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A generalized sequential subpredictor for input delayed systems under time-varying delay mismatches☆

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ABSTRACT

For input delayed systems, the sequential subpredictor (SSP) control scheme has the advantage that arbitrarily large delays can be tolerated in the control loop by introducing more and more subpredictors. However, an exact delay compensation is not possible in the presence of time-varying delay mismatches, and larger delays cannot therefore be obtained by increasing the number of subpredictors. To alleviate this limitation and enhance robustness against time-varying delay uncertainties, a generalized sequential subpredictor (SSP) control scheme is proposed by introducing new observer parameters that can be designed via Linear Matrix Inequalities (LMI) and Cone-Complementarity Linearization (CCL) algorithm. Finally, the effectiveness of the proposed method is illustrated by a simulation example.

1. Introduction

It is well known that the presence of input delays may jeopardize the closed-loop stability of the control system if they are not taken into account in control synthesis. Among other techniques, such as those based on classical Smith predictor (Smith, 1959) and Finite Spectrum Assignment (FSA) (Manitius & Olbrot, 1979), the sequential subpredictor (SSP) method (Hernández-Pérez, Fragoso-Rubio, Velasco-Villa, del Muro-Cuéllar, Márquez-Rubio, & Puebla, 2020; Najafi, Hosseinnia, Sheikholeslam, & Karimadini, 2013) was conceived to increase the maximum allowable delay in the control loop. SSP consists of a set of coupled subpredictors for a certain number of small pieces of a long delay, such that the convergence of each prediction error is driven to zero. Hence, SSP has the advantage that arbitrarily large delays can be allowed by introducing more and more subpredictors. This feature has recently motivated some notable works aimed at extending SSP to time-varying systems (Mazenc & Malisoff, 2017), discrete-time systems (Hao, Liu, & Zhou, 2019; Mazenc, Malisoff, & Bhogaraju, 2020), aperiodic sampled-data systems (Weston & Malisoff, 2018) and networked-control systems (Zhu & Fridman, 2021). However, to the best of author's knowledge, the design of SSP under time-varying delay uncertainties has not been fully investigated, which motivates the present work.

This note proposes a novel SSP control scheme with a more general structure in comparison to previous SSP methods. The objective is to increase the maximum allowable delay and robustness in the presence of time-varying delay mismatches by introducing more observer parameters whose design can efficiently be addressed via Linear Matrix Inequalities (LMI) and Cone-Complementarity Linearization (CCL) algorithm (El Ghaoui, Oustry, & AitRami, 1997).

2. Problem statement and preliminaries

Consider the following linear system:

$$\dot{x}(t) = Ax(t) + Bu(t - h(t)),$$
$$y(t) = Cx(t) \tag{1}$$

where $x(t) \in \mathcal{R}^n$, $u(t) \in \mathcal{R}^m$ and $y(t) \in \mathcal{R}^q$ are the state variable, the control input and the measured output, and $h_1 \leq h(t) \leq h_2$ represents any unknown, unmeasurable and bounded arbitrarily fast time-varying delay with known bounds $0 < h_1 \leq h_2$. To deal with time-varying delay mismatches for robust stability analysis, the following preliminary result is presented:

Lemma 1. *Given any arbitrary signal $z_1(t) \in \mathcal{R}^n$ and any time-varying delay function $h(t) \in \mathcal{R}$ satisfying $h_1 \leq h(t) \leq h_2$, define*

$$w_h(t) = \frac{2}{\tau}\left(z_1(t-h(t)) - \frac{1}{2}\left(z_1(t-h_1) + z_1(t-h_2)\right)\right),$$

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where $\tau = h_2 - h_1$ is the delay interval. Then, the time-varying operator $\Delta_h(t) : \dot{z}_1 \to w_h : \mathcal{R}^n \to \mathcal{R}^n$ renders:

$$w_h(t) = \frac{1}{\tau}\int_{t-h_2}^{t-h_1} \phi_d(s)\dot{z}_1(s)ds,$$

$$\phi_d(s) = \begin{cases} 1 & \text{if } s < t - h(t), \\ -1 & \text{otherwise.} \end{cases}$$

Moreover, $\Delta_h(t)$ satisfies $\|S\Delta_h(t)S^{-1}\|_\infty \leq 1$ for any invertible matrix $S \in \mathcal{R}^n$.

Proof. The proof can be obtained by a straightforward adaptation of a similar result given in Li and Gao (2011, Lemma 2) for continuous-time case.

3. Generalized SSP control scheme

This section presents the generalized SSP for robust stabilization of system (1). Let us introduce the control law:

$$u(t) = Kz_1(t) \tag{2}$$

where $K \in \mathcal{R}^{m\times n}$ is the controller gain, and $z_1(t)$ is an observer SSP state variable, which is obtained through the following generalized SSP with $N \geq 1$ subpredictors:

$$\begin{aligned}\dot{z}_i(t) &= \sum_{j=1}^{i} F_{i,j}z_j(t-(i-j)\tilde{h}_1) \\ &+ \sum_{j=1}^{i} G_{i,j}v_j(t-(i-j)\tilde{h}_1), \qquad 1 \leq i \leq N-1,\end{aligned} \tag{3}$$

$$\cdots$$

$$\begin{aligned}\dot{z}_N(t) &= \sum_{j=1}^{N} F_{N,j}z_j(t-(N-j)\tilde{h}_1) \\ &+ \sum_{j=1}^{N-1} G_{N,j}v_j(t-(N-j)\tilde{h}_1) \\ &+ L\left(Cz_N(t-\tilde{h}_1) + Cz_N(t-\tilde{h}_1-\tau) - 2y(t)\right),\end{aligned}$$

where $\tilde{h}_1 = h_1/N$, τ is the delay interval defined in Lemma 1, $F_{i,j}$, $G_{i,j} \in \mathcal{R}^n$, $L \in \mathcal{R}^{n\times q}$ are the observer gains to be designed, and

$$v_i(t) = z_i(t-\tilde{h}_1) - z_{i+1}(t), \qquad 1 \leq i \leq N-1. \tag{4}$$

The following lemma gives an equivalent interconnected closed-loop model for system (1) and the control law (2) where delay uncertainties are put in a feedback system Δ in order to allow stability analysis via small gain theory:

Lemma 2. *Let $\tilde{h}_2 = \tilde{h}_1 + \tau$. The closed-loop system formed by (1) and the control law (2) can be described by the following interconnected system:*

$$\mathcal{M} : \begin{cases} \dot{\bar{x}}(t) = \mathcal{A}\bar{x}(t) + \mathcal{A}_1\bar{e}(t-\tilde{h}_1) + \mathcal{A}_2 e_N(t-\tilde{h}_2) \\ +\tau\mathcal{B}_h w_h(t), \\ y_h(t) = \mathcal{C}\bar{x}(t) + \mathcal{C}_1\bar{e}(t-\tilde{h}_1) \end{cases} \tag{5}$$

$$\Delta : \left\{ w_h(t) = \Delta(t)y_h(t) \right.$$

where $\Delta(t)$ is a time-varying operator satisfying $\|S\Delta(t)S^{-1}\|_\infty \leq 1$ for any invertible matrix $S \in \mathcal{R}^n$, and

$$\bar{x}(t) = \begin{bmatrix} x^T(t) & \bar{e}^T(t) \end{bmatrix}^T, \quad \bar{e}(t) = \begin{bmatrix} e_1^T(t) & \cdots & e_N^T(t) \end{bmatrix}^T,$$

$$\mathcal{A} = \begin{bmatrix} A+BK & \mathcal{U}_1 \otimes BK \\ \bar{F} - \mathbf{1}_{1,N}\otimes(A+BK) & \mathcal{F} - \mathbf{1}_{1,N}\otimes(\mathcal{U}_1\otimes BK) \end{bmatrix},$$

$$\mathcal{A}_1 = \begin{bmatrix} 0 \\ \mathcal{G}\mathcal{V} \end{bmatrix}, \quad \mathcal{A}_2 = \begin{bmatrix} 0 \\ LC \end{bmatrix}, \quad \mathcal{B}_h = \begin{bmatrix} \frac{1}{2}BK \\ -\mathbf{1}_{N,1}\otimes\frac{1}{2}BK \end{bmatrix},$$

$$\mathcal{C} = \bar{\mathcal{U}}_1\begin{bmatrix} \bar{F} & \mathcal{F} \end{bmatrix}, \qquad \mathcal{C}_1 = \bar{\mathcal{U}}_1\mathcal{G}\mathcal{V}, \qquad \bar{\mathcal{U}}_1 = \mathcal{U}_1 \otimes I_n,$$

$$\mathcal{U}_1 = \underbrace{\begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}}_{N}, \; \mathcal{V} = W_1 - W_2, \; \bar{F} = \mathcal{F}\left(\mathbf{1}_{N,1}\otimes I_n\right),$$

$$W_1 = \begin{bmatrix} I_{nN} & 0_{nN\times n} \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0_{nN\times n} & I_{nN} \end{bmatrix}, \tag{6}$$

$$\mathcal{F} = \begin{bmatrix} F_{1,1} & 0 & \cdots & 0 \\ F_{2,1} & F_{2,2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ F_{N,1} & F_{N,2} & \cdots & F_{N,N} \end{bmatrix}, \mathcal{G} = \begin{bmatrix} G_{1,1} & 0 & \cdots & 0 \\ G_{2,1} & G_{2,2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ G_{N,1} & G_{N,2} & \cdots & LC \end{bmatrix},$$

where $\mathbf{1}_{m,n}$ denotes a $m \times n$ matrix with all entries equal to 1, and $e_i(t)$ are the observer errors defined as:

$$\begin{aligned} e_i(t) &= \frac{1}{2}z_i(t-h_1+(i-1)\tilde{h}_1) \\ &+ \frac{1}{2}z_i(t-h_1-\tau+(i-1)\tilde{h}_1) - x(t), \qquad 1 \leq i \leq N \end{aligned} \tag{7}$$

Proof. Let us reformulate $\dot{x}(t)$ in (1) to deal with time-varying delay mismatches as:

$$\dot{x}(t) = Ax(t) + \frac{BK}{2}\left(z_1(t-h_1) + z_1(t-h_2) + \tau w_h(t)\right)$$

where $w_h(t)$ (defined in Lemma 1) is useful to deal with time-varying delay mismatches by virtue of Lemma 1, as shown later in (9). From $e_i(t)$ given in (7), it can be seen that $e_1(t)$ satisfies:

$$\frac{BK}{2}\left(z_1(t-h_1) + z_1(t-h_2)\right) = BK\left(x(t) + e_1(t)\right).$$

The above equivalence allows to remove from $\dot{x}(t)$ the delayed terms $z_1(t-h_1)$ and $z_1(t-h_2)$ by substitution, obtaining:

$$\dot{x}(t) = (A+BK)x(t) + BKe_1(t) + \frac{\tau}{2}BKw_h(t) \tag{8}$$

By defining the time-constant delay operator $\Delta_\delta(t) : \dot{z}_1(t) \to \dot{z}_1(t-\delta) : \mathcal{R}^n \to \mathcal{R}^n$ with $\delta \in \mathcal{R} > 0$, applying Lemma 1, one can write $w_h(t)$ as

$$w_h(t) = \Delta(t)y_h(t) \tag{9}$$

where

$$\begin{aligned} y_h(t) &= \frac{1}{2}\dot{z}_1(t-h_1) + \frac{1}{2}\dot{z}_1(t-h_1-\tau) \\ &= F_{1,1}\left(x(t) + e_1(t)\right) + G_{1,1}\left(e_1(t-\tilde{h}_1) - e_2(t-\tilde{h}_1)\right) \end{aligned} \tag{10}$$

and $\Delta(t)$ is the composition of the time-varying delay operator $\Delta_h(t)$ and the sum of two time-constant delay operators $\Delta_\delta(t)$ weighted by 0.5 with $\delta = h_1$ and $\delta = h_2$. Hence, it can be deduced that $\|S\Delta(t)S^{-1}\|_\infty \leq 1$ for any invertible matrix $S \in \mathcal{R}^n$, in light of Lemma 1 and noting that $\Delta_\delta(t)$ is unitary norm-bounded for any $\delta > 0$. Also, from $v_i(t)$, $e_i(t)$ defined in (4) and (7) respectively, the following equivalences can be deduced for $1 \leq i \leq N-1$:

$$\begin{aligned} e_i(t-\tilde{h}_1) - e_{i+1}(t-\tilde{h}_1) &= \frac{1}{2}v_i(t-h_1+(i-1)\tilde{h}_1) \\ &+ \frac{1}{2}v_i(t-h_1-\tau+(i-1)\tilde{h}_1), \\ e_i(t-\tilde{h}_2) - e_{i+1}(t-\tilde{h}_2) &= \frac{1}{2}v_i(t-h_1-\tau+(i-1)\tilde{h}_1) \\ &+ \frac{1}{2}v_i(t-h_1-2\tau+(i-1)\tilde{h}_1), \end{aligned} \tag{11}$$

From the above equivalences, time-derivative of the observer errors $e_i(t)$ can be obtained as:

$$\dot{e}_i(t) = \sum_{j=1}^{i} F_{i,j}\left(x(t)+e_j(t)\right) + \sum_{j=1}^{i} G_{i,j}\left(e_j(t-\tilde{h}_1)-e_{j+1}(t-\tilde{h}_1)\right) - \dot{x}(t), \qquad 1 \le i \le N-1, \tag{12}$$

$$\dots,$$

$$\dot{e}_N(t) = \sum_{j=1}^{N} F_{N,j}\left(x(t)+e_j(t)\right) + \sum_{j=1}^{N-1} G_{N,j}\left(e_j(t-\tilde{h}_1)-e_{j+1}(t-\tilde{h}_1)\right) + LCe_N(t-\tilde{h}_1)+LCe_N(t-\tilde{h}_2)-\dot{x}(t)$$

Finally, the interconnected system (5) can be obtained writing in matrix form the expressions from (8), (10) and (12).

Remark 1. Notice that the particular choice for the SSP parameters $F_{i,j}$, $G_{i,j}$ as:

$$F_{i,j} = \begin{cases} A+BK & \text{if } i=1 \text{ and } j=1,\\ A & \text{if } i=j>1,\\ BK & \text{if } i>1 \text{ and } j=1,\\ 0 & \text{otherwise,}\end{cases} \tag{13}$$

$$G_{i,j} = \begin{cases} L_iC & \text{if } i=j\\ 0 & \text{otherwise}\end{cases}$$

leads to the SSP structure (Zhou, Liu, & Mazenc, 2017) adapted for systems with no state delay, and the SSP (Najafi et al., 2013; Zhu & Fridman, 2021) choosing the same value for L_i, $i=1,\dots,N$. Next section addresses the control design of the observer parameters $F_{i,j}$, $G_{i,j}$ and L in (3).

4. Robust stabilization

The following theorem provides a sufficient condition for closed-loop stability analysis given some delay bounds h_1, h_2 and parameters $F_{i,j}$, $G_{i,j}$ and L:

Theorem 1. *System (5) is stable if there exist matrices $P_i \in \mathcal{R}^n > 0$, $i=0,\dots,N$, $Q_1, Z_1 \in \mathcal{R}^{nN} > 0$, $Q_2, Z_2, S \in \mathcal{R}^n > 0$ such that the following LMI holds:*

$$\begin{bmatrix} \Xi_1 & \Xi_2^T Z & \Xi_3^T S\\ (*) & -Z & 0\\ (*) & (*) & -S \end{bmatrix} < 0 \tag{14}$$

where

$$\Xi_1 = \begin{bmatrix} \Xi_{1,1} & \Xi_{1,2} & \Xi_{1,3} & \Xi_{1,4}\\ (*) & -S & 0 & 0\\ (*) & (*) & \Xi_{3,3} & \bar{\mathcal{U}}_2^T Z_2\\ (*) & (*) & (*) & \Xi_{4,4}\end{bmatrix}, \ \Xi_2^T = \begin{bmatrix} \mathcal{A}^T\mathcal{W}^T\\ \tau\mathcal{B}_h^T\mathcal{W}^T\\ \mathcal{A}_1^T\mathcal{W}^T\\ \mathcal{A}_2^T\mathcal{W}^T\end{bmatrix},$$

$$\Xi_3 = \begin{bmatrix}\mathcal{C} & 0 & \mathcal{C}_1 & 0\end{bmatrix},$$

$$\Xi_{1,1} = \mathcal{A}^T\mathcal{P}+\mathcal{P}\mathcal{A} + \mathcal{W}^T\left(Q_1+\bar{\mathcal{U}}_2^T Q_2\bar{\mathcal{U}}_2 - Z_1 - \bar{\mathcal{U}}_2^T Z_2\bar{\mathcal{U}}_2\right)\mathcal{W}, \tag{15}$$

$$\Xi_{1,2} = \tau\mathcal{P}\mathcal{B}_h, \quad \bar{\mathcal{U}}_2 = \mathcal{U}_2\otimes I_n, \quad \mathcal{U}_2 = \underbrace{\begin{bmatrix}0 & 0 & \cdots & 1\end{bmatrix}}_{N},$$

$$\Xi_{1,3} = \mathcal{P}\mathcal{A}_1 + \mathcal{W}^T Z_1, \qquad \Xi_{1,4} = \mathcal{P}\mathcal{A}_2,$$

$$\Xi_{3,3} = -Q_1 - Z_1, \qquad \Xi_{4,4} = -Q_2 - Z_2,$$

$$Z = \tilde{h}_1^2 Z_1 + \tau^2\,\bar{\mathcal{U}}_2^T Z_2\bar{\mathcal{U}}_2, \qquad \mathcal{W} = \begin{bmatrix}0_{nN\times n} & I_{nN}\end{bmatrix},$$

$$\mathcal{P} = diag\,(P_0,\ P), \quad P = diag\,(P_1,\ \cdots,\ P_N),$$

Proof. The proof can be addressed by defining the Lyapunov–Krasovskii Functional $V(t) = V_1(t)+V_2(t)+V_3(t)$ with

$$V_1(t) = \bar{x}^T(t)\mathcal{P}\bar{x}(t),\ V_2(t) = \int_{t-\tilde{h}_1}^{t}\bar{e}^T(s)Q_1\bar{e}(s)ds + \int_{t-\tilde{h}_2}^{t} e_N^T(s)Q_2e_N(s), \tag{16}$$

$$V_3(t) = \tilde{h}_1^2\int_{-\tilde{h}_1}^{0}\int_{t+\theta}^{t}\dot{\bar{e}}^T(s)Z_1\dot{\bar{e}}(s)dsd\theta + \tau^2\int_{-\tilde{h}_2}^{-\tilde{h}_1}\int_{t+\theta}^{t}\dot{e}_N^T(s)Z_2\dot{e}_N(s)dsd\theta$$

Time-derivative of $V_1(t)$, $V_2(t)$ and $V_3(t)$ yield:

$$\dot{V}_1(t) = 2\bar{x}^T(t)\mathcal{P}\big(\mathcal{A}\bar{x}(t) + \mathcal{A}_1\bar{e}(t-\tilde{h}_1)+\mathcal{A}_2e_N(t-\tilde{h}_2)+\tau B_h w_h(t)\big), \tag{17}$$

$$\dot{V}_2(t) = \bar{e}^T(t)Q_1\bar{e}(t) - \bar{e}^T(t-\tilde{h}_1)Q_1\bar{e}(t-\tilde{h}_1) + e_N^T(t)Q_2e_N(t) - e_N^T(t-\tilde{h}_2)Q_2e_N(t-\tilde{h}_2),$$

$$\dot{V}_3(t) = \tilde{h}_1^2\dot{\bar{e}}^T(t)Z_1\dot{\bar{e}}(t) - \tilde{h}_1\int_{t-\tilde{h}_1}^{t}\dot{\bar{e}}^T(s)Z_1\dot{\bar{e}}(s)ds + \tau^2\dot{e}_N^T(t)Z_1\dot{e}_N(t) - \tau\int_{t-\tilde{h}_2}^{t-\tilde{h}_1}\dot{e}_N^T(s)Z_2\dot{e}_N(s)ds$$

Applying Jensen's inequality, we have that

$$-\tilde{h}_1\int_{t-\tilde{h}_1}^{t}\dot{\bar{e}}^T(s)Z_1\dot{\bar{e}}(s)ds \le \left(\bar{e}(t)-\bar{e}(t-\tilde{h}_1)\right)^T Z_1\left(\bar{e}(t)-\bar{e}(t-\tilde{h}_1)\right), \tag{18}$$

$$-\tau\int_{t-\tilde{h}_2}^{t-\tilde{h}_1}\dot{e}_N^T(s)Z_2\dot{e}_N(s)ds \le \left(e_N(t-\tilde{h}_1)-e_N(t-\tilde{h}_2)\right)^T \times Z_2\left(e_N(t-\tilde{h}_1)-e_N(t-\tilde{h}_2)\right)$$

The interconnected system (5) is proved to be stable if $\dot{V}(t) < 0$ holds and $\|S\Delta(t)S^{-1}\|_\infty \le 1$ for any invertible matrix S and the time-varying operator $\Delta(t)$ defined in (5). Applying small gain theorem, and taking into account that $\bar{e}(t) = \mathcal{W}\bar{x}(t)$ and $e_N(t) = \bar{U}_2\bar{e}(t)$, both conditions are true if

$$\dot{V}(t) + y_h^T(t)Sy_h(t) - w_h^T(t)Sw_h(t) \le \bar{\xi}^T(t)\left(\Xi_1 + \Xi_2^T Z\Xi_2 + \Xi_3^T S\Xi_3\right)\bar{\xi}(t) < 0 \tag{19}$$

where

$$\bar{\xi}(t) = \begin{bmatrix}x^T(t) & w_h^T(t) & \bar{e}^T(t-\tilde{h}_1) & e_N^T(t-\tilde{h}_2)\end{bmatrix}^T \tag{20}$$

Finally, applying twice Schur Complement, the equivalence between (19) and (14) is proven ■

The design of the SSP parameters $F_{i,j}$, $G_{i,j}$ and L in (3) can be addressed via CCL on the basis of the following corollary:

Corollary 1. *System (5) is stable if there exist matrices $P_i, \tilde{P}_i \in \mathcal{R}^n > 0$, $i=0,\dots,N$, $Q_1, Z_1, \tilde{Z}, X, \tilde{X} \in \mathcal{R}^{nN} > 0$, $Q_2, Z_2, S, \tilde{S}, Y, \tilde{Y} \in \mathcal{R}^n > 0$, and matrices $\tilde{F}_{i,j}, \in \mathcal{R}^n$, $i=1,\dots,N$, $j=1,\dots,N, j \le$*

i, $\tilde{G}_{i,j} \in \mathcal{R}^n$, $i = 1, \ldots, N$, $j = 1, \ldots, N-1, j \le i$, and $\tilde{L} \in \mathcal{R}^{n\times q}$ such that the following constraints hold:

$$\begin{bmatrix} \tilde{\Xi}_1 & \tilde{\Xi}_2^T & \tilde{\Xi}_3^T \\ (*) & -X & 0 \\ (*) & (*) & -Y \end{bmatrix} < 0, \begin{bmatrix} \tilde{X} & \tilde{P} \\ \tilde{P} & \tilde{Z} \end{bmatrix} \ge 0, \begin{bmatrix} \tilde{Y} & \tilde{P}_1 \\ \tilde{P}_1 & \tilde{S} \end{bmatrix} \ge 0, \quad (21)$$

$$P\tilde{P} = I, \quad Z\tilde{Z} = I, \quad S\tilde{S} = I, \quad X\tilde{X} = I, \quad Y\tilde{Y} = I$$

where

$$\tilde{\Xi}_1 = \begin{bmatrix} \tilde{\Xi}_{1,1} & \Xi_{1,2} & \Omega_1 + \mathcal{W}^T Z_1 & \Omega_2 \\ (*) & -S & 0 & 0 \\ (*) & (*) & \Xi_{3,3} & \bar{\mathcal{U}}_2^T Z_2 \\ (*) & (*) & (*) & \Xi_{4,4} \end{bmatrix}, \Xi_2^T = \begin{bmatrix} \Omega_0^T \mathcal{W}^T \\ \tau \mathcal{B}_h^T \mathcal{W}^T \\ \Omega_1^T \mathcal{W}^T \\ \Omega_2^T \mathcal{W}^T \end{bmatrix},$$

$$\tilde{\Xi}_3 = \begin{bmatrix} \tilde{\mathcal{C}} & 0 & \tilde{\mathcal{C}}_1 & 0 \end{bmatrix}, \quad \tilde{P} = diag\left(\tilde{P}_1, \ \ldots, \ \tilde{P}_N\right)$$

$$\tilde{\Xi}_{1,1} = \Omega_0 + \Omega_0^T \quad (22)$$
$$+ \mathcal{W}^T \left(Q_1 + \bar{\mathcal{U}}_2^T Q_2 \bar{\mathcal{U}}_2 - Z_1 - \bar{\mathcal{U}}_2^T Z_2 \bar{\mathcal{U}}_2\right) \mathcal{W},$$

$$\Omega_0 = \begin{bmatrix} P_0 A + \frac{1}{2} P_0 BK & \mathcal{U}_1 \otimes \frac{1}{2} P_0 BK \\ \tilde{\mathcal{F}}\left(\mathbf{1}_{N,1} \otimes I_n\right) - P\left(\mathbf{1}_{1,N} \otimes (A + \frac{1}{2}BK)\right) & \tilde{\mathcal{F}} - P\left(\mathbf{1}_{1,N} \otimes \left(\mathcal{U}_1 \otimes \frac{1}{2}BK\right)\right) \end{bmatrix},$$

$$\Omega_1 = \begin{bmatrix} 0 \\ \tilde{\mathcal{G}}\mathcal{V} \end{bmatrix}, \Omega_2 = \begin{bmatrix} 0 \\ \tilde{L}C \end{bmatrix},$$

$$\tilde{\mathcal{C}} = \bar{\mathcal{U}}_1 \begin{bmatrix} \tilde{\mathcal{F}}\left(\mathbf{1}_{N,1} \otimes I_n\right) & \tilde{\mathcal{F}} \end{bmatrix}, \qquad \tilde{\mathcal{C}}_1 = \bar{\mathcal{U}}_1 \tilde{\mathcal{G}} \mathcal{V}.$$

and $\tilde{\mathcal{F}}$, $\tilde{\mathcal{G}}$ are block-triangular matrices with the same structure as $\mathcal{F}$, $\mathcal{G}$ in (6) by replacing $F_{i,j}$, $G_{i,j}$, L by $\tilde{F}_{i,j}$, $\tilde{F}_{i,j}$, $\tilde{L}$. Moreover, if a feasible solution exists, the observer gains can be obtained as:

$$F_{ij} = P_i^{-1}\tilde{F}_{ij}, \quad G_{ij} = P_i^{-1}\tilde{G}_{ij}, \quad L = P_N^{-1}\tilde{L} \quad (23)$$

Proof. The proof can be outlined by first pre-and post multiply LMI (14) by

$$diag\left(I, \ I, \ I, \ I, \ Z^{-1}P, \ S^{-1}P_1\right) \quad (24)$$

and further denoting $\tilde{F}_{ij} = P_i F_{ij}$, $\tilde{G}_{ij} = P_i G_{ij}$, $\tilde{L} = \tilde{P}_N L$. Then, (21) is obtained after replacing the diagonal entries $-PZ^{-1}P$ and $-P_1 S^{-1} P_1$ in the obtained LMI by terms $-X$ and $-Y$, where X and Y are matrices satisfying:

$$PZ^{-1}P - X \ge 0, \qquad P_1 S^{-1} P_1 - Y \ge 0 \quad (25)$$

and finally applying Schur Complement in (25), and denoting $\tilde{P}_i = P_i^{-1}$, $\tilde{Z} = Z^{-1}$, $\tilde{S} = S^{-1}$, $\tilde{X} = X^{-1}$, and $\tilde{Y} = Y^{-1}$.

Remark 2. The equality constraints $P\tilde{P} = I$, $Z\tilde{Z} = I$, $S\tilde{S} = I$, $X\tilde{X} = I$ and $Y\tilde{Y}$ given in Corollary 1 can be treated applying CCL algorithm. The observer parameters can be obtained by (23), and the maximum bounds for h_1, τ can be obtained by slightly increasing their values at each CCL iteration starting from a feasible SSP controller with sufficiently small h_1, τ (as discussed below in Remark 3) until no feasible solution for LMI (14) is found.

Remark 3. One remarkable advantage of the CCL algorithm is that a starting condition is always guaranteed by choosing a sufficiently small value of h_1, τ, and any K, L such that $A+BK$ and $A+LC$ are Hurwitz. This property comes from the block-triangular structure of the delay-free representation of system (5) with the particular solution for $F_{i,j}$, $G_{i,j}$ depicted in (13) (Najafi et al., 2013).

Table 1
Maximum delay bound h_1 with $\tau = 0.3$ and $N = 2, \ldots, 6$.

N	2	3	4	5	6
h_1 (SSP (Zhou et al., 2017))	1.07	1.16	1.18	1.20	1.20
h_1 (Generalized SSP)	1.28	1.46	1.59	1.67	1.72

Table 2
Maximum delay interval τ with $h_1 = 2$ and $N = 2, \ldots, 6$.

N	2	3	4	5	6
τ (SSP (Zhou et al., 2017))	0.18	0.19	0.19	0.19	0.19
τ (Generalized SSP)	0.21	0.23	0.24	0.25	0.25

Remark 4. Corollary 1 can also be applied to design a SSP of the structure given in Remark 1 (Zhou et al., 2017) by defining

$$\tilde{F}_{i,j} = \begin{cases} P_i A + P_i BK & \text{if } i = 1 \text{ and } j = 1, \\ P_i A & \text{if } i = j > 1, \\ P_i BK & \text{if } i > 1 \text{ and } j = 1, \\ 0 & \text{otherwise,} \end{cases}$$

$$\tilde{G}_{i,j} = \begin{cases} \tilde{L}_i C & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

with decision variables $\tilde{L}_i$, $i = 1, \ldots, N$, where the observer gains can be designed as $L_i = P_i^{-1}\tilde{L}_i$.

5. Example

Consider the open-loop unstable system (1) borrowed from Zhu and Fridman (2021, Example 1) with matrices:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad (26)$$

For comparative analysis, two designs have been carried out via CCL algorithm for SSP (Zhou et al., 2017) and the generalized SSP with the objective to maximize h_1 assuming a time-varying delay interval $\tau = 0.3$. The maximum delay h_1 is depicted in Table 1 (see below) for $N = 2, \ldots, 6$. It can be appreciated that larger delays are obtained with the generalized SSP in comparison to SSP (Zhou et al., 2017) using the same number of subpredictors N. Note also that SSP (Zhou et al., 2017) cannot stabilize the system for delays larger than $h_1 = 1.20$ by introducing more than $N = 5$ subpredictors, whereas the generalized SSP reaches $h_1 = 1.28$ with $N = 2$, and even larger delays with $N > 2$. However, this is at the expense of an increment in the computation time due to the $n^2N^2 + (n^2 - np)N + (np - n^2)$ extra number of decision variables (NoV) corresponding to the new observer parameters $F_{i,j}$, G_{ij}. For instance, choosing $N = 2$, the average computation time at each CCL iteration with the generalized SSP is 3.07 s, which is slightly greater than 3.05 s with SSP (Zhou et al., 2017) due to the 18 extra NoV. With $N = 6$, the average computation time at each CCL iteration is 4.95 s, greater than 4.43 s with SSP (Zhou et al., 2017) due to the 154 extra NoV.

In order to compare the robustness against time-varying delay mismatches, Table 2 shows the maximum delay interval τ for a fixed delay $h_1 = 2$ obtained with both SSP schemes, where it can be seen that larger time-varying delay intervals are obtained for a certain number of subpredictors N with the generalized SSP.

In both cases, the controller gain has been chosen as $K = [-3.75, \ -11.5]$, and the observer parameters $F_{i,j}$, $G_{i,j}$ have been initialized as explained in Remark 1 with $L_i \equiv [-1.4, \ -0.36]^T$, $i = 1, \ldots, N$ and starting values $\tau = 0$ and $h_1 = 0.01$.

6. Conclusions and perspectives

This note has provided a novel control scheme based on sequential subpredictors for input delayed systems under time-varying delay mismatches, where all observer parameters can be designed via LMI and CCL algorithm. The generalized structure of the SSP has been shown to allow larger input delays and better robustness against time-varying delay uncertainties in comparison to other SSP-based methods.

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