, only one time slot is considered (t = t1). This time slot starts at 7:30 a.m. and ends at 10 p.m. Therefore, each subject could potentially be assigned to any lecturer.
- All the lecturers have the required competences to teach every subject.
- 4 different subjects (s) are considered: German A1 (geA1), German A2 (geA1), German B1 (geB1) and German B2 (geB2). Each one is made up of a maximum of 2 credits types: c1 (theory) and c2 (practice).

Table 12 shows the value of their main characteristics:

[Table 12 near here]

As it is shown in **Table 13**, each subject is made up of different groups, which are taught in one of the two semesters, as being determined by the second letter in g.

[Table 13 near here]

The following **Figures 7** and **8** show the multiple overlaps among subjects in both semesters, which makes the teaching assignment more complex. In both semesters, just one representative week with the highest number of overlaps was chosen (w4 in semester A and w3 in semester B). In this problem, the schedule is stable and it hardly varies among weeks.

[Figures 7-8 near here]

Finally, all the lecturers' preferences related data (*pref*) is shown in the following **Table 14.** (rating scales range from 1 to 10).

[Table 14 near here]

5.2.2. Experimentation

The objective of this section is to check the validity of the model, analyze its behavior within different scenarios and measure the quality of the obtained results as well as its computational efficiency.

To achieve it, different scenarios are proposed:

- **"Basic" scenario**: restrictions related to time compatibility among lecturers and students, all the credits assignment and the impossibility of lecturers to be assigned over their teaching load are considered. Regarding the objective function, lecturers' satisfaction is maximized while meeting their preferences. This scenario execution aims to validate all the hard restrictions as well as checking that all the input data related to the overlapping of the different subjects is properly read.
- **"Lecturers maximum saturation" scenario**: restrictions related to the impossibility of lecturers to be assigned credits over a α percentage of their teaching load availability are added to the first scenario. It may be seen that α is equal to 1 in the basic scenario. This scenario execution aims to analyze how the reduction of lecturers' teaching load affects in comparison with the basic scenario.
- **"Lecturers minimum saturation" scenario**: restrictions related to the impossibility of lecturers to be assigned credits under a percentage α of their teaching load availability are added to the first scenario. This scenario execution aims to analyze how the fact of sharing the credits among all the lecturers affects to their global satisfaction.
- **"Maximum number of lecturers per group" scenario**: restrictions related to the impossibility of assigning more than μ lecturers for each group of each credit type of each subject are added to the first scenario. Teaching assignment obtained by the basic scenario could assign different lecturers for a certain group, leading to a teaching quality deterioration.
- **"Maximum number of groups per lecturer" scenario**: it is limited to λ the maximum number of groups that may be assigned to a lecturer in a certain semester. This scenario execution aims to balance each lecturer teaching load by preventing them for being assigned all the credits in the same semester.

These five scenarios have a unique objective function that, as aforementioned in the basic scenario, aims to maximize the global lecturers' satisfaction while meeting their teaching preferences.

A sixth scenario considering a multi-objective function was also experimented, although due to space restrictions, it is not included in this paper. However, in section 3 "Problem Modeling" is briefly indicated how to formulate it. The obtained escalated function (equation **(3)**) considers either maximizing the global lecturers' satisfaction (equation **(1)**), used in the previous 5 scenarios analysis, or minimizing the number of different groups assigned to a certain lecturer (equation (2)).

The next **Figure 9** summarizes which specific equations from all the ones defined in former sections are included in each of the five described scenarios:

[Figure 9 near here]

In the following, the obtained results for each scenario are shown in detail: problem size (number of decision variables and restrictions), solution quality (objective function and assignment) and the computational efficiency (execution time and gap). Finally, a comparison among them is conducted.

A. “Basic” scenario

This first experiment aims to validate that lecturers are not assigned to two overlapping classes as well as all the credits of the subjects are distributed and that in no case lecturers are assigned over their teaching load availability.

This model is solved in two stages. **Table 15** shows the achieved improvements with the modification discussed above in the section “resolution methodology”. The column called "improvements" contains the various reductions incurred in deciding to solve the model in two steps instead of one.

[Table 15 near here]

It should be noted that in the case of the two-steps resolution, the data in the previous table refer to the second step, which is the one where the lecturers’ distribution is done. The first stage ends in 21.5 minutes. Therefore, the results show that modifying the resolution procedure saves 65% of time for the different running scenarios.

The inefficiencies identified in the proposed solution are:

- Lecturer l3 is not assigned any credit and l1, l2 and l5 lecturers are assigned most of them, instead.
- Only 4 out of 23 groups are taught by a unique lecturer.

In the following experiments these disadvantages will be overcome by adding some additional restrictions.

B. “Lecturers’ maximum saturation” scenario

In this scenario, a restriction regarding the maximum number of credits assigned to a lecturer is added to the first scenario. The parameter α determines the maximum percentage over the lecturers teaching load availability that may be assigned. In this case, a minimum number of credits that must correspond to each lecturer is not defined and therefore the parameter β (percentage of minimum teaching load) is equal to 0.

The results obtained running the model with different values of α are shown in **Table 16**. Parameter α is not allowed to take values less than 0.7. This is because if the maximum teaching load of all the lecturers is reduced by 30%, the value of the sum of all of them is less than the sum of the credits of the subjects, that is, the number of teaching hours are greater than the lecturers’ available hours, leading to an infeasible solution.

[Table 16 near here]

Regarding the obtained solution, every lecturer is assigned at least some credit although a lecturer minimum saturation level of credits is not defined. This is due to the fact that in the “basic” scenario there are two lecturers who are assigned credits reaching their maximum saturation level. Now, by limiting this maximum load, these lecturers can not cover as many credits as in the “basic” scenario model.

It is logical that the objective function has a decreasing trend since this restriction narrows the possibility of satisfying the lecturers’ preferences. For example, in the “basic” scenario lecturer l6 is assigned credits of 9 different groups among which it is AB1B1. This lecturer is ranked "r1" and gives the group AB1B1 a preference of 8 points. Therefore, the satisfaction weighted preference is 1.5*8, that is, 12 points. On the other hand, lecturer l5 is ranked "r2" and gives to that group a preference of 9 points, in total, 1.2*9 = 10.8 points. Therefore, in order to maximize the overall lecturers' satisfaction, such assignment would correspond to lecturer l6 as in the "basic" scenario. However, in this second scenario the lecturer l6 reaches its maximum saturation level and cannot teach in AB1B1, which is assigned to lecturer l5, being the one with the next highest satisfaction.

C. "Lecturers' minimum saturation" scenario

Unlike the above experiment, in this case β is the parameter that varies, while α remains equal to 1. As in the previous case, a **Table 17** is depicted to compare the results obtained with the various scenarios. The parameter α remains unchanged and takes the value of 1, that is, lecturers may be assigned credits until their maximum saturation level.

In this case, β may not exceed the value of 0.65 since the solution is infeasible due to the same reasons as the previous experiment.

[Table 17 near here]

Regarding the value of the decision variables, the distribution has the following structure: all the lecturers are assigned at least some credits but there are still groups of the same subject and credit type that are assigned to 4 different lecturers what is considered detrimental to the teaching quality. This inefficiency is taken into account in the fourth experiment.

D. "Maximum numbers of lecturers per group" scenario

In this scenario, the behavior of the objective function with the restriction that limits the number of lecturers who may teach in the same group of a certain credit type of a subject is analyzed.

As previously noted the schedule variability (non-regular time schedule) requires the lecturers' assignment to subjects on a daily basis. This restriction aims to improve the teaching quality by preventing more than μ lecturers for being assigned to a particular group of a subject. This inefficiency was detected in previous experiments. Additionally, as it may be seen in the data, the maximum number of credits of a group is 72.5 hours and one lecturer assignable teaching load may be, in some cases, 200 hours. It makes any lecturer being able to teach all the classes of a specific group.

[Table 18 near here]

In the **Table 18,** the statistical data of the solution are shown. It may be seen how the objective function for $\mu = 2$ and $\mu = 3$ cases coincides with that obtained in the "basic" scenario.

For example, in the basic scenario the AA1A1 group is assigned to three different lecturers while in the case $\mu = 2$ is only assigned to two lecturers. This verifies how adding a few restrictions may improve the solution quality without affecting the value of the weighted overall lecturers' satisfaction.

E. "Maximum numbers of groups per lecturer" scenario

As discussed in the "maximum number of lecturers by group" scenario, some of the results assign a lecturer more than 9 different groups. This is contrary to the objective of the proposed model: the overall lecturers' satisfaction while taking into account aspects that influence the structure and therefore the teaching quality.

In this experiment λ, maximum number of groups per lecturer each semester, must take the values of 3 and 4, since higher values lead to a teaching quality reduction.

As reflected in **Table 19**, the average lecturers' satisfaction is higher when λ takes the value of 3. This is because, by limiting the number of groups per lecturer, the distribution is fairer and is more likely to meet, on average, the lecturers' preferences. However, overall satisfaction is lower in this scenario because it takes into account the lecturers ranks, and without such restrictions, the model will tend to assign as many groups as possible to higher ranks lecturers.

[Table 19 near here]

It is assumed in this scenario that the greater the number of assigned classes are the greater the satisfaction is. This fact explains why the "average preference" is not ranged between 1 and 10.

5.2.3. Comparison of the results obtained with the different scenarios

The best results of the values of the objective functions are collected in **Table 20,** after running the different experiments. These values represent the overall lecturers' satisfaction, obtained in each of them.

[Table 20 near here]

It must be highlighted the minimum variation in the value of the objective function of the different scenarios. Furthermore, the obtained gap is in all the cases very small. Therefore, these results verify that minor modifications in the model in the form of restrictions have little impact on the objective of the problem (maximizing the overall lecturers' satisfaction). Such variations, such as the limitation on the maximum number of lecturers per group or the maximum number of groups per lecturer, have a positive impact on the teaching quality of both lecturers and students: the first ones because they can focus their teaching on a limited number of groups and the second ones because they can benefit from that. Besides, the fact that one group is assigned to various lecturers results, in practice, in different classes taught by different lecturers, deteriorating the teaching quality. Therefore, the limitation on the number of lecturers per group forces the model to assign to a lecturer the greatest number of classes within a group.

Based on all the above, it may be confirmed after the experimentation that the mathematical model has the expected behavior, therefore finishing the validation process.

Finally, with the aim of giving some insights on the scalability and speed characteristics of the resolution methodology, the model has also been solved for a larger problem: the teaching assignment of the Industrial Management Engineering Degree to the Dept. of Business Organization at the UPV. The parameters of the model were set to the following values: α=1, ß=0.3 and μ=2. The higher size of this problem (26 lecturers, 21 subjects and 28 groups) implied: for the first step, 21776 binary variables, 43552 constraints and a sparse time (reading time) of 33,05 min and, for the second step, 83720 binary variables, 370385 constraints with a sparse time of 187,62 minutes and a resolution time of 4,15 minutes. As it can be observed, also for this more complex case, the parsing time is much higher than the resolution one that remains acceptable. To finish, it is important to highlight that splitting the solution process into two steps also presents great advantages as regards the computational time in case the decision-maker desires to run the model for different scenarios with the same start and end dates of the subjects. The reason is that for this situation, the first step should be executed only once, independently of the number of scenarios to be tested and, the second step should be solved, as many times as scenarios defined. Through this two-steps resolution methodology, the total computational time is reduced as compared with solving the model in just one step since for this last one, the model should be entirely solved every time for each scenario with the corresponding increasing in the total sparse time.

6. Conclusions

This paper proposes a decision-making tool for an efficient teaching assignment. The developed tool is based on a mixed integer linear programming model whose one of the main novelty with respect to previous research is that such an assignment is done under a non-regular schedule. This fact makes this tool to be closer to reality features. In addition to the consideration of "non-regular schedule" aspects, it may be noted, from the literature review, that no study featuring characteristics such as the teaching assignment per number of hours (credits) instead of number of classes or the distinction within each subject between the different groups and credit types has been found. While it is true that the objective of the model is to maximize the lecturers' satisfaction while meeting their preferences about which subjects to teach, non-regular schedules have led to use creativity to get closer to the reality in the easiest manner. For this, both lecturers and subjects have been characterized by their start and end times to model both lecturers' time availability and classes' schedule. Moreover, the academic schedule is variable among weeks, so data must be detailed for each day. These features have made the problem modeling more difficult. However, the proposed model is versatile as it may also be used with regular time slots.

Maximizing the lecturers' satisfaction without considering other aspects can lead to an undesirable solution in which a subject is taught by many lecturers, which is detrimental to the teaching quality. Consideration of policies to increase the solution quality has been modeled through the inclusion of various restrictions and a new objective. These restrictions refer to the maximum and minimum saturation levels of each lecturer, the maximum number of lecturers who teach the same group and the maximum number of groups being taught by a lecturer in a semester. On the other hand, the additional objective seeks to minimize the number of lecturers assigned to each group. With the conducted experimentation, it has been proved how the inclusion of such restrictions greatly improves the quality of the solution while the value of the objective function has a minimum variation or even null.

The resolution of the model has resulted in a lengthy data reading time that in some cases has led the computer to be ran out of memory. That is why a methodology of resolution in two steps has been developed to shorten it. In the first step, once the subjects' schedules have been set, the value of the variables related to the existence of overlapping subjects in the schedule and those ones with a similar nomenclature are calculated. The values of these variables just depend on the start and end time of each group and the credit type of each subject, so that it is possible to obtain their values regardless of the objective function. In the second step, the original model has been run taking the value of the above decision variables as input data. The applied methodology shows a reduction of 65% in terms of reading time. This allows to speed up the experimentation of different scenarios after executing just once the first step for determining the existence of overlaps among subjects.

In order to analyze and validate the behavior of the model in different scenarios, a real case corresponding to the teaching of the German subject that ETSII faculty requests to the linguistics department has been applied. In order to provide with a deeper analysis on the on the scalability and speed characteristics of the resolution methodology, a second real case corresponding to the teaching assignment of the Industrial Management Engineering Degree to the Business Organization department has been also conducted.

Although the equations to contemplate the possibility of a multi-objective scenario were initially pointed out, just five scenarios have experimentally being considered, with just a single objective, aiming to maximize the overall lecturers' satisfaction. The obtained results verify the need to include policies that increase the solution quality, through the addition of "soft" constraints. Moreover, the value of the objective function in these cases hardly changes, which means that higher quality solutions without this negative impact on the overall lecturers' satisfaction may be achieved. On the other hand, as this overall satisfaction is weighted by lecturers' ranks, the higher the rank is the higher the probability is to assign credits to that lecturer. In this case, restrictions that limit the feasibility area are required which leads the solution to level the average teaching load assigned per lecturer.

Ethical Statement

Conflict of Interest: The authors declare that they have no conflict of interest.

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AUTHORS (YEAR)	SCHEDULED EVENT	PROBLEM TYPE	MODEL	RESOLUTION METHOD	RESOLUTION PROCEDURE
Badri (1996)	University Schedule	L-S-T-C	BP	Exact: simplex extension for this type of models	2-steps: Step 1: L-S Step 2: S-T
Hultberg and Cardoso (1997)	University Schedule	L-S	MILP	Exact: solver CPLEX Heuristic: branch-and-bound algorithm	1-step
Badri et al. (1998)	University Schedule	L-S-T-C	BP	Exact: simplex extension for this type of models	1-step
Dimopoulou and Miliotis (2001)	University Schedule	S-T-C	BP	Exact: MPCODE and XPRESS-MP of Windows	1-step
Daskalaki et al. (2004)	University Schedule	S-T-C	BP	Exact: solver CPLEX 5.1	1-step
Daskalaki & Birbas (2005)	University Schedule	S-T-C	BP	Exact: solver CPLEX 5.1	2-steps: Step I: initial solution Step II: multiperiod consecutive classes option is added after step 1
Al-Yakoob & Sherali (2006)	University Schedule	L- S-T-C	MILP	Exact: solver CPLEX Heuristic:LP-based algorithm	2-steps: Step I: S-T-S Step II: L-S, with the possibility of modify by 15% the step 1 solution
Ismayilova et al. (2007)	University Schedule	L-S-T	BP	Exact: solver CPLEX	1-step
Birbas et al. (2009)	School Schedule	S-T	BP	Exact: solver CPLEX 10.1	2-steps: Step I: lecturers shift-assignment Step II: specify shifts-periods
Santos et al. (2012)	University Schedule	S-T	MILP	Exact: CPLEX 10.1 solver cutting and column generation algorithm	1-step
Tassopoulos & Beligiannis (2012)	School Schedule	L-S-T	IP	Heuristic: particle swarm optimization (PSO)	1-step
Ceschia et al. (2014)	University Schedule	S-T	IP	Exact: CPLEX 12.2 solver Heuristic: Simulated annealing	2-steps
Katsaragakis et al. (2015)	School Schedule	L-S-T	MIP	Heuristic: Particle Swarm Optimization & Artificial Fish Swarm	1-step
Mühlenthaler & Wanka (2016)	University Schedule	S-T-C	NLP	Heuristic: Simulated annealing, with man-mix fairness and Jain's fairness index application	1-step
Fonseca et al. (2017)	School & University Schedules	L-S-T	IP	Exact: Cuts and reformulation of the originals IPs	2-steps: Step I. IP Model with all hard constraints Step II: IP model with all soft constraints. The solution process warm-started from its previous state

Skoullis et al. (2017)	School Schedule	L-S-T	MIP	Heuristic: hybrid cat swarm optimization based algorithm	1-step
Saviniec et al. (2020)	School Schedule	S-T	2 MILPs	Exact Parallel metaheuristic based algorithm	1-step: MILP 1 Fenchel cuts 1-step: MILP 2 Column generation framework
Tassopoulos et al. (2020)	School Schedule	L-S-T	MIP	Exact: Gurobi & CPLEX	1-step (monolithic method) 2-steps (decomposition method) Step I. Model with all hard and some soft constraints Step II: 5 submodels (one per week day) with the rest of soft constraints
OUR PAPER	University Schedule	L-S	MILP	Exact: Gurobi 6.0.3	2-steps: Step I: calculation of dependent variables of input data Step II: main model is solved

Table 1 Review of works based on the dimensions “scheduled event”, “problem type”, “model”, “resolution method” and “resolution procedure”

AUTHORS (YEAR)	OBJECTIVES
Badri (1996)	Meet the teaching load assigned to each lecturer. Maximize the lecturer preferences regarding which subjects to teach and when. Do not exceed the maximum nº of classes in the last hours of the day, allowed to each lecturer
Hultberg & Cardoso (1997)	Minimize the average number of different subjects taught by each lecturer.
Badri et al.(1998)	Schedule of the required subjects. Meet the teaching load assigned to each lecturer. Maximize the lecturer preferences regarding which subjects to teach and when. Fit daily classrooms availability.
Dimopoulou & Miliotis (2001)	Maximize the lecturer preferences regarding when to teach the subjects.
Daskalaki et al. (2004)	Minimize the times in which a subject is taught in an unwanted period, taking into account lecturer preferences Minimize gaps in the students' schedule as well as the number of classroom changes. Minimize the number of multi-period classes that are scheduled on unwanted days.
Daskalaki &Birbas (2005)	Minimize the times in which a subject is taught in an unwanted period, taking into account lecturer preferences Minimize gaps in the students' schedule as well as the number of classroom changes. Minimize the number of multi-period classes that are scheduled on unwanted days.
Al-Yakoob & Sherali (2006)	Minimize the individual dissatisfaction of lecturers regarding subjects and periods of time. Minimize the difference between lecturer dissatisfactions with the same workload. Minimize the nº of multi-period classes exceeding the max. nº of consecutive periods allowed. Minimize the nº of conflicts between subjects intended to be scheduled in the same period.
Ismayilova et al. (2007)	Maximize the satisfaction regarding the lecturer-subject and subject-period of time. Minimize the deviation for exceeding the upper workload limit assigned to a lecturer
Birbas et al. (2009)	Maximize lecturer shift preferences
Santos et al. (2012)	Maximize the schedule compatibility and the time availability of every lecturer. Minimize the schedule gaps. Minimize the number of double classes required for a lecturer and not met.
Tassopoulos and Beligiannis (2012)	Minimize the nº of gaps in lecturer schedules. Minimize the nº of gaps in lecturer schedules, which are not uniformly distributed among their available time. Minimize the class hours of lecturers not uniformly distributed among their availability. Minimize the nº of subjects which are assigned more than once in the same day.
Ceschia et al. (2014)	Minimize the deviation between the ideal and the obtained credit distribution among periods. Maximize the preferences satisfaction of the lecturers regarding to time periods. Minimize the heterogeneity of students from different academic years attending the same class.
Katsaragakis et al. (2015)	Minimize the dispersion in the distribution of the hours of the same lesson for each class in the days it's taught expressed as a cost. Minimize the dispersion in the distribution of each teacher's hours in the days he/she is available expressed as a cost. Minimize the number of idle hours each teacher has available between his/her teaching hours expressed as a cost.
Mühlenthalerand Wanka (2016).	Balance the penalty assigned to the timetables distributing it among the different curricula, based on its "fairness".
Fonseca et al. (2017)	Three alternative formulations: Linear: Minimize the sum of constraints deviations Quadratic: Minimize the sum of the squares of constraints deviations Step: Minimize the penalizations of the number of deviations, regardless their magnitudes
Skoullis et al. (2017)	Minimize the dispersion in the distribution of the hours of the same lesson for each class in the days it's taught expressed as a cost. Minimize the dispersion in the distribution of each teacher's hours in the days he/she is available expressed as a cost. Minimize the number of idle hours each teacher has available between his/her teaching hours expressed as a cost.
Saviniec et al. (2020)	Minimize violations of soft requirements
Tassopoulos et al. (2020)	The soft constraints have been incorporated in the objective function, the value of which we aim to minimise under the hard constraints: teacher idle periods, teacher dispersion, class dispersion.
OUR PAPER	Maximize the overall lecturer´s satisfaction and the teaching quality. Minimize the number of different lecturers teaching in the same group.

Table 2 Review of works based on the dimension "objectives"

AUTHOR	CREDIT TYPE DISTINCTION	TIMETABLE	AVAILAB	LECTURER PREFERENCES	RANK	SATURATION
Badri (1996)	No	Regular	No	Yes	No	No
Hultberg and Cardoso (1997)	No	Regular	No	No	No	Yes
Badri et al.(1998)	No	Regular	No	Yes	No	No
Dimopoulou and Miliotis (2001)	No	Regular	No	Yes	No	No
Daskalaki et al. (2004)	Yes	Regular	Yes	Yes	No	No
Daskalaki and Birbas (2005)	Yes	Regular	Yes	Yes	No	No
Al-Yakoob and Sherali (2006)	Yes	Regular	No	Yes	Yes	Yes
Ismayilova et al. (2007)	No	Regular	No	Yes	No	Yes
Birbas et al. (2009)	No	Regular	Yes	Yes	Yes	Yes
Santos et al. (2012)	No	Regular	Yes	No	No	Yes
Tassopoulos and Beligiannis (2012)	No	Regular	Yes	No	No	Yes
Ceschia et al. (2014)	No	Regular	No	Yes	No	No
Katsaragakis et al. (2015)	No	Regular	Yes	No	No	Yes
Mühlenthaler & Wanka (2016)	No	Regular	No	No	No	No
Fontseca et al. (2017)	Yes	Regular	Yes	Yes	No	Yes
Skoullis et al. (2017)	No	Regular	Yes	No	No	Yes
Saviniec et al. (2020)	No	Regular	Yes	No	No	Yes
Tassopoulos et al. (2020)	No	Regular	Yes	No	No	Yes
OUR PAPER	Yes	Non-regular	Yes	Yes	Yes	Yes

Table 3 Review of works based on the dimensions “timetable”, “credit type”, “lecturers´ availability time”, “preferences”, “ranks” and “saturation”

FEATURES	REVIEWED WORKS	PROPOSED MODEL
Objective: meeting the preferences regarding the assignment lecturer - subject	Secondary	Principal
Rank	Restricted to two levels	Hierarchical with any number of levels
Academic schedule and Lecturers' availability	Regular (just one week is scheduled)	Non-regular (it is necessary to distinguish between weeks)
Comparison: lecturer availability and academic schedule	Restricted to the number of daily uniform time periods	Exact time slots are compared
Credit types distinction	Not considered	Considered

Table 4 Differences between the reviewed works and the proposed model

l	Lecturers	d	Days
r	Ranks	d'	Days
s	Subjects	w	Weeks
s'	Subjects	w'	Weeks
c	Credit types	z	Semesters
c	Credit types	z'	Semesters
g	Groups	t	Available time intervals
g'	Groups		

Table 5 Indexes

$R\ (l,\ r)$	Rank r of the lecturer l
$C\ (s,c)$	Credit types c of the subject s
$C'\ (s',c')$	Credit types $c´$ of the subject $s´$
$G(s,c,g)$	Groups g from credit type c of the subject s
$G'(s',c',g')$	Groups $g´$ from credit type $c´$ of the subject $s´$
$S_{possible}(l,s)$	Set of subjects s that lecturer l could be assigned
$S'_{possible}(l,s')$	Set of subjects $s´$ that lecturer l could be assigned
$W(w,\ z)$	Weeks w corresponding to semester z
$W'(w',\ z')$	Weeks $w´$ corresponding to semester $z´$
$Z(s,\ z)$	Set of subjects s taught in semester z
$Z'(s',\ z')$	Set of subjects $s´$ taught in semester $z´$
$D(s,\ c,\ g,\ z,\ w,\ d)$	Set of groups g from credit type c of the subject s taught the day d of the week w of the semester z
$D'(s',c',g',z',w',\ d')$	Set of groups $g´$ from credit type $c´$ of the subject $s´$ taught the day d´ of the week $w´$ of the semester $z´$
$T(l,\ z,\ w,\ d,\ t)$	Available time intervals t of lecturer l the day d of the week w of the semester z

Table 6 Sets

$stassig_{scgzwd}$	Starting time of group *g* from credit type *c* of the subject *s* the day *d* of the week *w* of the semester *z*
$'stassig_{s'c'g'z'w'd'}$	Starting time of group *g´* from credit type *c´* of the subject *s´* the day *d´* of the week *w´* of the semester *z´*
$enassig_{scgzwd}$	Ending time of group *g* from credit type *c* of the subject *s* the day *d* of the week *w* of the semester *z*
$'enassig_{s'c'g'z'w'd'}$	Ending time of group *g´* from credit type *c´* of the subject *s´* the day *d´* of the week *w´* of the semester *z´*
$stavail_{lzwdt}$	Starting time of lecturer *l* the day *d* of the week *w* of the semester *z* in the available time interval *t*
$enavail_{lzwdt}$	Ending time of lecturer *l* the day *d* of the week *w* of the semester z in the available time interval *t*
φ	This parameter will take value of the duration of the shortest span between classes throughout the academic schedule

Table 7 Parameters (I)

$credits_{scg}$	Number of credits corresponding to group g from credit type c of the subject s
$load_l$	Maximum number of credits (hours) potentially assigned to lecturer l
α	Maximum saturation level: maximum percentage of $load_l$
β	Minimum saturation level: minimum percentage of $load_l$
$pref_{lscg}$	Preference degree expressed by lecturer l to teach the group g from credit type c of the subject s (values rank from 1 to 10).
$rank_r$	Factor prioritizing the satisfaction of lecturers preferences depending on the rank r
λ	Maximum number of different groups from any credit type c of a subject s which might be assigned to a unique lecturer in a semester
μ	Maximum number of lecturers' teaching classes from any group g of credit type c of the subject s
n	Maximum number of classes (credit types * groups) of any subject
M	Value indicating the difference between the latest time of the last class and the start time of the earliest class, throughout the academic schedule

Table 8 Parameters (II)

$X_{lscgzwd}$	Binary variable with a value of 1 if the lecturer l teaches the group g from credit type c of the subject s the day d of the week w of the semester z, and 0 otherwise.
$X_{ls'c'g'z'w'd'}$	Binary variable with a value of 1 if the lecturer $l´$ teaches the group g´ from credit type $c´$ of the subject $s´$ the day $d´$ of the week $w´$ of the semester $z´$, and 0 otherwise.
Y_{lscg}	Binary variable with a value of 1 if the lecturer l teaches a group g from credit type c of the subject s, without regard the value of g.
$CASSIG_l$	Continuous variable indicating the number of credits (hours) assigned to lecturer l.
$DEVST_{lscgzwdt}$	Binary variable with a value of 1 if the group g from credit type c of the subject s scheduled the day d of the week w of the semester z, starts later or at the same time than the start time of the lecturer l in the available time interval t ($stassig_{scgzwd} \geq stavail_{lzwdt}$), and 0 otherwise.
$DEVEN_{lscgzwdt}$	Binary variable with a value of 1 if the group g from credit type c of the subject s scheduled the day d of the week w of the semester z, ends earlier or at the same time than the end time of the lecturer l in the available time interval t ($enassig_{scgzwd} \leq enavail_{lzwdt}$), and 0 otherwise.

Table 9 Decision variables (I)

$OSTSSTS'_{ss'cc'gg'zz'ww'dd'}$	Binary variable with a value of 1 if the subject *s* starts earlier or at the same time than the subject *s´* scheduled the day *d* of the week *w* of the semester *z* ($stassig_{scgzwd} \leq 'stassig_{s'c'g'z'w'd'}$), and 0 otherwise. This variable is only calculated when the value of *s* is not the same as *s'*, *z* is equal to *z'*, *w* is equal to *w'* and *d* is equal to *d'*.
$OSTS'ENS_{jj'kk'gg'zz'ss'dd'}$	Binary variable with a value of 1 if the subject *s´* starts earlier or at the same time than the end time of the subject *s* scheduled the day *d* of the week *w* of the semester *z* ($stassig_{s'c'g'z'w'd'} \leq enassig_{scgzwd}$), and 0 otherwise. This variable is only calculated when the value of *s* is not the same as *s'*, *z* equal to *z'*, *w* equal to *w'* and *d* equal to *d'*.
$OSTSENS'_{ss'cc'gg'zz'ww'dd'}$	Binary variable with a value of 1 if the subject *s* starts earlier or at the same time than the end time of the subject *s´* scheduled the day *d* of the week *w* of the semester *z* ($stassig_{ccgzwd} \leq enassig_{s'c'g'z'w'd'}$), and 0 otherwise. This variable is only calculated when the value of *s* is not the same as *s'*, *z* equal to *z'*, *w* equal to *w'* and *d* equal to *d'*.
$OENS'ENS_{ss'cc'gg'zz'ww'dd'}$	Binary variable with a value of 1 if the subject *s´* ends earlier or at the same time than the subject *s* scheduled the day *d* of the week *w* of the semester *z* ($'enassig_{s'c'g'z'w'd'} \leq enassig_{scgzwd}$), and 0 otherwise. This variable is only calculated when the value of *s* is not the same as *s'*, *z* equal to *z'*, *w* equal to *w'* and *d* equal to *d'*.
$OSTCSTC'_{ss'cc'gg'zz'ww'dd'}$	Binary variable with a value of 1 if the credit type *c* starts earlier or at the same time than the credit type *c´* of the subject *s* scheduled the day *d* of the week *w* of the semester *z* ($stassig_{scgzwd} \leq 'stassig_{s'c'g'z'w'd'}$), and 0 otherwise. This variable is only calculated when *s* is equal to *s'*, *c* is not the same as *c'*, *z* equal to *z'*, *w* equal to *w'* and *d* equal to *d'*.
$OSTC'ENC_{ss'cc'gg'zz'ww'dd'}$	Binary variable with a value of 1 if the credit type *c´* starts earlier or at the same time than the end time of credit type *c* of the subject *s* scheduled the day *d* of the week *w* of the semester *z* ($'stassig_{s'c'g'z'w'd'} \leq enassig_{scgzwd}$), and 0 otherwise. This variable is only calculated when *s* is equal to *s'*, *c* is not the same as *c'*, *z* equal to *z'*, *w* equal to *w'* and *d* equal to *d'*.
$OSTCENC'_{ss'cc'gg'zz'ww'dd'}$	Binary variable with a value of 1 if the credit type *c* starts earlier or at the same time than the end time of credit type *c´* of the subject *s* scheduled the day *d* of the week *w* of the semester z ($stassig_{scgzwd} \leq 'enassig_{s'c'g'z'w'd'}$), and 0 otherwise. This variable is only calculated when *s* is equal to *s'*, *c* is not the same as *c'*, *z* equal to *z'*, *w* equal to *w'* and *d* equal to *d'*.
$OENC'ENC_{ss'cc'gg'zz'ww'dd'}$	Binary variable with a value of 1 if the credit type *c´* ends earlier or at the same time than the credit type *c* of the subject *s* scheduled the day *d* of the week *w* of the semester *z* ($'enassig_{s'c'g'z'w'd'} \leq enassig_{scgzwd}$), and 0 otherwise. This variable is only calculated when *s* is equal to *s'*, *c* is not the same as *c'*, *z* equal to *z'*, *w* equal to *w'* and *d* equal to *d'*.

Table 10 Decision variables (II)

$OSTGSTG'_{ss'cc'gg'zz'ww'dd'}$	Binary variable with a value of 1 if the group g starts earlier or at the same time than the group g' from the credit type c of the subject s scheduled the day d of the week w of the semester z ($stassig_{scgzwd} \leq stassig_{s'c'g'z'w'd'}$), and 0 otherwise. This variable is only calculated when s is equal to s', c is equal to c', g is not the same as g', z equal to z', w equal to w' and d equal to d'.
$OSTG'ENG_{ss'cc'gg'zz'ww'dd'}$	Binary variable with a value of 1 if the group g' starts earlier or at the same time than the end time of group g from the credit type c of the subject s scheduled the day d of the week w of the semester z ($'stassig_{s'c'g'z'w'd'} \leq enassig_{scgzwd}$), and 0 otherwise. This variable is only calculated when s is equal to s', c is not the same as c', z equal to z', w equal to w' and d equal to d'.
$OSTGENG'_{ss'cc'gg'zz'ww'dd'}$	Binary variable with a value of 1 if the group g starts earlier or at the same time than the end time of group g´ from the credit type c of the subject s scheduled the day d of the week w of the semester z ($stassig_{scgwd} \leq {}'enassig_{s'c'g'z'w'd'}$), and 0 otherwise. This variable is only calculated when s is equal to s', c is not the same as c', z equal to z', w equal to w' and d equal to d'.
$OENG'ENG_{ss'cc'gg'zz'ww'dd'}$	Binary variable with a value of 1 if the group g' ends earlier or at the same time than the group g from the credit type c of the subject s scheduled the day d of the week w of the semester z ($'enassig_{s'c'g'z'w'd'} \leq enassig_{scgwd}$), and 0 otherwise. This variable is only calculated when s is equal to s', c is not the same as c', z equal to z', w equal to w' and d equal to d'.

Table 11 Decision variables (III)

Lecturer	l1	l2	l3	l4	l5	l6	l7	l8	l9
Rank	r1	r2	r3	r1	r2	r1	r3	r1	r3
Load (hours)	100	150	200	100	150	200	100	150	200

Table 12 Lecturers teaching load

Subject	Credit	Group	Credits (hours)
geA1 (13762)	c1	AA1A1	60
geA1 (13762)	c1	AA1A2	15
geA1 (13762)	c2	AA1A2	45
geA1 (13762)	c1	AA1A3	45
geA1 (13762)	c2	AA1A3a	15
geA1 (13762)	c2	AA1A3b	15
geA1 (13762)	c1	AA1A4	43.5
geA1 (13762)	c2	AA1A4a	21
geA1 (13762)	c2	AA1A4b	21
geA1 (13762)	c1	AA1A5	32
geA1 (13762)	c2	AA1A5	28
geA1 (13762)	c1	AA1B1	57.5
geA1 (13762)	c1	AA1B2	32.5
geA1 (13762)	c2	AA1B2	27.5
geA2 (13763)	c1	AA2A1	60
geA2 (13763)	c1	AA2AB	60
geA2 (13763)	c1	AA2B2	60
geB1 (13764)	c1	AB1A1	72.5
geB1 (13764)	c1	AB1B1	45
geB1 (13764)	c2	AB1B1	15
geB2 (13765)	c1	AB2A1	32.5
geB2 (13765)	c2	AB2A1	27.5
geB2 (13765)	c1	AB2B1	60

Table 13 Data about the subjects

Lecturer	Subject	Credit	Pref	Lecturer	Subject	Credit	Pref
l1	geA1 (13762)	c1	10	l5	geB1 (13764)	c2	9
l1	geA1 (13762)	c2	10	l5	geB2 (13765)	c1	10
l1	geA2 (13763)	c1	7	l5	geB2 (13765)	c2	10
l1	geB1 (13764)	c1	5	l6	geA1 (13762)	c1	8
l1	geB1 (13764)	c2	5	l6	geA1 (13762)	c2	8
l1	geB2 (13765)	c1	5	l6	geA2 (13763)	c1	8
l1	geB2 (13765)	c2	5	l6	geB1 (13764)	c1	8
l2	geA1 (13762)	c1	7	l6	geB1 (13764)	c2	8
l2	geA1 (13762)	c2	7	l6	geB2 (13765)	c1	8
l2	geA2 (13763)	c1	9	l6	geB2 (13765)	c2	8
l2	geB1 (13764)	c1	9	l7	geA1 (13762)	c1	5
l2	geB1 (13764)	c2	9	l7	geA1 (13762)	c2	10
l2	geB2 (13765)	c1	10	l7	geA2 (13763)	c1	5
l2	geB2 (13765)	c2	10	l7	geB1 (13764)	c1	5
l3	geA1 (13762)	c1	5	l7	geB1 (13764)	c2	10
l3	geA1 (13762)	c2	5	l7	geB2 (13765)	c1	5
l3	geA2 (13763)	c1	8	l7	geB2 (13765)	c2	10
l3	geB1 (13764)	c1	8	l8	geA1 (13762)	c1	6
l3	geB1 (13764)	c2	8	l8	geA1 (13762)	c2	10
l3	geB2 (13765)	c1	10	l8	geA2 (13763)	c1	3
l3	geB2 (13765)	c2	10	l8	geB1 (13764)	c1	3
l4	geA1 (13762)	c1	10	l8	geB1 (13764)	c2	3
l4	geA1 (13762)	c2	10	l8	geB2 (13765)	c1	3
l4	geA2 (13763)	c1	7.5	l8	geB2 (13765)	c2	3
l4	geB1 (13764)	c1	7.5	l9	geA1 (13762)	c1	5
l4	geB1 (13764)	c2	7.5	l9	geA1 (13762)	c2	5
l4	geB2 (13765)	c1	2	l9	geA2 (13763)	c1	5
l4	geB2 (13765)	c2	2	l9	geB1 (13764)	c1	5
l5	geA1 (13762)	c1	3	l9	geB1 (13764)	c2	5
l5	geA1 (13762)	c2	3	l9	geB2 (13765)	c1	5
l5	geA2 (13763)	c1	8	l9	geB2 (13765)	c2	5
l5	geB1 (13764)	c1	9				

Table 14 Lecturers' preferences for German subjects of academic course 2015/2016

FACTORS		1 STEP	2 STEPS	IMPROVEMENTS
Objective Function		4964	4964	-
Number of Variables	**Binaries**	19292	13860	28%
	Continuous	9	9	-
Number of Constraints		49549	38730	22%
Reading Time		55.243 min	19.084 min	65%
Optimization Time		6.65 s	2.08 s	69%
Gap		0 %	0 %	-

Table 15 One-step & Two-steps resolution (experiment A)

α		0.9	0.8	0.7
Objective Function		4856.4	4710	4454.2
Number of Variables	**Binaries**	13860	13860	13860
	Continuous	9	9	9
Number of Constraints		38730	38730	38730
Number of Iterations		91814	22886	135324
Optimization Time		46.15 s	21.81 s	62 s
Gap		0 %	0.0085%	0 %

Table 16 Experiment B results

β		0.2	0.4	0.6	0.65
Objective Function		4871	4730.2	4488.6	4369.4
Number of Variables	**Binaries**	13860	13860	13860	13860
	Continuous	9	9	9	9
Number of Constraints		38739	38739	38739	38739
Number of Iterations		2905	6426	1087	1008
Optimization Time		1.29 s	2.63 s	1.25 s	1.4 s
Gap		0 %	0 %	0 %	0 %

Table 17 Experiment C results

μ		1	2	3
Objective Function		4898.80	4964	4964
Number of Variables	**Binaries**	14067	14067	14067
	Continuous	9	9	9
Number of Constraints		39167	39167	39167
Number of Iterations		478	592394	14907
Optimization Time		1.85 s	900 s (time limit)	15.92 s
Gap		0 %	0.0483%	0 %

Table 18 Experiment D results

λ=3		λ=4	
Lecturer	**Average satisfaction**	**Lecturer**	**Average satisfaction**
l1	196.7	l1	107.5
l2	165	l2	137.5
l3	160	l3	105
l4	133.3	l4	127.5
l5	159	l5	123.5
l6	183.3	l6	160
l7	60	l7	60
l8	130	l8	132.5
OVERALL AVERAGE SATISFACTION = 148.4125		**OVERALL AVERAGE SATISFACTION = 119.18**	

Table 19 Experiment E results

SCENARIOS	Parameter/s Values	Objective Function	Optimization Time	Gap
Basic	$\alpha = 1$	4964	2.08 s	0 %
Lecturers maximum saturation	$\alpha = 0.9$	4856.4	46.15 s	0 %
Lecturers minimum saturation	$\alpha = 1$; $\beta = 0.2$	4871	1.29 s	0 %
Maximum number of lecturers per group	$\alpha = 1$; $\mu = 3$	4964	15.92 s	0 %
Maximum number of groups per lecturer	$\alpha = 1$; $\lambda = 4$	4867.8	286.8 s	0.0082%

Table 20 Comparison of the best results obtained with the different scenarios

FIGURES CAPTIONS