SLAB TRACK OPTIMISATION USING METAMODELS TO IMPROVE RAIL CONSTRUCTION SUSTAINABILITY

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Abstract

Railways are an efficient transport mode, but building and maintaining railways tracks has a significant environmental impact in terms of CO_2 emissions and use of raw materials. This is particularly true for slab tracks, which require large quantities of concrete. They are also more expensive to build than conventional ballasted tracks, but require less maintenance and have other advantages that make them a good alternative, especially for high-speed lines. In order to contribute to a more sustainable railways, this paper aims to optimise the design of one of the most common slab track typologies: RHEDA 2000. The main objective is to reduce the amount of concrete required to build the slab without compromising its performance and durability. To do so, a Finite Elements (FEM) model of the track has been used, paired with a kriging meta-model to allow analysing multiple options of slab thickness and concrete strength in a timely manner. By means of the kriging, optimal solutions have been obtained and them validate through the FEM model to ensure that predefined mechanical and geometrical constraints are met. Starting from an initial setup with a 30 cm slab made of concrete with a characteristic strength of 40 MPa, an optimised solution has been reached, consisting on a 24 cm slab

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made of concrete with a strength of 45 MPa, which yields a cost reduction of 17.5%. This process may be now applied to other slab typologies to obtain more sustainable designs.

Keywords: Slab track; optimization; latin hypercube, kriging; finite elements

1 Introduction

Transport is one of the most essential human activities, and as such has a significant impact on Greenhouse Gasses (GHG) emissions. For instance, in the European Union, transport accounts for 24.6% of CO_2 equivalent emissions, the biggest share among all economic sectors except the energy sector (European Commission, 2020). Therefore, promoting a more sustainable transport and reducing its carbon footprint are crucial steps towards a cleaner society.

Railways are a rather efficient transport mode, as they contribute only 0.5% of the CO_2 equivalent emissions of the transport sector in the EU-27, despite carrying 6.9% of passengers and 12.4% of freight (European Commission, 2020). This explains the current trend of promoting railways across the EU as a way of reducing the environmental impact of transport (European Commission, 2018), which implies construction of new lines and renewal and maintenance of existing ones.

However, building and maintaining railway tracks have also a significant environmental impact in terms of GHG emissions and use of raw materials. For instance, according to Kortazar et al. (2021), the development of the Spanish High-Speed network requires up to 196.66 tonnes of CO_2 equivalent per year for construction and maintenance (compared to an estimated 355.68 tonnes of CO_2 equivalent per year for operation), considering the four main high-speed corridors. While studying the environmental impacts of the Beijing-Shanghai high-speed line, Kaewunruen et al. (2020) estimated that construction accounts for 64.86% of the total carbon emissions of the line (including operation and maintenance), and that producing the cement required to build the slab track accounts for 60% of the emissions due to construction.

These examples point out that constructing and maintaining railway tracks greatly contribute to the overall carbon footprint of this transport mode, to the extent that they may reduce (but not cancel) the reduction of GHG emissions that may be achieved by shifting to rail from other, less efficient modes (Åkerman, 2011). Therefore, optimising the track layout (and particularly, the amount of materials used to build the track) is of paramount importance to mitigate GHG emissions and to make railways an even better option for a more sustainable transport.

Although the conventional ballasted track remains the most common track layout across the world, ballastless (or slab) tracks are a rather interesting alternative, particularly for new high-speed lines. Slab tracks replace the ballast-and-sleeper package with a reinforced concrete slab (either made in situ or prefabricated). They are usually between 1.5 and 2 times more expensive to build than ballasted tracks (Lichtberger, 2011). However, if properly built, require less maintenance than ballasted tracks over the years. Several studies have shown slab track maintenance costs to be up to 30% lower than those of ballasted tracks (Michas, 2012), which means that slab tracks are economically more efficient in the long term (Esveld, 2001; Michas, 2012). Additionally, slab tracks present other advantages such as longer life-cycle or better load distribution (Esveld, 2001; Michas, 2012), and may even be more sustainable in the long term, assuming service life over 75 years (Pons et al., 2020).

There are many different designs of slab tracks that have been used and tested over the last five decades. They may be roughly classified into two categories depending on whether the slab is prefabricated or cast in situ. Among the former are worth mentioning the Japanese Shinkansen slab, the German Bögl, the Austrian ÖBB-Porr or the Italian

IPA. Among the latter, the German RHEDA and Züblin or the French STEDEF (Gautier, 2015; Michas, 2012). However, from an environmental point of view, all of them require large amounts of concrete (and thus cement) which, as pointed out previously (Kaewunruen et al., 2020), greatly contribute to their carbon footprint. To put this into context, a standard RHEDA 2000 layout (see Figure 1) requires approximately 1540 m^3 of reinforced concrete for the slab plus 2280 m^3 of mass concrete for the supporting layer per km of single track, hence a global figure of 3,82 m^3/m. For comparison, the approximate amount of concrete used to build large rail bridges is 32 m^3/m (Tuchschmid et al., 2011), but bridges are singular structures while tracks may cover hundreds of kilometres.

Therefore, optimising current slab designs to reduce their cost and environmental impact would have a noticeable effect on their sustainability. However, established slab designs have been carefully devised to offer long-term durability, resistance and reliability (Esveld, 2001), and thus optimising them taking into account environmental concerns is a complex task that should take into account several criteria.

In fact, in contrast to other areas of civil engineering, there has been little effort in recent years to optimise rail cross-sections and track typologies. Research has focused mostly on optimising vehicles, studying for instance their shape and aerodynamic performance (Jakubek and Wagner, 2016), wheel profiles or damping systems (Ye et al., 2021). A remarkable amount of work has also been devoted to eco-driving and energy efficiency (Eaton et al., 2017; Fernández et al., 2015; Martínez Fernández et al., 2019b).

However, optimisation of the rail infrastructure (and, particularly, the track) is a much less developed area of study, and has been applied basically to rail profiling and maintenance (Ye and Sun, 2021) or to reduce noise and vibration (Zhao et al., 2017). From an environmental point of view, other studies have focused on reusing waste materials, particularly those produced in large quantities such as scrap tyres (Ferdous et al., 2021), to build track elements (Hidalgo Signes et al., 2016). Traditionally, rail track cross-section design has been based on experience and full-scale tests implemented by railway administrators. Thus, tracks are generally designed to meet operational and economical requirements, such as higher speeds or axle loads, with little regard paid to environmental concerns. A thorough optimisation of the track cross-section to reduce its carbon footprint, particularly for slab tracks, remains a task to be carried out within the railway sector.

In order to start filling this gap and contribute to a more sustainable railway, this paper aims to carry out an optimisation of the RHEDA 2000 slab design, one of the most common and tested slab tracks (Esveld, 2001; Michas, 2012). The main objective is to apply optimisation methods well tested in other areas of civil engineering to reduce the amount of concrete used to build the slab as well as its characteristic strength (and thus both its cost and GHG emissions) without compromising its performance and long-term durability. In this way, the paper pretends to test the applicability of said methods to optimise slab track cross-sections and to offer a first approach to an optimised RHEDA 2000 slab track.

In order to do so, a Finite Element Model (FEM) of a ballastless railway track built with a RHEDA 2000 slab has been used. Although several approaches and methodologies are already available, the Finite Element Method (FEM) has proved to be a useful and effective tool for research on railway track elements (Esveld, 2001; Selig and Waters, 1994; Zhao et al., 2017).

However, as calculating each and every alternative with the FEM model is unfeasible, a meta-model has also been used. Meta-models are simpler approximations to more complex models (such as the FEM in this case) which allow sifting out the solution space in a timely manner and picking up optimal solutions (or rather, roughly optimal ones) than may be then checked with the FEM model.

There are many different meta-models and ways of creating them. Although many of them have been extensively compared in the past, deciding which one is better depends on the specific problem to be solved. That said, there are certain models that are more commonly used, such as polynomic regression, neural networks (García-Segura et al., 2018; Martínez Fernández et al., 2019a) or kriging (Penadés-Plà et al., 2020b). In this paper, kriging has been chosen as it is more flexible than polynomials and is less time-consuming than neural networks (Simpson et al., 2001). Moreover, it is already been used for structural optimisation in other areas of civil engineering such as bridge construction (Penadés-Plà et al., 2020a), or even in railways to optimise wheel profiles and train suspension (Ye et al., 2021).

The paper is organised as follows: first, a detailed description of the optimisation problem is given, and the FEM model, the meta-model (kriging) and other tools used are explained. Then, the results obtained from the optimisation process are given and discussed, and finally the main conclusions achieved are explained, together with insights for further research.

2 Materials and methods

2.1 Optimisation problem description

As explained before, the main objective of this paper is to carry out an optimisation of an existing slab track design (RHEDA 2000), seeking to reduce its costs and environmental impact without compromising its performance and durability. The RHEDA 2000 is, as stated before, one of the most common slab track typologies, but the optimisation process presented in this paper could be applied to other slab tracks such as Shinkansen, Bögl, Züblin or Stedef, as they all are based on a monolithic concrete slab (be it prefabricated or built in situ) that replaces the ballast layer (Michas, 2012).

The RHEDA 2000 is a slab track system with discrete rail support: the continuous rail is fastened to concrete sleepers which are, in turn, embedded in a monolithic concrete slab (Michas, 2012). The original RHEDA system was first used in 1972 in Germany, and has been updated and improved over the years. The current RHEDA 2000 version was first used for parts of the high-speed line between Leipzig and Halle (Esveld, 2001), and has since become one of the most used slab systems in the world (Michas, 2012).

In this study, a standard RHEDA 2000 layout has been considered (figure 1). The slab has a thickness of 30 cm and is made of concrete with a characteristic strength of 40 MPa. The slab rests over a layer (thickness: 30 cm) of mass concrete (strength: 20 MPa) and an embankment coronation layer (thickness: 30 cm) of soil cement. The rails are UIC60 and the fastening system consists on IOARV 300 fasteners with Vossloh clips (model SKL 15). It would be interesting to also analyse the economic and environmental impact of track fastening elements, but this is a complex task in itself and beyond the scope of this paper.

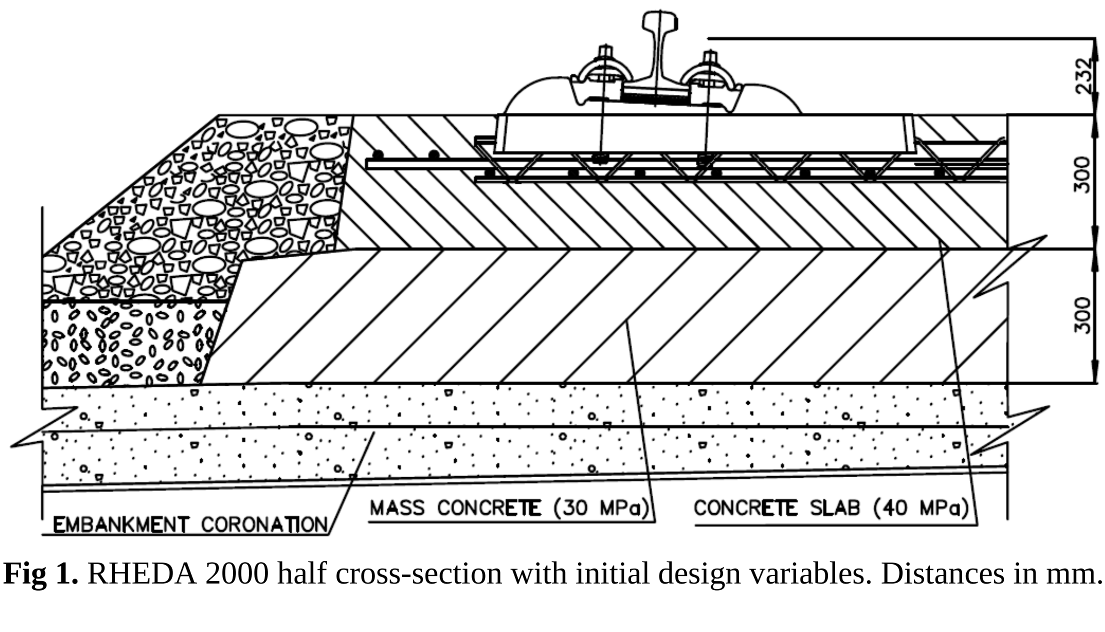

Fig 1. RHEDA 2000 half cross-section with initial design variables. Distances in mm.

The main assumption on which the optimisation process will be based is that reducing the cost of the track section (and, specifically, of the slab) will in turn yield a drop of its environmental impact and an improvement of its sustainability (owing to the reduction of raw materials used). This is justified by the large quantities of concrete required to build a single km of RHEDA 2000 (roughly 1540 m^3/km only for the slab, considering the layout shown in Figure 1), and the significant impact that concrete and cement production have on the environment (Mohamad et al., 2021).

Therefore, our objective function is expressed as:

$$\min\{C(e, f_{ck}) = \frac{e}{100} \cdot 2.6 \cdot 1000 \cdot P(f_{ck}) = 26 \cdot e \cdot P(f_{ck})\} \tag{1}$$

Where C(e, f_{ck}) is the total cost in € of 1 km of slab track, e is the slab thickness in cm, f_{ck} is the characteristic strength in MPa of the concrete used to build the slab and P(f_{ck}) is the unitary cost in €/m^3 of said concrete. The reason for choosing these two variables is that slab thickness is the only slab dimension that may be modified to achieve a substantial reduction of the amount of concrete used (as slab width is constrained by track geometric requirements). Moreover, a thinner slab may require a stronger concrete to resist traffic loads, which increases costs and also implies higher quantities of cement. Therefore, these two variables not only have a direct impact on slab cost, but are also correlated to each other.

That said, the objective is to minimise Equation (1), which means dealing with those two design variables: slab thickness (e) and concrete characteristic strength (f_{ck}). Considering usual concrete characteristics as per Eurocode 2 (CEN, 2004) and usual geometric requirements for the slab based on RHEDA 2000 tracks in service (Cortina Ruiz, 2013; Michas, 2012), these two variables will be within the following ranges:

$$\begin{cases} e \in [10,40] cm \\ f_{ck} \in [30,45] MPa \end{cases} \tag{2}$$

Where e will be varied in intervals of 1 cm and f_{ck} in intervals of 5 MPa. Of course, as explained before, there are several criteria to be considered to ensure that any combination of e and f_{ck} values within the ranges of Equation (2) that minimise Equation (1) yield a resistant and durable slab. In order to set up such criteria, several Spanish and international standards for railway construction have been considered, including Spanish Recommendations for Railway Infrastructures (Ministerio de Fomento, 1999) and ADIF Standards (ADIF, 2000), as well as UIC 719R (UIC, 2008). From these regulations and recommendations, the following requirements have been defined:

- Maximum vertical displacement in rail: $\delta_y \leq 3$mm.
- Maximum stress in rail (Von Mises criterion): $\sigma_{vm} \leq 137.5$ MPa
- Vertical track stiffness (based on (Pita et al., 2004)): $k \in [75,85]$ kN/mm
- Maximum traction stress in slab: $\sigma_{adm} \leq f_{tck}/4.1$ (where f_{tck} is the concrete characteristic tensile strength).

Therefore, any optimal solution obtained during the process should comply with all these constraints to ensure that the resultant slab does not compromise track performance.

In order to assess the different combinations of slab thickness and concrete strength and calculate the track performance (and all the aforementioned displacements and stresses), a FEM model of the track was used. FEM models have been extensively used to model track dynamics (Connolly et al., 2013; Martínez Fernández et al., 2013; Villalba Sanchis et al., 2021) and are thus a reliable tool to analyse the behaviour of any variation of the RHEDA system. However, FEM models are also time-consuming because, in order to properly model the track section (and considering that a sufficient length of track must be modelled to avoid boundary disturbances), they usually consist on hundreds of thousands of elements (Hall, 2003; Sayeed and Shahin, 2016) even accounting for symmetry simplifications (which are not always possible). Considering also moving, dynamic loads (to simulate the passing of a train), FEM simulations require indeed high computational costs and running time, although the exact figure depends on model complexity and computer assets available (Jin et al., 2018; Li et al., 2018). Therefore, it is not feasible to directly use a FEM model to test all the possible combinations of e and f_{ck} within the ranges shown in Equation (2). Consequently, a meta-model was used instead to carry out the bulk of the optimisation process, leaving the FEM model as a validation check for the few, best solutions obtained via the meta-model. As explained before, meta-models are simpler approximations to more complex models (in this case, the FEM model of the track) which provided a rough and fast estimation of the results that the main model would yield for a given input. This is expressed in Equation (3):

$$y = f(x) = g(x) + \varepsilon \tag{3}$$

Where x are the input variables for the model, $f(x)$ is the output of the main model, $g(x)$ is the output of the meta-model and ε is the meta-model error. There are several mathematical approaches to define and create the metamodel $g(x)$, but in this case the kriging model was used because it has lower computational requirements than other similar approaches (Simpson et al., 2001) and thus provides solutions faster (which is, as explained before, one of the reasons for using meta-models instead of the FEM full model). Another advantage is that kriging is more flexible due to its formulation, but this is further explained in section 2.3.

Figure 2 shows a flow chart of the optimisation process, whose different steps and elements are explained in more detail in the following subsections.

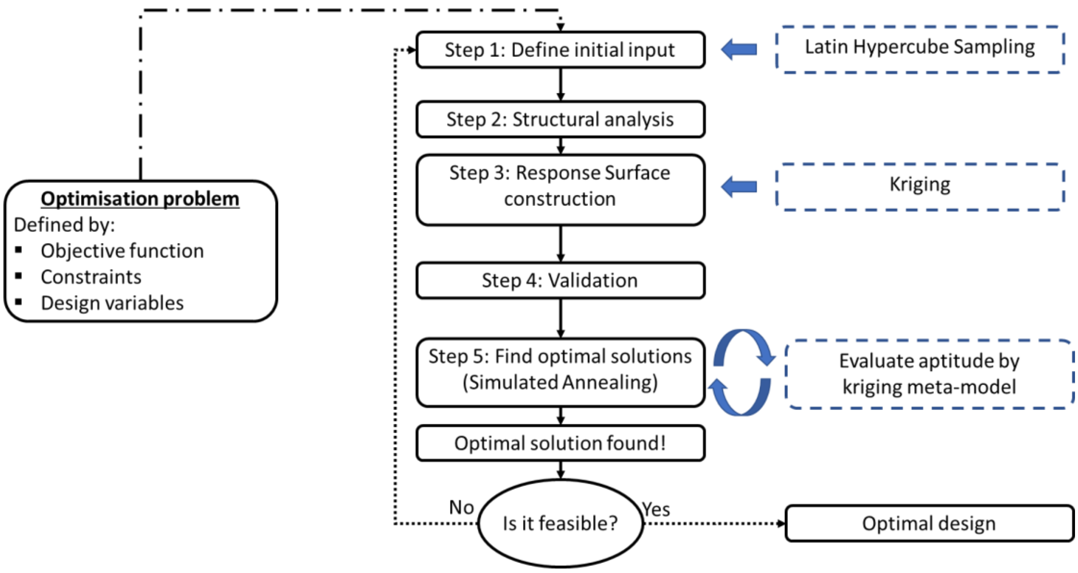

Fig. 2. Flow chart of the optimisation process. Adapted from Penadés-Plà et al. (2019).

2.2 Input selection

As shown in Figure 2, once the optimisation problem is defined (with objective function, design variables and constraints), the first step of the process is to obtain an initial population that will be then used to build the meta-model. How this input set is obtained will have a crucial impact on the meta-model accuracy, as it must be representative of the solution space (i.e. the space defined by all the possible values of each input variable). The input set is defined by its size and location across the solution space. Input size is directly related to the number of variables, which in our case amounts to two (namely, slab thickness e, and concrete characteristic strength f_{ck}). As for how to sample each variable across its respective range to obtain the most representative input set, a form of Design of Experiments (DOE) should be used.

There are many different techniques that belong to the DOE approach, from classical methods (such as Box-Behnken or D-optimal (Myers et al., 2016)) to space filling methods, more apt for complex meta-models as they cover the whole solution space to account for local phenomena. Among the latter, the most common is the Latin Hypercube (Dette and Pepelyshev, 2010). This is the one chosen in this paper, as it has been used in previous studies of structural optimisation (Penadés-Plà et al., 2020b, 2019) and it is comparatively more efficient (i.e. provides samples faster) than other common sampling techniques (Olsson et al., 2003; Woods and Lewis, 2017).

The Latin Hypercube determines a number n of non-overlapping intervals for each variable v and number of input points n. This divides the solution space into $n \times v$ regions so that each initial input point is placed in one region, thus ensuring that all variables are sampled across their whole range.

The population sampled through the Latin Hypercube is then analysed to obtain not only the value of the objective function (Equation 1) but also the corresponding values for each of the predefined constraints. The latter is done by means of the FEM model described in section 2.5.

2.3 Kriging meta-model

With the input sampled using the Latin Hypercube as described above, it is possible to generate the meta-model. As explained before, the one chosen for this study is the kriging model, which was first proposed by Danie Kirge for geo-statistical applications and later formalised by Matheron (Matheron, 1963). The kriging meta-model is formulated according to Equation (4):

$$y(x) = f(x) + Z(x) \quad (4)$$

Where $y(x)$ is the deterministic output to be obtained, $f(x)$ is a defined approximation function (similar to a regression model) and $Z(x)$ is a stochastic process with mean $\mu = 0$ and variance $\sigma^2 \neq 0$. The purpose of the stochastic addition is to generate local variations for the kriging to interpolate between the initial input data.

There are different versions of the kriging approach formulated in Equation (4). The most common (i.e. ordinary kriging) uses a constant $f(x)$ term. Furthermore, if $f(x)$ tends to zero (which means that *y(x)* as a mean value close to zero too), the model is known as simple kriging (Simpson et al., 2001).

Regarding the precise formulation of *f(x)*, the most common form (and the one used in this paper) is that of a weighted linear combination as shown in Equation (5):

$$f(x) = \sum_{i=1}^{n} \beta_i \cdot f_i(x) \tag{5}$$

Where $f_i(x)$ are already known outputs (that is, the dataset used to build the meta-model) and β_i are the corresponding weights (Biles et al., 2007). As for the stochastic term *Z(x)* of Equation (4), its usually defined as follows:

$$\begin{cases} cov\left[Z(x_i), Z\left(x_j\right)\right] = \sigma^2 \cdot R\left(x_i, x_j\right) \\ \quad R\left(x_i, x_j\right) = e^{-\sum_{k=1}^{m} \theta \left|x_k^1 - x_k^j\right|^2} \end{cases} \tag{6}$$

Note that σ^2 scales the spatial correlation function $R(x_i, x_j)$, which in turn is usually (Simpson et al., 2001) expressed as a gaussian correlation function with a single parameter θ. A lower value of θ means that all the points in the sample are highly correlated and thus the *Z(x)* term in Equation (4) remains similar in the whole solution space. On the other hand, as the θ value increases, the more correlated points are closer and the *Z(x)* term will vary across the solution space. This allows for a more flexible approach than other similar meta-models which do not allow tuning the spatial correlation (Simpson et al., 2001).

Finally, the kriging model requires an associated method to fit the initial input data, which in this case, based on previous research (Penadés-Plà et al., 2019), is the Best Linear Unbiased Prediction method (BLUP).

2.4 Optimisation process, surfaces

In order to solve the optimisation problem defined in section 2.1 by means of the kriging meta-model, the process summarised in Figure 2 is followed. Therefore, once the initial output set has been sampled using the Latin Hypercube, the meta-model described in section 2.3 may be used to generated a response surface, which is an estimation of the output of Equation 1 by the meta-model.

However, as Equation 1 only calculates the cost of the slab considering two design variables (thickness e, and concrete strength f_{ck}), the response surface will take the form shown in figure 3a. Of course, in that case the cheaper solution will be that with the shallower slab (e = 10 cm) and weakest concrete (f_{ck} = 30 MPa), but this slab layout might not ensure a proper performance. In order to take into account the constraints defined in section 2.1, penalties are applied to the response surface in the form of a coefficient that multiplies the value of the objective function (in this case, the cost of the slab). This coefficient is defined as the quotient between the exceeding value and the admissible value for each constraint, raised to the power of 5 (as usually this quotient is only slightly over 1, and without raising it to a high power, it would barely affect the outcome). By applying this coefficient to the kriging output, the response surface is altered, taking a shape similar to that in figure 3b.

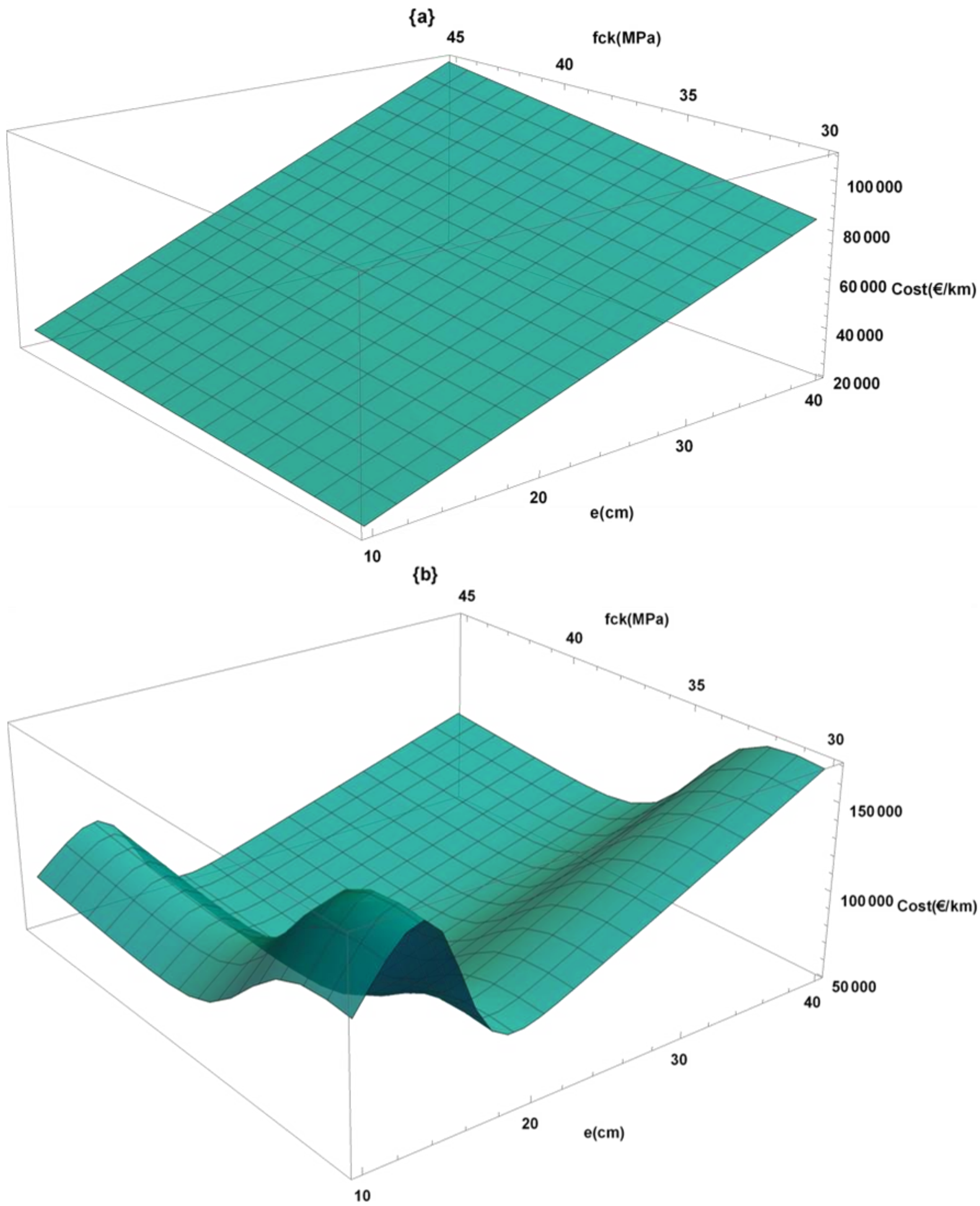

Fig. 3. Response surface obtained with kriging. (a) Surface without constraints. (b) Surface with constraints applied through penalties.

This is now a complex surface with local maxima (solutions that do not comply with constraints) and minima (solutions that do comply all constraints). The surface may take different shapes depending on the input dataset used to build the meta-model and may present many local minima. Therefore, identifying the global minimum (which would yield an optimal slab layout) is done by means of a meta-heuristic algorithm. The one used in this work is Simulated Annealing (SA) as it is a versatile algorithm capable of finding global optimums (Kirkpatrick et al., 1983).

SA is an algorithm inspired by the annealing process in metallurgy. It is an iterative process where the current state of the objective function (in this case, the cost of the slab) is compared to nearby states in the response surface, looking for lower values. A detailed description of the algorithm may be found in Ghasemalizadeh et al. (2016) and Kirkpatrick et al. (1983). Regarding the SA parameters, they have been calibrated following the process found in Medina (2001) to define the initial temperature, adopting a cooling coefficient (k) of 0.8.

Of course, introducing those penalties to account for mechanical and geometrical constraints causes uncertainty to the cost estimation done by the kriging meta-model. To address this issue, up to nine response surfaces have been generated, each one based on a different sampling of initial pairs of e and f_{ck} obtained with the Latin Hypercube. This will, in turn, yield nine optimal solutions for our optimisation problem that can be then analysed in more detail and validated using the whole model i.e. the FEM track model.

The kriging model creation, the generation of response surfaces and the calculation of optimal solutions by means of a heuristic algorithm were all carried out using *MATLAB* 2018a (The Mathworks, Inc.) with the DACE toolbox developed by Lophaven et al. (2002).

2.5 Validation with FEM

As explained before, FEM models have been extensively used to model the mechanical behaviour of the track (Esveld, 2001; Selig and Waters, 1994; Zhao et al., 2017). These models can simulate the tensional and deformation state by discretizing the track structure to finite elements and solving the resulting mathematical equations. Thus, the track superstructure and substructure, subjected to specific boundary conditions and restrictions, may be analysed as a whole, considering interactions between all components.

In the context of this study, the FEM model represents the most accurate simulation of the track behaviour, and it is used in two stages: First, to evaluate the constraints of the initial datasets sampled by the Latin Hypercube and used to generate the kriging meta-model, and secondly, to validate the optimal solutions provided by identifying the global minimum of each response surface generated by the kriging. The latter is required because the meta-model, as an approximation of the FEM model, provides optimal solutions in a timely manner but, in doing so, assumes a higher degree of uncertainty. The FEM model, which provides more accurate values of stress and strain in every element of the track, helps checking that the optimal solutions provided by the kriging meta-model do fulfil all the technical constraints defined in section 2.1.

The RHEDA 2000 slab track is modelled as a parametrized three-dimensional track, where rails are discretely supported above each sleeper, which in turn are directly embedded in the concrete, forming a monolithic slab (Figure 4). Rails are modelled as a uniaxial three-dimensional solid element with twenty nodes with tensile, compression, bending and torsion behaviour. The rail pad is represented by a spring-damper element with two nodes that behaves in uniaxial tension-compression. In addition, the sleepers and the multilayer system use a three-dimensional solid element with three degrees of freedom per node: translations in X, Y and Z. Conventional values of stiffness and strength are used for concrete and steel elements. Finally, the values given by the UIC soil classification are used as a reference for the embankment.

The model has a total length of 5.4 metres, covering 9 sleepers with a separation of 0.6 m. The slab width is 2.6 m (equivalent to a single track with standard gauge). However, as the track is completely symmetrical along the longitudinal axis, only half track is modelled. Regarding the loads, a high-speed TALGO 102 series is considered with a static axle load of 167 kN (and hence a wheel load of 83.5 kN). However, in order to account for dynamic effects, this static load is raised using the Eisenmann criterion, yielding a total wheel load of 116.9 kN (considering a train speed of 300 km/h).

The FEM model was created and run in *ANSYS Mechanical APDL* 17.2 (Ansys, Inc.).

3 Results and discussion

3.1 Surfaces obtained and table of optimal results for each one.

The whole optimisation process described in Figure 2 has been applied to obtain optimal combinations of slab thickness (e) and concrete characteristic strength (f_{ck}) than minimise cost (Equation 1) while complying with all the constraints described in section 2.1.

In order to ensure that the whole solution space is analysed, the Latin Hypercube method was applied nine times to obtain nine different input sets, which in turn were used to build nine meta-models and obtain the same number of surfaces from which the heuristic algorithm could get an optimal solution. The results are shown in Table 1.

The first noteworthy result is that every optimal solution obtained from each of the nine surfaces generated through kriging complies with the four main constraints, and thus all of them are feasible solutions that do not compromise the slab performance. This justifies the use of penalties to modify the response surfaces and avoid optimal solutions that are not constraint-compliant.

In terms of cost (the parameter to be minimised), Table 2 shows the cost value for each optimal solution as given by the kriging model, and the comparison with the real cost calculated directly from Equation 1. There is an evident discrepancy between real and predicted costs due to the penalties introduced to the surfaces as explained in section 2.4. This points out that using the meta-model implies a certain degree of error (as expressed in Equation 3), but as the mean error is below 4.35% it is deemed acceptable for our purposes.

In any case, according to the results of Table 2, the solution which provides the greater cost reduction of the slab when compared to the original setup (i.e. a slab thickness of 30 cm and a concrete strength of 40 MPa) is the one obtained from the ninth surface (e = 24 cm and f_{ck} = 45 MPa). This solution yields a 17.15% cost reduction. Note that the same optimised setup was also obtained from the sixth surface, only in this case the estimation of cost given by the meta-model was higher.

It is also worth noting that the optimal solution from the 8th surface yields a cost variability of 0 (which means that the cost figure given by the meta-model is equal to the real cost). This is due to the fact that, in that case, the optimal value belonged to the initial set sampled by the Latin Hypercube and used to build the meta-model.

3.2 Validation with FEM

In order to validate the optimal setup obtained (i.e., e = 24 cm and f_{ck} = 45 MPa), the complete FEM model was used to check all constraints with more detail and accuracy. Only the slab thickness and concrete characteristic strength are modified to represent the optimal solution, with all other geometric and mechanic parameters of the FEM model fixed as defined in section 2.5. Figure 5 shows an excerpt of the main results yielded by the model in terms of vertical displacements (5a) and stresses (5b).

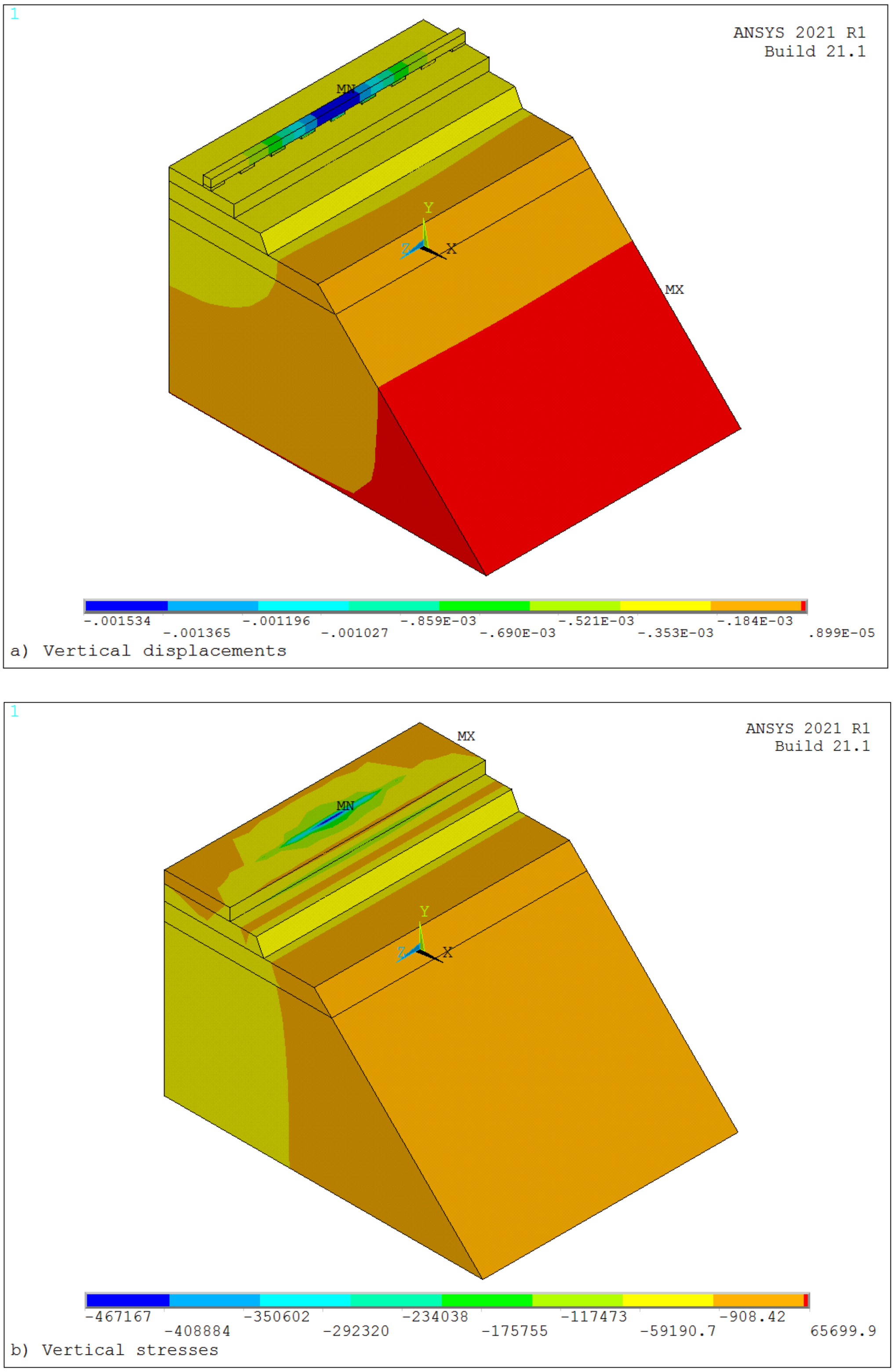

Fig. 4. FEM model results. a) Vertical displacements. b) Vertical stresses.

According to the FEM model, by using a slab with a thickness of 24 cm, made of a concrete with a characteristic strength of 45 MPa, the resultant track complies with all the predefined constraints. Specifically, the vertical displacement in the rail is 1.5 mm, thus below the threshold value of 3 mm. Moreover, the overall track vertical stiffness is 78 kN/mm (well within the range of [75,85]) and the maximum stress is below the predefined threshold for every material (i.e. rail, slab, platform). Therefore, the optimal setup obtained using the kriging meta-model is a feasible solution that ensures a correct slab performance. Figure 6 shows a track cross-section with the optimised variables.

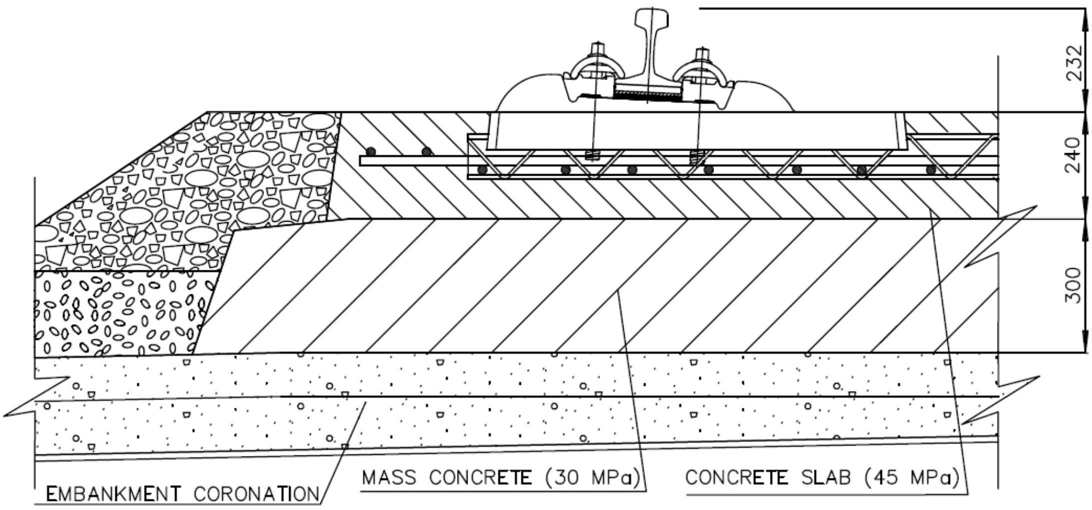

Fig. 5. Track half cross-section with optimised variables. Distances in mm.

Finally, to further assess the potential advantages of the optimal solution, compared to the original setup, a life-cycle analysis has been carried out. This analysis is based on the *ReCiPe* methodology proposed by Goedkoop et al. (2009), with data from the ECOINVENT database (Frischknecht and Rebitzer, 2005), and was carried out using *OpenLCA* open source software. The ReCiPe method combines both midpoint and endpoint indicators, but in this study only the latter have been considered as they provide a more global and easier to compare assessment. These indicators consist on four categories that evaluate the impact of each alternative studied by grouping many different specific parameters:

- Impact in resources, measured in US dollars.
- Impact on ecosystems, measured in lost species per year.
- Impact on human health, measured in disability-adjusted life years.

- Impact on safety, measured in hours of average risk.

This methodology has already been used by other authors to perform life cycle assessments in civil engineering (Ata-Ali et al., 2021; Pons et al., 2018). The results are shown in Table 3.

As the table shows, the optimised cross-section not only reduces costs by 17.2%, but also yields a 15.27% reduction in the use of resources and a 37.50% reduction in the impact to ecosystems. Moreover, the new solution also improves health and safety.

Overall, the results obtained prove that the chosen methodology, based on a FEM model and a kriging meta-model, may be applied to track optimisation. This contributes to start filling the gap previously identified, namely the lack of thorough optimisation studies of track cross-sections (particularly slab designs) that consider environmental concerns. By using an optimisation methodology similar to the one proposed in this paper, which has already been tested in other areas of civil engineering, it is possible to obtain refined slab designs with reduced costs and use of materials. In this way, well-established slab typologies such as the RHEDA 2000 analysed in this paper could be modified to reduce their environmental impact and help improving the overall sustainability of railways, particularly with regard to their construction and maintenance.

Of course, the optimised RHEDA 2000 achieved in this paper is but a first approach to a complex matter, and further studies should be carried out to incorporate other factors into the analysis and to account for elements such as track subgrade or fastening systems, which have been omitted. Another noteworthy line of research would be to consider recycled aggregates or concrete for the construction of the slab, as using such materials may reduce the environmental impact of the infrastructure (Wang et al., 2021). Nevertheless, this approach paves the way for further optimisation in railways construction and sustainability, as it provides a fast and useful tool for railway engineers to introduce environmental concerns into rail track design without requiring complex and time-consuming FEM models. Moreover, as slab tracks are becoming a noteworthy choice for new high-speed lines in many countries, the methodology presented in this paper may help achieving more optimised designs which will reduce the carbon footprint of rail construction and will help making railways an even more sustainable transport mode.

4 Conclusions

This paper aims to apply an optimisation methodology based on a FEM model and a kriging meta-model to offer a new approach into the reduction of the environmental impact linked to track construction. This methodology is tested with one of the most used slab track designs: RHEDA 2000. This is a monolithic slab track with discrete supports that has been extensively built for high-speed lines in countries such as Germany. Building a slab track implies a remarkable environmental impact in terms of raw materials (particularly, cement), which reduces to some extent the environmental benefits of building new railway tracks to shift passengers and freight to rail from other, less sustainable transport means.

In order to reduce the carbon footprint of the RHEDA 2000 slab track, the slab design has been optimised, trying to minimise its costs as an indirect way of alleviating the amount of concrete used as well as its characteristic strength (which in turn reduces the amount of cement). The methodology used to do so consists on a FEM model of a railway track with a RHEDA 2000 slab, created to test different slab designs with varying slab thickness (e) and concrete characteristic strength (f_{ck}). However, as FEM models are time-consuming, a meta-model has been used instead to analyse the whole solution space, saving the FEM model only for final validation.

Using a kriging meta-model, built upon nine different initial datasets (all sampled using a Latin Hypercube method), nine response surfaces have been created. Each one yielded an optimal solution (obtained by means of Simulated Annealing), that is, a pair of values e and f_{ck} that minimise slab cost while also complying with certain, predefined mechanical and geometrical constraints. These constraints were applied to ensure that any optimal solution found does not compromise the track performance.

The best of the nine solutions obtained has a thickness of 24 cm and is made of concrete with a characteristic strength of 45 MPa. Its cost is 52,965.12 €/km, which represents a 17.15% reduction when compared to the original setup (e = 30 cm, f_{ck} = 40 MPa). This slab setup has been then validated using the FEM model, and complies with all the predefined constraints. In order to further assess its environmental benefits, a life-cycle assessment has been carried out. According to this analysis, the optimised slab setup yields a 15.27% reduction in the use of resources and a 37.50% reduction in the impact to ecosystems, among other benefits.

This work aims to offer a first approach to slab track optimisation, filling a clear gap in the literature with regard to rail infrastructure optimisation. The proposed approach uses meta-models (i.e. kriging) as a useful methodology that does not required the amount of time and computing resources that whole track models (mainly based on FEM) would require. Although an optimised setup for a well-known and used slab design (RHEDA 2000) has been achieved, the study could be further extended to other track typologies (both ballasted and ballastless), and improved by including other factors and criteria in the optimisation process.

Data availability statement

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request. (Kriging code in MATLAB, FEM model in ANSYS.)

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Table 1. Optimal values of slab cross-section from each kriging surface and constraints checking.

Kriging surface	Slab thickness (cm)	f_{ck} (MPa)	f_{tck} (MPA)	Rail displacement (mm)	Rail stress (Von Misses) (MPa)	Track vertical stiffness (kN/mm)	Slab max. stress (MPa)
1	29	40	2.46	2.872	68.3	81.82	0.464
2	26	45	2.66	2.879	68.3	81.63	0.497
3	25	45	2.66	2.882	68.4	81.54	0.509
4	27	45	2.66	2.877	68.3	81.68	0.484
5	30	40	2.46	2.869	68.3	81.91	0.453
6	24	45	2.66	2.886	68.4	81.43	0.523
7	34	35	2.25	2.861	68.2	82.14	0.424
8	32	35	2.25	2.866	68.2	82.00	0.440
9	24	45	2.66	2.886	68.4	81.43	0.523
Constraint	–	–	–	≤ 3	≤ 137.5	[75, 85]	$\leq \frac{f_{tck}}{4.1}$
Checks?	–	–	–	Yes	Yes	Yes	Yes

Table 2. Optimal values of slab cross-section from each kriging surface and comparison of cost values.

Kriging surface	Slab thickness (cm)	f_{ck} (MPa)	f_{tck} (MPA)	Predicted cost (€/km)	Real cost (€/km)	Cost variability	Cost reduction
1	29	40	2.46	59,530.98	61,797.84	3.67%	3.33%
2	26	45	2.66	54,445.45	57,378.88	5.11%	10.25%
3	25	45	2.66	52,654.37	55,172.00	4.56%	13.70%
4	27	45	2.66	55,097.51	59,585.76	7.53%	6.79%
5	30	40	2.46	64,342.17	63,928.80	0.64%	0.00%
6	24	45	2.66	56,823.80	52,965.12	6.79%	17.15%
7	34	35	2.25	62,036.69	66,609.40	6.86%	-4.19%
8	32	35	2.25	62,691.51	62,691.20	0.00%	1.94%
9	**24**	**45**	**2.66**	**50,880.00**	**52,965.12**	**3.94%**	**17.15%**
Mean	–	–	–	57,611.39	59,232.68	4.35%	7.35%

Table 3. Comparison of costs and impacts between the original cross-section and the optimised one.

Parameter	Original section	Optimised section	Reduction
Thickness (cm)	30	24	–
f_{ck} (MPa)	40	45	–
f_{tck} (MPa)	2.46	2.66	–
Real cost (€/km)	63,928.80	52,965.12	17.15%
Impact on resources ($)	62,425.93	52,894.84	15.27%
Impact on ecosystems (species.year)	0.00328	0.00205	37.50%
Impact on human health (DALY)	0.47278	0.38938	17.64%
Impact on safety (hours of average risk)	480,760.10	386,774.24	19.55%