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Capilla Tamborero, E.; Calvo Roselló, V.; Gómez Collado, MDC. (2022). Geometrical Analysis of Oval Domes through Architectural and Mathematical Methods. The Case of the Dome of the Camarin of the Virgin of El Puig (Valencia, Spain). *International Journal of Architectural Heritage*. 16(10):1516-1530. <https://doi.org/10.1080/15583058.2021.1896820>



The final publication is available at

<https://doi.org/10.1080/15583058.2021.1896820>

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# **Geometrical analysis of oval domes through architectural and mathematical methods. The case of the dome of the *Camarín* of the Virgin of El Puig (Valencia, Spain)**

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# **Geometrical analysis of oval domes through architectural and mathematical methods. The case of the dome of the *Camarín* of the Virgin of El Puig (Valencia, Spain)**

The study of oval geometry domes encompasses a broad and interesting field of research that can be approached from multiple points of view. This paper shows the multidisciplinary approach developed for the analysis of the geometry of this type of domes from the architectural and mathematical points of view.

This research paper focuses on the geometric study of the dome of the *Camarín* of the Virgin, of the Monastery of El Puig de Santa Maria (Valencia, Spain), an emblematic building of Valencian history and architecture. It begins with a survey by means of a multi-image photogrammetry and other surveying methods obtaining a point cloud that has served as a base for the geometric analysis.

The approach to this analysis is through graphic and mathematical gradients, which allows to determine the graphic layouts and equations that best define the shape of the dome as well as drawing up a geometric construction hypothesis, which is consistent with the data. The conclusion obtained is that the dome is generated by an elliptical plan and five-centered oval arches in the vertical sections and the methodological process developed can applied to other cases.

Keywords: oval domes; *El Puig de Santa María* Monastery; *Camarín Virgen de El Puig*; Juan Bautista Lapiedra; José Vergara; mathematical modelling.

## **1. Introduction**

Oval geometry domes are a widely used typology to cover spaces in numerous buildings in different historical periods. With respect to the geometrical analysis of masonry domes, numerous studies exist on hemispherical domes in addition to arches and domes built from ellipses and ovals, whether in plan or elevation. The study of these types of domes has been developed in multiple publications by different authors from different points of view: historical, typological, geometrical, mathematical, constructive, structural, etc.

From a geometric or mathematical point of view to the elliptical and oval shape we refer the following works. Ragazzo (1995) showed how it is possible to decide, among an infinite number of possibilities, which oval to draw once the measures of the two axes are given. The Euclidean proof of it for eggs and polycentric curves is the subject in Mazzoti (2014a) and the application to ovals and the possibility to classify the consequent constructions given different choices of parameters is the subject of Mazzoti (2014b). Mazzoti (2017) contains a collection of geometrical constructions and mathematical equations on the properties of ovals and the main parameters for managing them. A chapter (co-written with Margherita Caputo) is dedicated to new hypotheses on the project of Borromini's oval dome of the church of San Carlo alle Quattro Fontane in Rome. Another one presents the case study of the Colosseum as an example of ovals with eight centers. Dotto (2001) studies the constructions of four-center ovals and in Mazzoti (2019) a detailed study of ovals with  $4n$  centers is presented. Rosin (2005) analyses both Serlio's constructions of ovals and some of the many possible alternatives and evaluates their accuracy in terms of the oval's approximations to an ellipse. Simona (2005) focuses on Borromini's choice to use ovals according to the tradition of architecture treaties. Hatch (2015) speaks, among other things, about Kepler's methodology and interpretation of the universe as a source of inspiration for Borromini's divine geometry. Gómez-Collado, Calvo and Capilla (2018) approach the mathematical modeling of oval arches of  $n$  centers and present an analytical study of their geometry given the mathematical expressions of the elements that define them and then apply these results to the Orleans Bridge over the River Loire and the Neuilly Bridge over the River Seine. Piemonte et al. (2018) present an interesting methodology for planar development of a frescoed dome with an oval plan, from a geometric study, carried out with a photogrammetry and laser scanner; the object

of study is the dome of Pisa Cathedral. Cipriani, Fantini and Bertacchi (2020) make a comparison between the two domes of Hadrian's Villa in Tivoli with 3D surveys.

Some authors focus on structural analysis in addition to constructive, geometrical and historic aspects. Feizolahbeigi, Lourenço et al. (2021) highlight the role of geometry, proportion and construction techniques in seismic behavior of bulbous discontinuous double shell domes (DDDs) of 16<sup>th</sup> to 18<sup>th</sup> century, through the study of four special cases in central Iran by means of numerical analysis. Bennati et al. (2020) focus their research on the structural response of the oval-shaped dome of Pisa Cathedral from a multidisciplinary study including the use of *Mathematica* software. The equilibrium problem of the circular, pointed and elliptical arches that exist in numerous historic masonry buildings and bridges, is studied in Aita, Barsotti and Bennati (2011). In Galassi et al. (2017), the Eddy-Lévy graphical method is analysed and proposed as an assessment tool for masonry domes. Huerta (2004) studies the traditional calculation of masonry arches and vaults and its structural behavior and in Huerta (2007) the study focuses on the history, geometry and mechanics of oval domes. Chiorino et al. (2008) develop modeling strategies related to the structural reliability for the Sanctuary of Vicoforte, which has the largest elliptical dome ever built. Brencich, Gambarotta and Ghia (2014) analyze the mechanical response of the dome of the Basilica of S. Maria of Carignano; they also study the crack pattern and perform FEM analyses and highlight the importance of the detailed historical research in order to understand many problems. Cavalagli and Gusella (2015a) show a structural investigation on the ogival masonry domes as well as the contributions in the design of masonry domes of the architects Carlo Fontana and Bernardo Vittone through an analytical approach. Cavalagli and Gusella (2015b) show the application of the graphic statics to masonry domes, through three different methods applied to the dome of the

Basilica of Santa Maria degli Angeli in Assisi (Italy). Valibeig et al. (2017) analyze, simultaneously, the structural elements and the geometric shapes of double-shell domes. Deshpande, Amalkar and Jagadish (2012) carry out an experimental study on ellipsoidal domes built with masonry using molded table bricks and reinforced concrete (R.C). Pavani and Rossi-Costa (2019) and Pavani (2020) highlight the importance of ancient and modern Geometry and Mathematics in the construction of buildings with arches and domes throughout the centuries. The latter one also includes a mathematical study of the equilibrium properties of the catenary versus the parabola used as generating curves to lighten the structural load.

Several authors address studies related to Spanish oval domes. Gentil (1994, 1996) deals with the construction of oval and elliptic trace in architecture and analyses, in the second article, particularly, the geometry of the dome of the Chapter Hall of the Cathedral of Seville from a photogrammetric survey. López (2011) studies oval layouts for any given proportion and applies them to the ovals in the vaults of *El Escorial*. García Jara (2008) collects the historical antecedents of the domes from Antiquity to the 19th century; including an analysis and 3D modeling of domes of different types (spherical intrados, ellipsoidal intrados and oval base) and also provides a catalog of domes of the province of Alicante. García Jara (2010) deals with the oval and elliptical trace by means of an exhaustive graphical study. In the particular case of Valencian domes, we will mention some authors and their works. Soler (1995) addresses the study of domes in modern Valencian architecture from the sixteenth to the eighteenth centuries. Soler-Verdú and Soler-Estrela (2015) describe the typology of Valencian tile vaults in the 18<sup>th</sup> century. The study the oval dome of the *Basilica de la Virgen de los Desamparados* of Valencia was carried out by Huerta (2002, 2012a, 2012b); Capilla and Calvo (2014) and Calvo, Capilla and Navarro (2020). Huerta approaches his study

from the structural analysis of the masonry domes applied to this case. The other authors carried out an interdisciplinary, architectural and mathematical study, also using *Mathematica* software, to determine the shape of the dome.

Our main contribution with this research is to provide a methodology for the study and investigation of the geometry of oval domes, from an interdisciplinary point of view, architectural and mathematical. This particular methodology was developed by analyzing the specific case of the dome of the *Camarín* of the Virgin of the church of Santa María de El Puig, considered one of the most emblematic building of Valencian history and architecture. It may, however, be easily extrapolated to the study of any other domes.



Figure 1. Dome of the *Camarín* of the Virgin of the church of *Santa María de El Puig*.  
Font: Authors

The dome that covers the *Camarín* of the Virgin was built between 1766 and 1784 (Fig. 1). The main objective has been to determine the shape, the spatial geometry

and the design process of the dome, since we have not found any work that contributes a precise analysis of its geometry. This geometry has been obtained through architectural and mathematical analysis. The architectural one has been fundamentally graphic, also supported by constructive aspects. To do this, we have started with point clouds obtained by topographic and multi-image photogrammetry means survey methods. The results of this proves that it is definitely an oval geometry dome, both in plan and elevation. These same results have been obtained from the study of the geometry of its plan, the cornice and the horizontal and vertical sections (both longitudinal and transversal). In the Results and discussion section, the exact geometry and whether it is generated from ovals and/or ellipses are described, as well as the methodological process developed.

### ***1.1. The monastery of El Puig de Santa María***

The monastery, an emblematic building of Valencian history and architecture, was founded by King Jaime I *the Conqueror* in 1238 and declared an artistic-historic monument in 1969 (Fig. 2).



Figure 2. Monastery of *Santa María* of El Puig. Font: Authors.



The church is dedicated to the Virgin of *Santa María*. King Jaime I ordered the building of a sanctuary in the place where in 1237, the image of the Virgin was discovered by Saint Pedro Nolasco, founder and Grand Master of the *Orden de la Merced* and declared the Virgin of El Puig the patron saint of the old *Kingdom of Valencia*. In 1300 the sanctuary collapsed. The author Fray Joaquín Millán (1968) considers that the old sanctuary was intentionally demolished almost in its entirety as it became too small due to the increase in devotees to the Virgin and because the new designed building did not adapt to the primitive church. According to Domínguez (1992, p. 143), various Mercedarian historians such as Fathers Boyl, Nolasco Martín, Anselmo Dempere..., among others, affirm that in the year 1300 the sanctuary simply collapsed and the causes were not mentioned, although there is no written record of this event. However, according to the information recently provided to the authors by the chronicler of El Puig de Santa María, Julio Samuel Badenes Almenara, in the year 1551, the chronicler Pedro Antonio Beuter in the *Segunda parte de la Coronica general de España, y especialmente de Aragon, Cathaluña y Valencia*, printed in Valencia, Folio XC, mentions the causes of the construction of the new church on which Jaime I had built. Literally, Beuter says: “And as some arcades fell due to the water, Don Roger de Loria son of Doña Margarita de Loria (...), rebuilt the entire Yglesia (which is the one we see today), year one thousand three hundred; and mother and son were buried in a chapel of this Yglesia” (own translation). A few years later, in 1591, Fray Felipe de Guimerán, in his work *Breve Historia de la Orden de Nuestra Señora de la Merced* (page 282), refers to the old age of the old church and attributes the construction of a new one to the *Countess Doña Margarita de Lauria*. He literally says: “that having come the first church that King *Don Jayme* built for us at a very old age, the most

illustrious Condessa, who with great beauty and greatness, has built the one we see today.” (own translation).

According to Beuter (1551, Folio XC), as already mentioned above and other more recent authors as Benito (1983, 55) or Garín (1986, 427), Admiral Roger de Lauria (†1305), in the fourteenth century, began the construction of a new church. Its construction was continued by his wife *Doña Saurina de Entenza* and his daughter Margarita. In the 15<sup>th</sup> century the primitive conventual building was expanded and at the end of the 16<sup>th</sup> century, in 1590, works began in Renaissance style inspired by the Escorial and the Valencian Monastery of San Miguel de los Reyes which would give it its current configuration, with some interventions in the following centuries that would modify some of the spaces (Benito 1983, 58-64). The dome of the Virgin, located behind the presbytery, was built between 1766 and 1784 under the supervision of the architect Juan Bautista Lapiedra (Devesa 1982, 28), and was covered by a decorated dome with frescos made by the Academist painter José Vergara (1726-1799). In 1835, at the root of the *Desamortización* by Mendizábal (confiscation of church property), the monastery was abandoned and experienced a progressive deterioration. In 1842 it was given to the Town Hall and since then has housed different uses. In 1948 it was returned to Mercedarian monks.

There are various studies regarding the monastery and the church, among others, Millán (1968); Devesa (1982); Benito (1983); Domínguez (1984, 1992); Garín (1986); Badenes (2010, 2017); Capilla et al. (2018) and some Final Degree Projects (Pastor 2018; Plaza 2018; Sanchis 2018; Magán 2019).

### ***1.2. The Camarín of the Virgin. Description***

The Camarín is behind the presbytery. It is delimited by the east façade of the monastery and by two internal rooms that overlook the church. In Fig. 3, on the left, its

location can be seen in a plan from 1970 archived in the El Puig Monastery Archive. No name appears on this map but rather a signature, which we identify as that of Juan Segura de Lago, the architect who carried out some interventions in the 20th century.

The space where the *Camarín* of the Virgin is located is of trapezoidal plan, it is covered by a dome and has pendentives in the four angles and oval arches in each one of the walls. A detailed description and interpretation of these exceptional frescos, by José Vergara, that decorate all these elements is described by Badenes (2010).

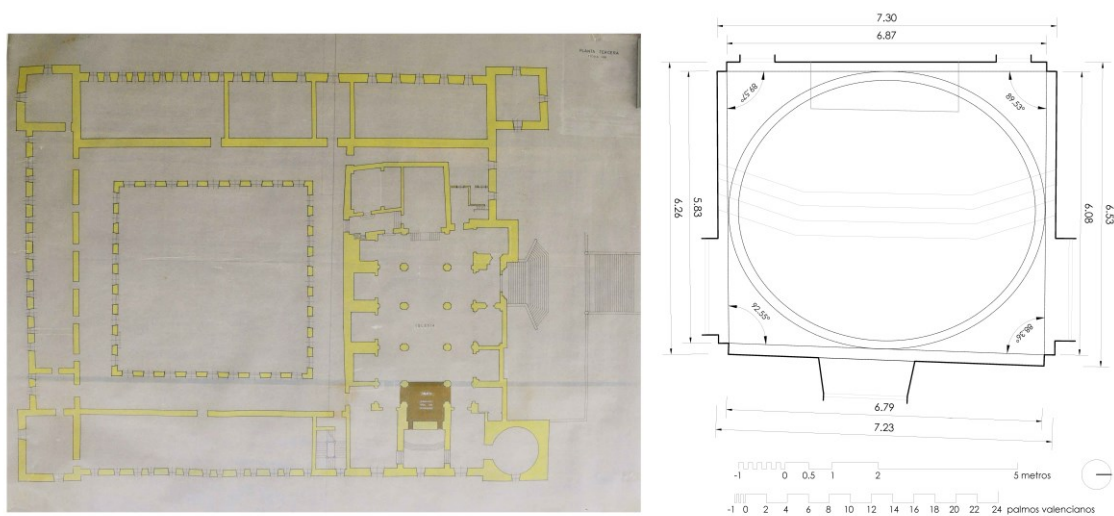


Figure 3. On the left, Monastery plan, 1970. Juan Segura de Lago. Font: El Puig Monastery Archive (AMP). On the right, *Camarín* of the Virgin plan. Font: Authors.

The dimensions of trapezoidal plan of *Camarín* are: longitude on the east and west sides, 7.23 and 7.30 meters, respectively. The North and South sides measure 6.53 m and 6.26 m, in this order. The exact angles that form together the interior walls have been determined from the survey carried out and described in the *Methodology* section; they are:  $89.53^\circ$  and  $89.57^\circ$  between the west wall and the north and south enclosures, respectively; practically  $90^\circ$ . The greatest difference is in the angles that form the east wall with the north ( $88.36^\circ$ ) and with the south ( $92.55^\circ$ ). Even though the angles are

almost 90°, the length of the sides do reflect a difference in measurements that should be considered in the geometric analysis (Fig. 3, on the right).

The dome has an oval geometry, in the plane and as well as elevation. The transition from quadrangular to oval or elliptical of the dome is done through the pendentives located in the four angles. These are delimited by the cornice of the dome and the oval arches that are on each one of the walls.

It is logical then to consider that to build the curve of the horizontal springing plane of the dome – hidden by the cornice- it is drawn in rectangle. Given the trapezoidal form described in the plan of the *Camarin*, an adjustment is needed between both geometries, which is solved through the cornice.

## **2. Material and Methods**

To determine the geometry of the *Camarin* of the Virgin and the dome that covers it, traditional and direct surveys were used as well as topographical instruments - total station- and multi-image photogrammetry.

As a first approach, data was gathered *in situ* by means of dimensional sketches that show the general dimensions of the *Camarin* and its architectonic elements. These data were supplemented with the survey carried out with the total station; with it, 526 points were taken to obtain dimensional information in plan and height (Fig. 4, on the left).

The topographical support carried out by the total station was equally important to position and scale the point cloud obtained through *multi-image photogrammetry* using the *Photoscan* program by *Agisoft* (Fig. 4, centre and right). The photogrammetric survey was made from 483 pictures taken with a NIKON D40 camera. Their resolution is 3,008x2,000 pixels. The point cloud obtained for the entire *Camarin* is almost 4,000,000 points and that of the dome is 708,720 with an approximate density of about

2-3 points/cm<sup>2</sup>. Multi-image photogrammetry provides a high number of points for which the exact correspondence to specific points in the real model is not known a priori. The total station gives us a more limited number of points, but these correspond to specific points chosen from the real model, so the topographic support made with the total station was used for the orientation and scaling of the point cloud and also to define the geometry of the cornice more closely.

For the geometric analysis, from the point clouds, the *AutoCAD* program of *Autodesk* was chosen, and *Mathematica* software was used for the mathematical analysis.



Figure 4. Cloud of 526 points from *Camarín* chapel taken with a total station (left). Dispersed point cloud (center) and point cloud with texture (right) of *Camarín* obtained with *Photoscan* program of *Agisoft*.

The 708,720 points of the dome were imported to *Excel* and a refinement was made, reducing the initial quantity to one-eighth resulting on 88,590 points. From the cornice 29,604 points were obtained. The cornice starts at a height of 6.97 m and ends at 7,54 m. The initial data that is available of the dome starts at 7.77 m and reaches 9.46 m in its highest point. Between 7.54 m and 7.77 m data is not available due to the shade created by cornice.

### 3. Results and discussion

We present here the scientific method developed for the realization of geometric analysis from architectural and mathematical methods of point clouds obtained through the architectural survey carried out with the modes, systems and measurement techniques previously described. The analytical method, in both cases, has been developed using different procedures and software. In all cases, the objective has been the same: to obtain a methodology to determine the shape, the spatial geometry and the design process of any dome. Throughout the process, the interaction between the partial results obtained with each analysis method has been essential for achieving the final results. The method of comparative analysis has been, therefore, key in developing the research presented.

The comparative analytical method has been developed from the case study of the dome of the *Camarin* of the Virgin in the church of the monastery of El Puig. Infinite curves were obtained from the point clouds that make up the shape of the dome. It was necessary to choose the curves that best suited our study. It was here that the relationship between mathematical and architectural analysis was crucial. With the architectural study, based on a first stage on a fundamentally graphic analysis, we were able to determine what type of curve is closest to the point cloud. The mathematical analysis, enabled us to calculate distances and to obtain the optimum parameters which define these curves, demonstrating with data, the validity of this hypothesis.

In the mathematical procedure, the point clouds were processed in matrices within *Excel* files in order to organize them systematically into groups at different heights. The *Excel* files were used to study which of the curves adapted better to the geometry of the dome dividing it into horizontal and vertical planes to then study the surface that best approximates the geometry.

The number of these groups and the height of courses were different in both graphical and mathematical methods, since the data and the operations were performed differently. With the graphical method, each course is worked separately; however, the mathematical method can be programmed, so fewer groups were made in the first method than in the second one.

The first step in data management was to center it. Since the central axis is a hypothetical straight vertical line in the air, we took the average of the values of the centers in each horizontal plane. As a result, we observed that the centers move slightly as a function of the height (but no more than 2-3 cm on both axes). Taking this observation into account we have corrected the point cloud appropriately and made our calculations.

### ***3.1. Graphical analysis of the geometry of the dome***

Prior to determining the geometry of dome in plane, a study of the cornice is carried out (Fig. 5) as well as the situation with the axes of both with respect to the trapezoidal plane (Fig. 3, on the right).

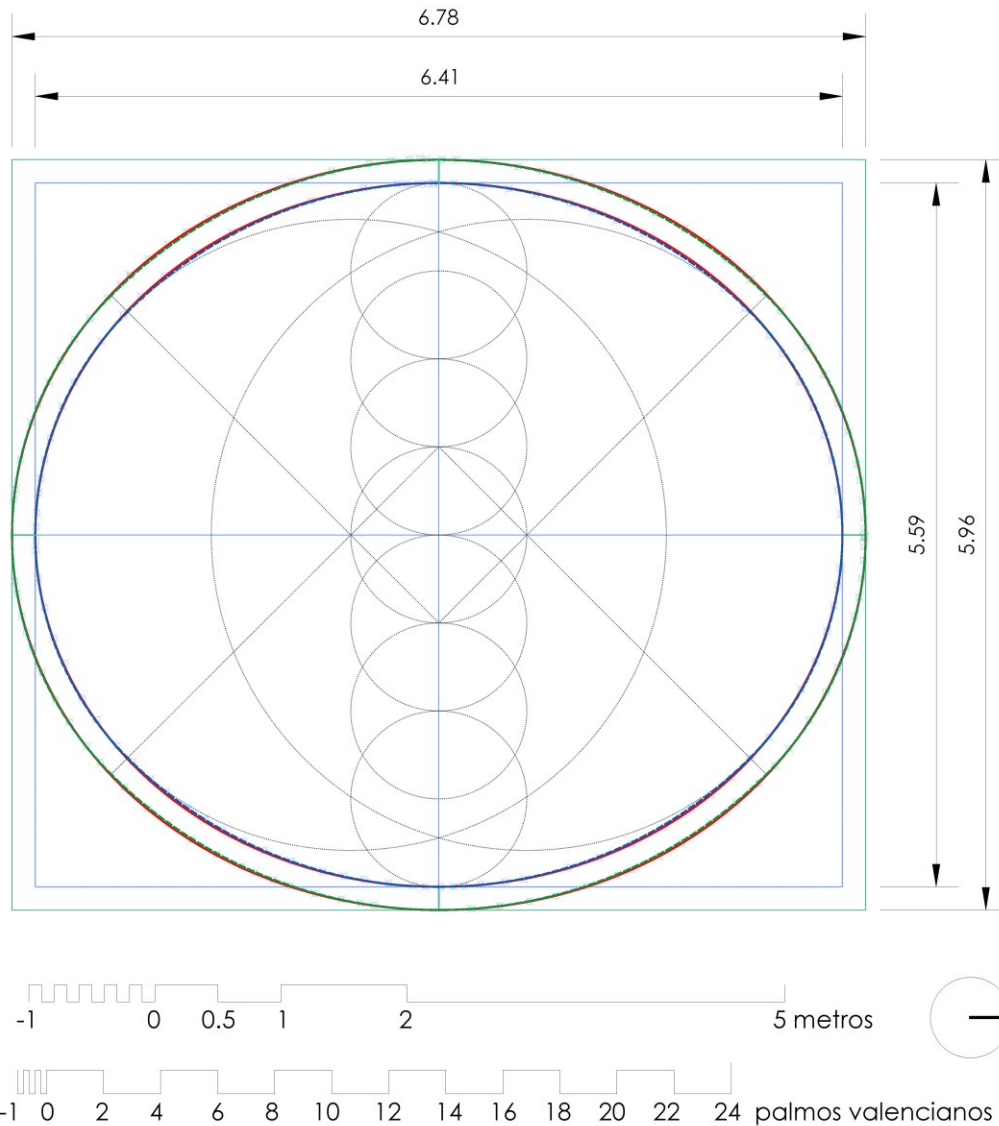


Figure 5. Graphical representation of the ellipses (in red and solid line) and the four-centered ovals (dashed) obtained corresponding to the upper moldings (in blue) and lower (in green) of the cornice, superimposed on the points obtained with the total station projected on a horizontal plane (in magenta).

It can be deduced from our study that the plane of the cornice is elliptical, although it is close to a four-centered oval (Fig. 5). The major axis of the ellipse is parallel to the west wall, where the *Camarin* is placed overlooking the presbytery. The projection on the horizontal plane of the points of the upper molding –the most interior– coincides with an ellipse of axes 5.59 m (east–west) and 6.41 m (north–south) and the



lower molding – the most exterior– with an ellipse of axes 5.96 m (east-west) and 6.78 m (north-south) (Fig. 3, on the right, and Fig. 5).

It was not possible to carry out a test hole to determine what the dome was composed of. However, based on other studies on contemporary Valencian domes such as those of Soler (1995); Huerta (2012a, 2012b) or Soler-Verdú and Soler-Estrela (2015), we can presume that it is a thin-tile vault consisting of one leaf with two-layers of brick arranged by “breaking joints”, that is, the joints in one course shall not coincide with those in the preceding course. The thickness of this brickwork, we can suppose is, approximately, half a Valencian palm (11.325 cm) considering that the thickness of the brick is 40 mm and the layers of plaster are of 10 mm (1 Valencian palm is the unit of measure applied at the time of construction and it is equivalent to 22.65 cm). To this we must add the thickness of the frescoes, approximately 20 mm.

Although the springing of the dome is not visible, we surmise based on the data that it should be level with the upper molding of the cornice. The intrados of the dome are set back approximately 26 cm – a little more than a Valencian palm - from the upper molding of the cornice and almost flush with the interior face of the oval arches of the walls. The extrados is almost flush with the inner face of the walls that delimit the *Camarín*. The height of the start of the dome is 7.54 m from the ground. The highest point of the dome is 9.46 m being, therefore, the total height or rise of the dome is 1.925 m equivalent to 8½ Valencian palms.

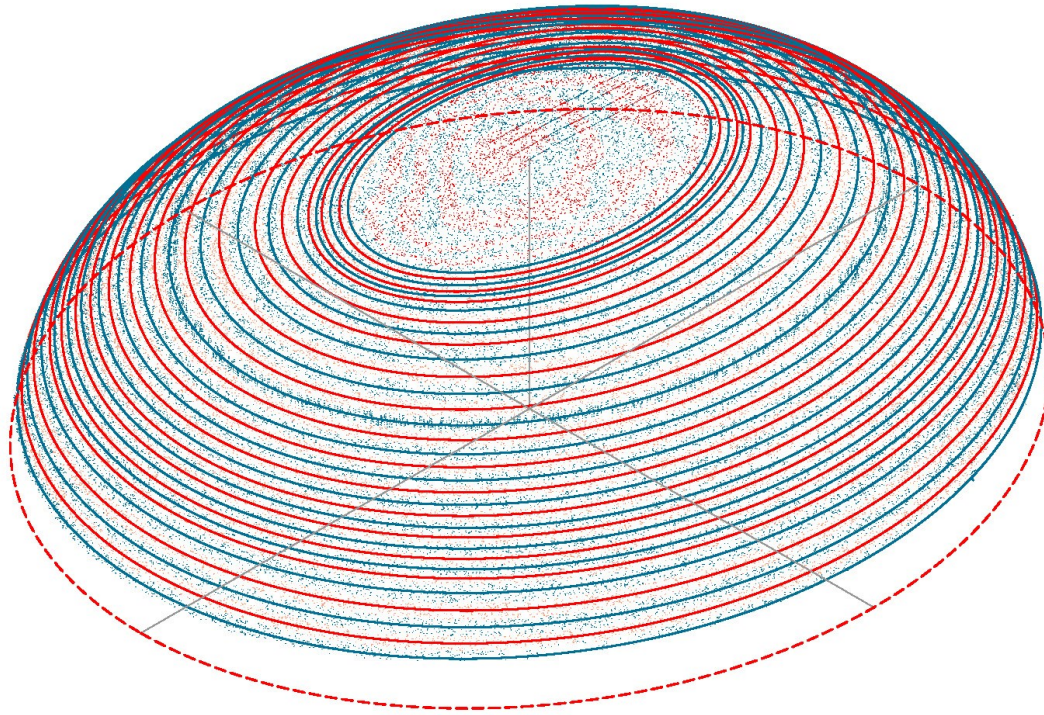


Figure 6. Graphical representation of the point cloud and the ellipses obtained by projecting the points grouped in the different rows; with dashed line the ellipse deduced on the starting plane.

For the graphic analysis of the geometry of the dome in horizontal planes, the grouping the points in rows is carried out. The heights of these rows are:

- 10 cm, from where there is data until 80 cm above the springing plane.
- From there, progressively, while it gained height, several groups of 5 cm, 2,5 cm and 1 cm.

The points of each row were projected onto the horizontal plane centered on each row. From 1.70 m in height, there are deformations in the dome, and it was verified that in the upper area of the closure of the dome, there was a flattening or reduction of the height on both sides of the axis (Fig. 6). This could be accounted for due to the existing mechanical deformations or rather we believe due to the difficulty in

building the closure of the dome especially since it is lowered; but the analysis of these elements exceeds the scope of this paper.

Once the analysis of all the rows was carried out, it can be confirmed that ellipses as well as four-centered ovals are pretty close to the projected point cloud (Fig. 5). However, the ellipse is always the best approximates as clarified in the mathematical analysis and in Table 1. The semi-axis of the first ellipse reliably defined –at 35 cm from the springing plane- are 3.42 and 3.00 m in north-south and east-west direction, respectively. They could also be four-centered oval arches, but we have opted for ellipses. The mathematical analysis confirms the dimensional differences (Table 1) and therefore justifies this decision. The dimension of the axes in the springing plane, (hidden by the cornice) has been determined from the construction of the central vertical sections, the result being 6.12 m (27 Valencian palms the minor axis and the 6.91 m of the major axis (30.5 Valencian palms) (Fig. 7).

In the vertical sections, initially, the points were grouped into sections 10 cm thick, both longitudinally and transversely. In a first approximation, the geometry in several of the rows was studied, but once it was verified that in plan the curve that most closely approximated the point cloud was the ellipse in any horizontal plane, only the geometry in the central, longitudinal and transversal rows was studied.

The geometry obtained in the two central vertical sections corresponds to five-centered oval arches, that best approximate the projected point cloud on the central vertical plans and follow clear geometric laws. In figure 7 the graphic construction can attest to this. An overlapping in red has been applied with broken lines, the possible ellipse for both planes in each vertical section.

In the longitudinal section, the horizontal axis or the major axis corresponding to the arch span as already mentioned, measures 30.5 Valencian palms (6.91 m) and the

vertical axis or rise, 8.5 Valencian palms (1.925 m). The data of the arches of the circumferences that compose the five-centered oval arch are: the central circumference has a radius equal to 6 times the rise (11.55 m); the minor circumference has the center in the horizontal at a distance from the origin equal to the rise and radius 1.53 m; the intermediate circumference tangent to both, has its center in the bisector of the angle that forms the vertical axis with the line that joins the center of the previous circumference and its radius 3.49 m (Fig. 7, on the left).

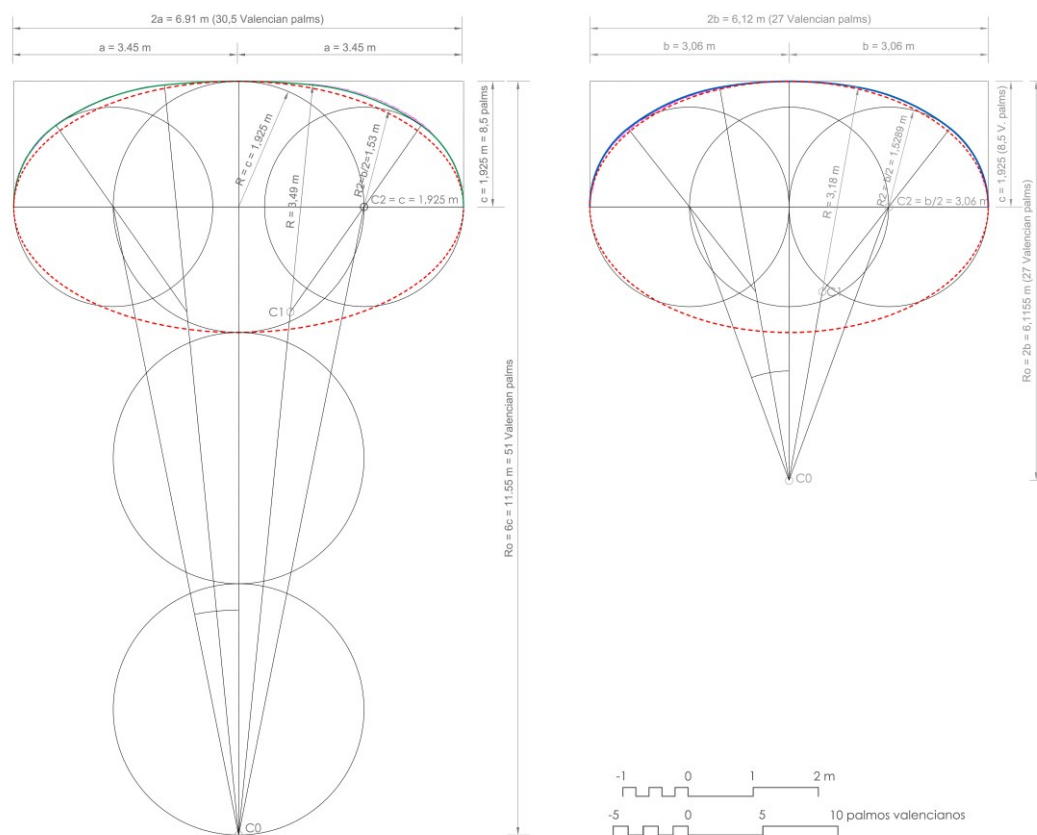


Figure 7. Graphic representation of the oval arch (in blue) and the geometric layout that most closely approximates the points of the longitudinal section, on the left, and transversal vertical section, on the right. Drawing of the ellipse with the same axes (in red and dashed). Overlay of both layouts on the point cloud (magenta).

In the transversal section, the major axis measures 6.12 m (27 Valencian palms), as already advanced. The vertical axis coincides, naturally, with the one in the

transversal section, that is, it measures 1.925 m (8.5 Valencian palms). The central circumference has a radius equal to the length of major axis and the minor circumference has its center in the horizontal axis in the half of each semi-axis and therefore its radius is half of the semi-axis (1.53 m). The intermediate circumference, tangent to both, has its center in the bisector of the angle that forms the vertical axis with the line that joins the two centers of the previous circumference and its radius is 3.18 m (Fig. 7, on the right).

In the mathematical analysis, the distances to the data from the point cloud are calculated between those ovals and the corresponding ellipses, which will confirm our construction hypothesis.

With regards to the spatial generation of the dome, we have seen that in plan we could consider both ellipses as well as the four-centered ovals, while in elevation in both central vertical sections of the five-centered oval arches are the curves that most approximates the point cloud (Fig. 8).

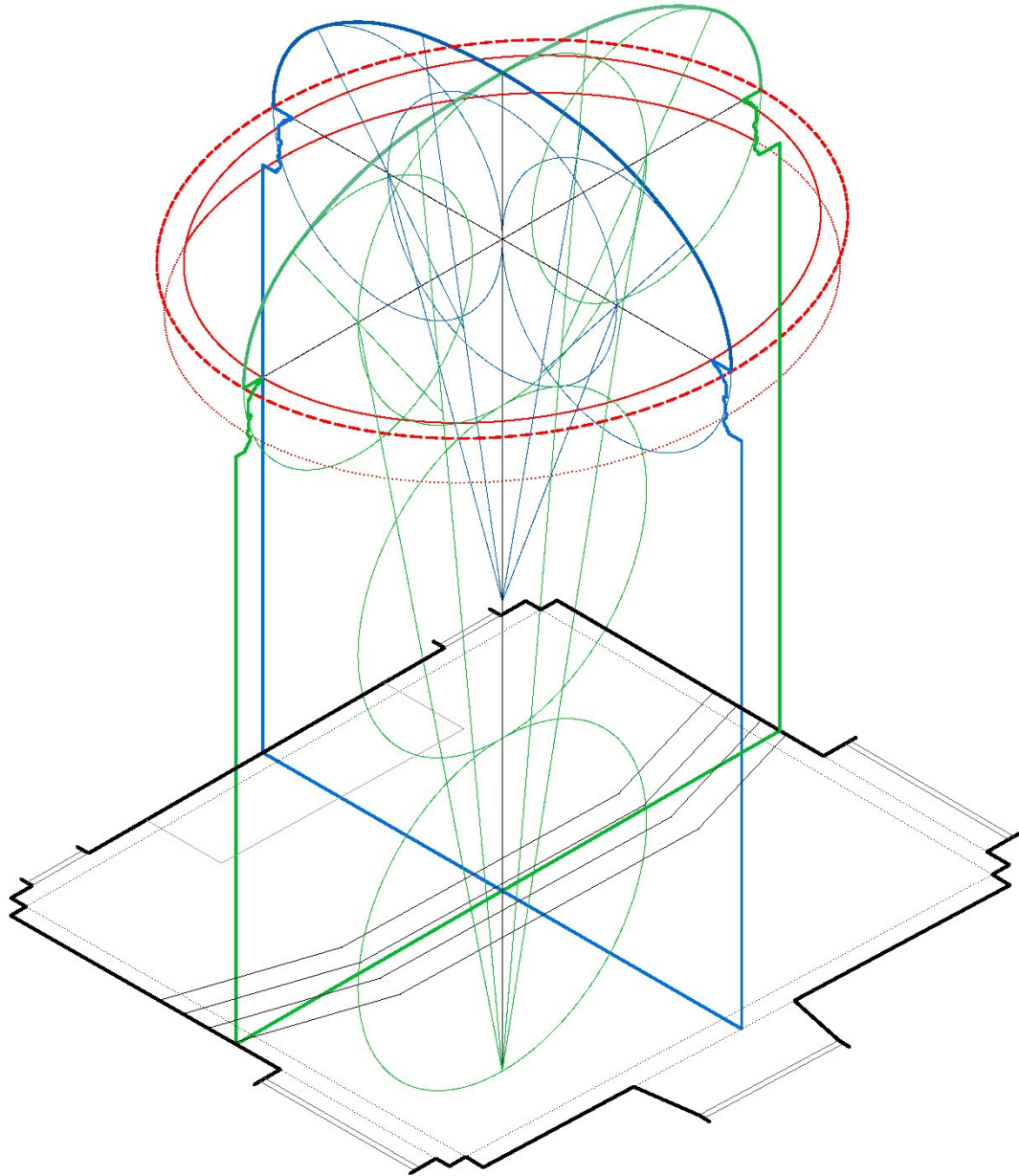


Figure 8. Representation of the spatial geometry of the dome with the vertical sections (longitudinal, in green and transversal in blue), the ellipses of the lower and upper molding of the cornice (red) and the ellipse in the horizontal starting plane (red dashed) with the axes deduced from the graphical construction of the vertical sections.

We believe that the author used ellipses to build the dome in plan and the oval arches in elevation for the following reasons:

- The ellipse is the curve that best approximates the point cloud in each horizontal plane. In the central vertical planes, however, it is the five-centered oval arches that are best suited. The reason could be that the ellipse is easy to build in plan, but not in elevation.
- Drawing an ellipse on the ground is easy with the Gardener's method, either on the ground or on a scaffold. Spatially, in this case, it is a simple construction, and since the dimensions of the axes are known, there is only one possible ellipse. In this case, the dimensions of the axes are given by the five-centered oval arches that define the geometry of the central vertical sections (Fig. 8).
- The plan layout of a four-centered oval with all of its centers within the outline of the same oval it is also simple. Spatially, the construction would be simplified if these centers were all in the same vertical, but we have verified that they are not, therefore, the geometric construction of the dome would be complicated.

### ***3.2. Mathematical analysis of the geometry of the dome***

For the mathematical analysis, as was carried out for the geometric study, the cornice and the dome were dealt with separately. The procedure followed is similar to that of Calvo, Capilla and Navarro (2020).

Firstly, an analysis of the horizontal sections was performed. In the dome, the total number of points was divided into 66 groups at 3cm height intervals in order to study in each group the ellipse and oval that best approximate the point cloud. The cornice was subdivided into 31 groups, each one of 2 cm in height. Figure 9 shows the point cloud of the dome in red and the section produced by a horizontal plane corresponding to group 26, in blue.

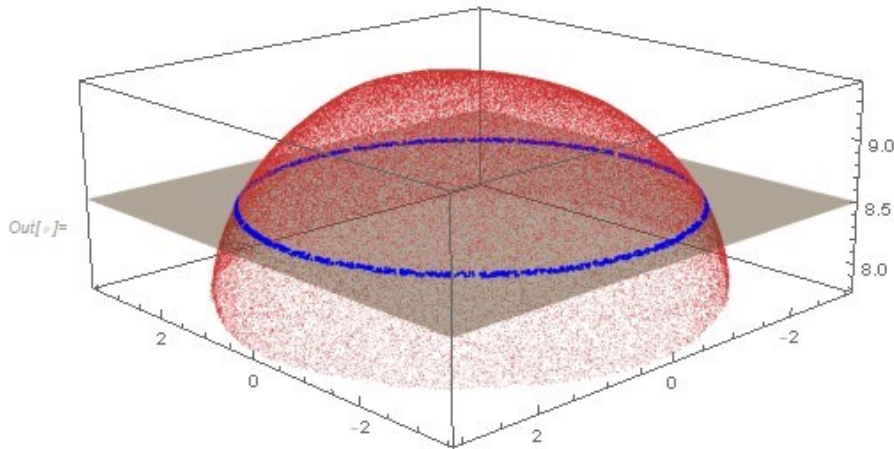


Figure 9. Point cloud of the dome (in red) and horizontal section of group 26 (in blue).

Once an initial analysis of the data was completed, a study was carried out of the horizontal curves to verify whether they were ellipses or ovals. Firstly, the approximation was proposed using ellipses. In Excel, for each group, the furthest points were calculated with respect to the  $x$  coordinate and also with respect to the  $y$  coordinate. The distances between these points define the major and minor semi-axes of the ellipse that we consider as the first approximation to that point cloud. Interactive graphics were programmed with *Mathematica* and the ellipse that best approximates the point cloud was also considered. The search for this ellipse was carried out using the method of least square, as is described in Capilla and Calvo (2014). We found that there was little difference between the two ellipses. As an example, figure 10 (top left) shows the graph obtained for group 15 of the cornice (at 7.25 m in height) where the point cloud is represented in blue, the first ellipse considered, of semi-axis major  $a=3.374$  m and minor  $b=2.961$  m centered in the origin in red and the second ellipse considered, in green, centered in  $(0, 0.01)$  and semi-axis 3.37 m and 2.96 m. Moreover,

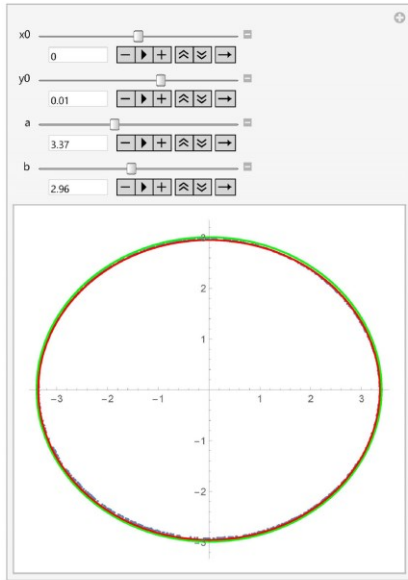


it is shown also the graph obtained for group 26 of the dome located at 8.51m in height and the ellipse that approximates with semi-axis 3.09 m and 2.69 m (Figure 10, bottom left).

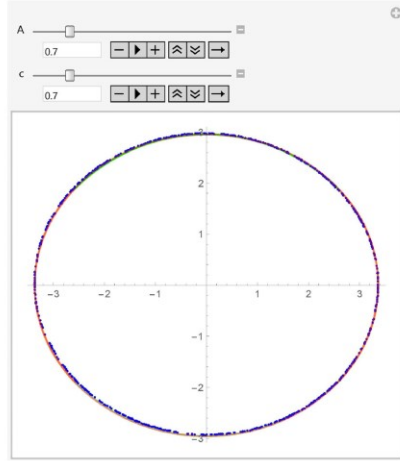
To study the approximation of the point cloud by ovals, we programmed a subroutine in *Mathematica* creating an interactive graph where, once the points of each group were represented, the different centers that define the circumferences of the oval could be manipulated to study those that best adapt. The condition given *a priori* was that the half-span and half rise of the ovals coincide with the major and minor semi-axes of the ellipse. For different groups of points, a study was carried out of possible arches that satisfied the condition and after analyzing the distances of the cloud points to these arches, the hypothesis of considering an oval of four centers was chosen, in which, if  $A (\pm a, 0)$  and  $B (0, \pm b)$  are the centers of the circumferences that make up the arch, so  $a = b$ . In this case, the relation of the coordinates of the centers with the rise  $f$  and the half span  $l$  is:

$$a = b = \frac{l-f}{2-\sqrt{2}} \quad (1)$$

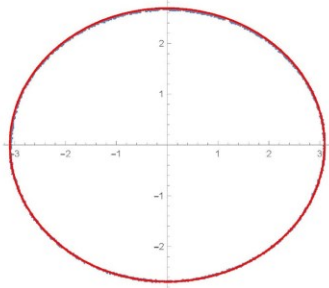
In the horizontal sections, the ellipses as well as the four-centered oval are good approximations to the point cloud. From a mathematical point of view, it is not surprising that this happens because in the case in which the imposed conditions are met, the oval and the ellipse are so close, they are difficult to differentiate. In Fig. 10, top right, the Mathematica's graph shows the point cloud of group 15 of the cornice, with the oval arch of four centers  $(0, \pm 0'7)$ ,  $(\pm 0'7, 0)$ , span 6'76 m and sagitta rise 2'97 m and the ellipse that approaches it; and on the lower right, the ellipse and four-centered oval in group 26 of the dome.



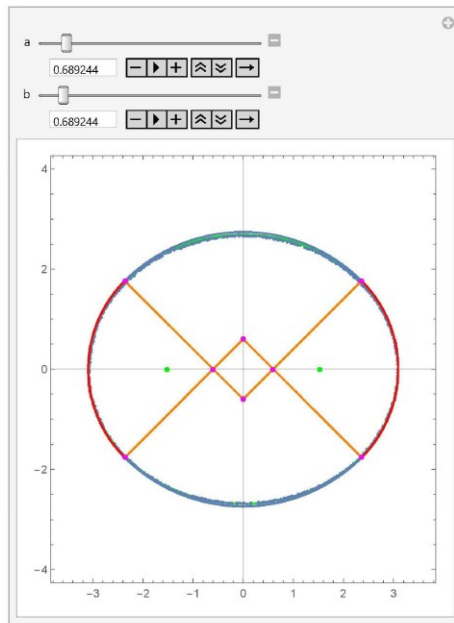
Graph of the point cloud of group 15 of the cornice and of two ellipses that approximate it.



Graph of the point cloud of group 15 of the cornice with the oval arch of four centres and the ellipse that approaches it.



Graph of the group 26 of the dome and the ellipse that approximate it.



Graph of the group 26 of the dome and the ellipse and four-centred oval.

Figure 10. Mathematica's graph for the group 15 of the cornice and group 26 of the dome comparing the point cloud with the ellipse and the four-centered oval.

With regards to the dome, Table 1 shows the maximum distances of the data in each horizontal plane to the ellipse with semi-axes  $a$  and  $b$  obtained from these data and to the four-centered oval described in the methodology. It can be seen that the ellipse better approximates the data.

<i>Grup</i>	<i>Height</i>	<i>Major semi-axis</i>	<i>Minor semi-axis</i>	<i>Number of points</i>	<i>max. ellipse distance</i>	<i>max. oval distance</i>
1	7,7679	3,3997	2,9513	312	0,1089	0,1189
2	7,7901	3,4205	2,9325	491	0,1046	0,1377
3	7,82	3,4398	2,9718	666	0,1145	0,0980
4	7,85	3,4432	3,0022	840	0,1352	0,1030
5	7,8801	3,4169	3,0089	925	0,0972	0,0759
6	7,91	3,4069	3,0041	1023	0,0745	0,0906
7	7,94	3,4019	2,9923	1126	0,0697	0,0921
8	7,9701	3,3929	2,9834	1191	0,1153	0,1303
9	8	3,3850	2,9741	1227	0,1087	0,1247
10	8,03	3,3741	2,9669	1235	0,1322	0,1540
11	8,0601	3,3645	2,9575	1231	0,0487	0,0779
12	8,09	3,3522	2,9475	1237	0,0254	0,0559
13	8,12	3,3395	2,9365	1262	0,0244	0,0592
14	8,1501	3,3269	2,9228	1263	0,0242	0,0601
15	8,18	3,3149	2,9145	1299	0,0309	0,0566
16	8,21	3,3011	2,8982	1319	0,0318	0,0533
17	8,2401	3,2858	2,8836	1331	0,0301	0,0514
18	8,27	3,2693	2,8653	1362	0,0321	0,0613
19	8,3	3,2500	2,8496	1394	0,0318	0,0502
20	8,3301	3,2299	2,8296	1418	0,0336	0,0582
21	8,36	3,2111	2,8078	1442	0,0354	0,0554
22	8,39	3,1895	2,7878	1448	0,0368	0,0596
23	8,42	3,1689	2,7652	1479	0,0404	0,0573
24	8,4501	3,1439	2,7425	1485	0,0403	0,0586
25	8,48	3,1196	2,7160	1523	0,0399	0,0665
26	8,51	3,0945	2,6907	1561	0,0446	0,0671
27	8,5401	3,0695	2,6646	1554	0,0493	0,0613
28	8,57	3,0435	2,6343	1555	0,0473	0,0653
29	8,6	3,0136	2,6034	1568	0,0476	0,0764
30	8,6301	2,9865	2,5713	1564	0,0421	0,0699
31	8,66	2,9584	2,5380	1589	0,0423	0,0773
32	8,69	2,9316	2,5019	1609	0,0480	0,0865
33	8,7201	2,8994	2,4685	1615	0,0472	0,0782
34	8,75	2,8601	2,4348	1621	0,0521	0,0808
35	8,78	2,8295	2,3978	1655	0,0550	0,0756
36	8,8101	2,7944	2,3575	1664	0,0531	0,0756
37	8,84	2,7532	2,3138	1695	0,0542	0,0776
38	8,87	2,7097	2,2730	1718	0,0585	0,0791
39	8,9001	2,6658	2,2254	1754	0,0592	0,0788
40	8,93	2,6135	2,1802	1784	0,0660	0,0788
41	8,96	2,5641	2,1266	1210	0,0505	0,0863
42	8,98	2,5277	2,0941	1222	0,0540	0,0816
43	9,0001	2,4899	2,0569	1218	0,0612	0,0850
44	9,0201	2,4502	2,0161	1205	0,0575	0,0983
45	9,0401	2,4147	1,9768	1195	0,0601	0,0975
46	9,0601	2,3729	1,9392	1191	0,0615	0,0944
47	9,0801	2,3329	1,9018	1194	0,0617	0,0955
48	9,1	2,2931	1,8613	1216	0,0611	0,0892
49	9,12	2,2471	1,8190	1221	0,0666	0,1007
50	9,14	2,2049	1,7720	1206	0,0639	0,1034
51	9,1601	2,1613	1,7269	1197	0,0759	0,1135
52	9,1801	2,1155	1,6808	1191	0,0795	0,1183
53	9,2001	2,0642	1,6292	1235	0,0834	0,1308
54	9,2201	2,0089	1,5772	1284	0,0917	0,1353
55	9,2401	1,9516	1,5180	1313	0,0947	0,1496
56	9,26	1,8894	1,4590	1378	0,1060	0,1552
57	9,28	1,8265	1,3923	1397	0,1039	0,1665
58	9,3	1,7571	1,3248	1394	0,1150	0,1746
59	9,3201	1,6847	1,2329	1377	0,1137	0,2209
60	9,3401	1,6082	1,1461	1430	0,1325	0,2417

Table 1. Maximum distances –in meters- between the point cloud, the ellipse and the four-centered oval in each horizontal group of the dome.

These distances are calculated from each point to the corresponding curve, oval or ellipse. Obviously, for these calculations it was necessary to execute a great number of subroutines in *Mathematica*. To do this, we have used the Rigel cluster and the computers, working in parallel, of the Calculation Centre of the *Universitat Politècnica de València*. Without this technical support it would have been impossible to perform the enormous quantity of calculations required in a reasonable computing time.

In the analysis of the vertical sections of the point clouds, in transversal as well as longitudinal, the same procedure was applied as in the horizontal sections. In this case, and after considering the graphs obtained and finding the distances from the point cloud to the ellipse and oval arches of different centers, the best approximation has been obtained using the five-centered oval arches described in the graphic analysis section.

The distances obtained are less than 3 cm in the ovals (except in some specific cases where deformations are observed) while the ellipses are separated by up to more than 11 cm in the longitudinal section. In the transversal section these distances are smaller, but still larger than with the oval. This gives an idea of the validity of the fit of the ovals formed with our construction hypothesis to the real data and confirms the conclusion obtained in the graphical analysis that the vertical sections are five-centered ovals and not ellipses.

The calculation of the distances to the ovals is complex because you must calculate the distance from each point to the corresponding circumference of the five that make up the oval. We have done this by grouping the data from the point cloud in each vertical section in five groups, one for each circumference. The division is obtained considering that the relationship  $y/x$  at every point  $(x, y)$  of the section is greater or less than that corresponding to the points of tangency of the circumference. These points, obtained as described in Gómez-Collado et al. (2018), are:

In the transversal section:

- Point of tangency between the central and the contiguous circumference, (1.0831, 6.0193).
- Point of tangency between the circumference with the smallest radius and the adjacent one, (2.4661, 5.4045).

In the longitudinal section, these points are (1.1383, 11.4938) and (2.804, 10.8747), respectively.

To obtain all the distances described, which have allowed us to determine the shape of the dome, we have used the Rigel cluster of the *Universitat Politècnica de València*.

#### **4. Conclusions**

After the graphical and mathematical analysis of the geometry of the dome described, we have obtained the following results:

- (1) The *Camarín* space in which the dome is located has a trapezoidal plan. The projection of the dome on a horizontal plane is inscribed in a rectangle with sides of 6.91 m and 6.12 m (equivalent to 30.5 and 27 Valencian palms, with the longest side parallel to the wall of the presbytery. These dimensions have been deduced from the study of the geometry in the vertical planes since they have not been physically determined (Fig. 7). The start of the dome takes place at a height of 7.54 meters and its highest point is at 9.46 m. The rise of the dome is, therefore, 1.925 m (8 ½ Valencian palms (Fig. 7 and 8).
- (2) The sections in the horizontal plane are ellipses, although they could also be four-centered ovals. For the reasons stated in the document, we believe that

ellipses were constructed, whose axes in the starting plane coincide with the sides of the rectangle defined in point 1 of these conclusions.

- (3) The sections of the vertical central planes – longitudinal and transversal – are five-centered oval arches (Fig. 11, 12 and 13).
- (4) There are some deformations and a flattening or reduction of the height in the upper part of the dome (from 1.70 to 1.925 m), due to the existing mechanical deformations or more probably due to the difficulty in closing it, however they are not important in order to determine the shape of the dome since they only involve the last 22.5 cm.

Our conclusion is that the dome is generated by ellipses in the horizontal planes and five-centered oval arches in the central verticals, which define the axes of these ellipses. This hypothesis solves the construction of the dome in a relatively simple way, since the dimensions of the axes are known, there is only one possible ellipse: in addition, in this way, solving the problem of constructing the curves in vertical sections with circumferential arches (Fig. 8).

Another conclusion that we consider relevant is that the methodological procedure used could be applied to the study of other domes of oval geometry.

## **5. Acknowledgments**

- Monks of the Monastery of Santa María de El Puig: Fathers M. Azcárate, J. Sesma, C. Peña and M. Anglés.
- Julio Samuel Badenes Almenara, the chronicler of El Puig de Santa María.
- Jacinta Harrington-Flynn for the participation in the English translation.
- *Àrea de Sistemes de la Informació i les Comunicacions (ASIC), Institut Universitari de Matemàtica Pura i Aplicada (IUMPA), Dpto. de Matemàtica Aplicada, Dpto. de Expressió Gràfica Arquitectònica and Centro de Informació Arquitectònica (CIA) of Escuela Técnica Superior de Arquitectura, in the Universitat Politècnica de València.*

- This work was supported by the R+D+i Project GV/2020/001 and the research of the third author was also partially supported by the Project MTM2016-76647-P.

## 6. Declaration of interest: none

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