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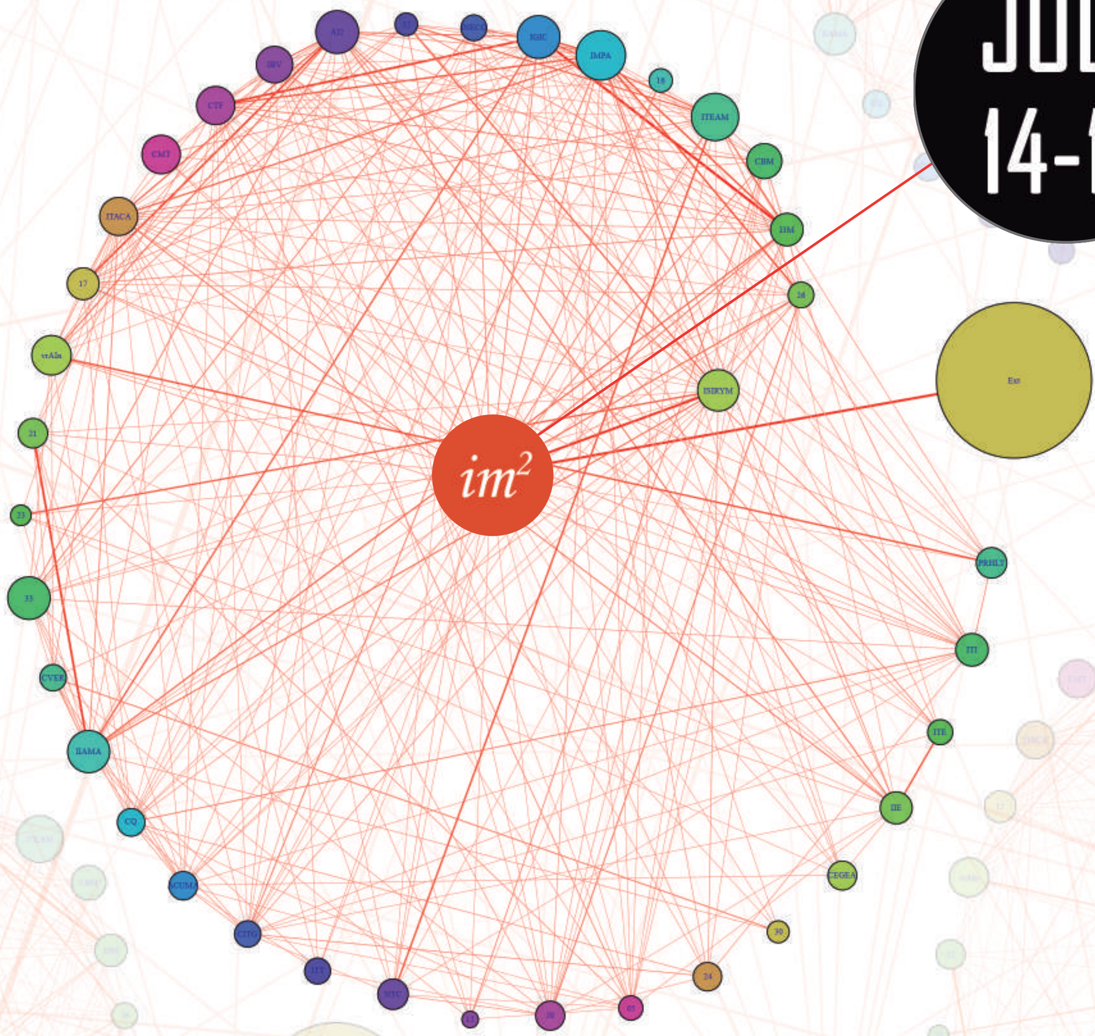

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SP$_N$ Neutron Noise Calculations

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1 Introduction

Nowadays, non-invasive detection of anomalies in nuclear power plants is possible through neutron noise monitoring. The early detection of such anomalies gives the possibility to take proper actions before they lead to safety concerns or impact on the plant availability. In this context, neutron noise is studied to analyse the effects of small perturbations in nuclear reactors that are generally produced by stochastic fluctuations of the coolant properties, such as its density or temperature, and mechanical vibration of fuel elements, controls rods and other structures in the reactor. Normally, these fluctuations are expressed as perturbations in the cross-sections of the materials of the reactor. This makes necessary to develop accurate simulations of the neutron noise against cross sections pertubations.
In this work, a full formulation of the neutron noise multigroup SP$_N$ equations in the frequency-domain is developed. This formulation differs from the classical diffusive formulations because it keeps all the terms of the time derivatives of the neutron flux [?].

2 Neutron noise in the full SP$_N$ approximation

Generally, the simplified spherical harmonics equations (SP$_N$ equations) are derived from the one-dimensional spherical harmonics equation (P$_N$ equations) where the spatial derivatives are substituted by gradients [1]. Different assumptions on the time derivatives of the neutron moments yield different formulations of the time-dependent SP$_N$ equations [3]. This Section derives the full SP$_N$ equations in frequency-domain (FSP$_N$ -FD equations) from the full SP$_N$ equation in time-domain (FSP$_N$ -TD equations), which takes into account all time-derivatives of the neutron moments. The FSP$_N$ -TD equations can be expressed as

$$\begin{aligned}\mathcal{V}\frac{\partial}{\partial t}\Phi^n + \nabla\left(\frac{n}{2n+1}\Phi^{n-1} + \frac{n+1}{2n+1}\Phi^{n+1}\right) + \Sigma^n\Phi^n \\ = \delta_{n,0}\left((1-\beta)\mathcal{R}^p\mathcal{F}\Phi^0 + \sum_{k=1}^{K}\lambda_k\mathcal{R}_k^d C_k\right), \quad n = 1,\ldots,N.\end{aligned} \tag{1}$$

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The equations for the delayed neutron precursor concentration are

$$\frac{\partial}{\partial t}\mathcal{C}_k = -\lambda_k \mathcal{C}_k + \beta_k \mathcal{F}\Phi^0\,, \qquad k = 1,\ldots,K, \tag{2}$$

where

$$\Phi^n = \left(\Phi_1^n, \Phi_2^n, \ldots, \Phi_G^n\right)^T\,, \quad \mathcal{F} = \left(\nu_1\Sigma_{f1} \quad \nu_2\Sigma_{f2} \quad \ldots \quad \nu_G\Sigma_{fG}\right), \tag{3}$$

$$\Sigma^n = \begin{pmatrix} \Sigma_{t1} - \Sigma_{s11}^n & -\Sigma_{s21}^n & \ldots & -\Sigma_{sG1}^n \\ -\Sigma_{s12}^n & \Sigma_{t2} - \Sigma_{s22}^n & \ldots & -\Sigma_{sG2}^n \\ \vdots & \vdots & \ddots & \vdots \\ -\Sigma_{s1G}^n & -\Sigma_{s2G}^n & \ldots & \Sigma_{tG} - \Sigma_{sGG}^n \end{pmatrix}, \tag{4}$$

$$\mathcal{V} = \mathrm{diag}(1/v_1, 1/v_2, \cdots, 1/v_G), \qquad \mathcal{R}^p = \left(\chi_1^p \quad \chi_2^p \quad \ldots \quad \chi_G^p\right)^T, \tag{5}$$

$$\mathcal{R}_k^d = \left(\chi_{k,1}^d \quad \chi_{k,2}^d \quad \ldots \quad \chi_{k,G}^d\right)^T. \tag{6}$$

The magnitude $\Phi_g^n = \Phi_g^n(x,t)$ is defined as the nth-moment of the neutron flux in the spherical harmonics expansion. $\mathcal{C}_k$ denotes the delayed neutron precursor concentration. Subindex g ($g = 1,\ldots,G$) refers to the energy group while subindex k ($k = 1,\ldots,K$) refers to the neutron precursors group. The total and the fission macroscopic cross-sections are denoted by Σ_{tg} and Σ_{fg}, respectively. The value of $\Sigma_{sgg'}^n$ is the nth-component of the scattering cross-section in the spherical harmonics expansion. The value of ν_g is the mean number of neutrons produced by fission. The value of v_g is the neutron velocity. The spectrum of the prompt and the delayed neutrons are χ_g^p and $\chi_{g,k}^d$. The fraction of the delayed neutrons is β_k such that the total delayed neutron fraction $\beta = \sum_k^K \beta_k$. Finally, the neutron precursor delayed constants are represented by λ_k. Usually, it is assumed that the scattering is isotropic, therefore $\Sigma_{s,gg'}^n = 0$, for $n > 1$. Moreover, the total cross-section, Σ_t, is approximated by the transport cross-section Σ_{tr}.
Following the previous notation, the steady-state multigroup SP$_N$ equations are expressed as

$$\nabla\left(\frac{n}{2n+1}\phi^{n-1} + \frac{n+1}{2n+1}\phi^{n+1}\right) + \Sigma^{n,0}\phi^n = \frac{\delta_{n0}}{k_{\mathrm{eff}}}\mathcal{R}\mathcal{F}^0\phi^0 \quad n = 1,\ldots,N, \tag{7}$$

where $\phi^n = \Phi^n(x,0)$ is the steady-state neutron flux moments

$$\mathcal{R} = \left(\chi_1 \quad \chi_2 \quad \ldots \quad \chi_G\right)^T, \tag{8}$$

where χ_g is defined as $\chi_g = \chi_g^p(1-\beta) + \sum_k^K \chi_g^{d,k}\beta_k$. Superindex 0 in $\Sigma^{n,0}$ and $\mathcal{F}^0$ indicates that the operators correspond to the value of them at initial conditions, $t = 0$. At the beginning of the transient, fission cross-sections are divided by k_{eff} when the steady-state problem is solved to assume that the reactor is in a critical state. Therefore, in the following, it is supposed that $k_{\mathrm{eff}} = 1$.
To develop the neutron noise equations, the quantities are split into the sum of the mean and the oscillatory part as

$$\Phi^n = \phi^n + \delta\Phi^n, \qquad \Sigma^n = \Sigma^{n,0} + \delta\Sigma^n,$$

$$\mathcal{F} = \mathcal{F}^0 + \delta\mathcal{F}, \qquad \mathcal{C}_k = \mathcal{C}_k^0 + \delta\mathcal{C}_k\,, \tag{9}$$

where for all the quantities p, it is assumed that $\|\delta p\|\lambda\|p^0\|$, i.e. the fluctuations are much smaller than the magnitudes in steady-state.
Substituting the neutron noise separations (9) into the equations (1) and (2), using the steady-state equations (7) and removing the second order terms, we obtain the first-order neutron noise FSP_N equations

$$\begin{aligned}\mathcal{V}\frac{\partial}{\partial t}\delta\Phi^n + \nabla\left(\frac{n}{2n+1}\delta\Phi^{n-1} + \frac{n+1}{2n+1}\delta\Phi^{n+1}\right) + \Sigma^{n,0}\delta\Phi^n + \delta\Sigma^n\phi^n \\ = \delta_{n,0}\left((1-\beta)\mathcal{R}^p(\mathcal{F}^0\delta\Phi^0 + \delta\mathcal{F}\phi^0) + \sum_{k=1}^{K}\lambda_k\mathcal{R}_k^d\delta C_k\right), \quad n = 1,\dots,N,\end{aligned} \tag{10}$$

where the equations associated with the neutron precursors are

$$\frac{\partial}{\partial t}\delta\mathcal{C}_k = -\lambda_k\delta\mathcal{C}_k + \beta_k\left(\mathcal{F}^0\delta\Phi^0 + \delta\mathcal{F}\phi^0\right), \qquad k = 1,\dots,K. \tag{11}$$

To obtain the neutron noise equations in the frequency domain, the following step is to apply the Fourier Transform, defined as,

$$f(\omega) = \mathcal{F}[f(t)] = \int_{-\infty}^{\infty}\exp\left(-i\omega t\right)f(t)dt, \tag{12}$$

to the previous equations, which permits isolating the concentration of precursors from the Equation (11) and substituting this term into the Equation (10). The full frequency-domain equation, once the precursors term is removed, has the form

$$\begin{aligned}i\omega\mathcal{V}\delta\Phi_g^n + \nabla\left(\frac{n}{2n+1}\delta\Phi^{n-1} + \frac{n+1}{2n+1}\delta\Phi^{n+1}\right) + \Sigma^{n,0}\delta\Phi^n - \delta_{n,0}\Gamma\mathcal{F}^0\delta\Phi^0 \\ = -\delta\Sigma^n\phi^n + \delta_{n,0}\Gamma\delta\mathcal{F}\phi^0, \qquad n = 1,\dots,N,\end{aligned} \tag{13}$$

where

$$\Gamma = (1-\beta)\mathcal{R}^p + \sum_{k=1}^{K}\frac{\lambda_k\beta_k}{i\omega + \lambda_k}\mathcal{R}_k^d. \tag{14}$$

3 Numerical results

The full and diffusive SP_N equations in frequency-domain are spatially discretized by using a continuous Galerkin finite element method with Lagrange polynomials of order 3. This discretiazation is implemented in `C++` by using structures from the open-source libraries `deal.II` [4] and `PETSc` [5]. This part has been developed as an extension of the open source neutronic code `FEMFFUSION` [6]. More details about the implementation of the time-dependent SP_3 equations with the finite element method are found in [3].
To test the frequency-domain SP_N equations proposed, we solve a two-dimensional a 2-dimensional simplified UOX fuel assembly for Pressurized Water Reactors (PWRs). The simplified fuel assembly is shown in Figure 1. The system includes 264 homogeneous square fuel pins and 25 homogeneous water holes. The size of the system is 21.58 cm × 21.58 cm, the size of the fuel pin is 0.7314 cm × 0.7314 cm, and the size of the water hole is 1.26 cm × 1.26 cm . The assembly is surrounded by a water blade of thickness equal to 0.08 cm. The boundary conditions are reflective. The nuclear data are generated with respect to 2 energy groups, and scattering is assumed to be isotropic. The cross sections for this benchmark can be found in [7].

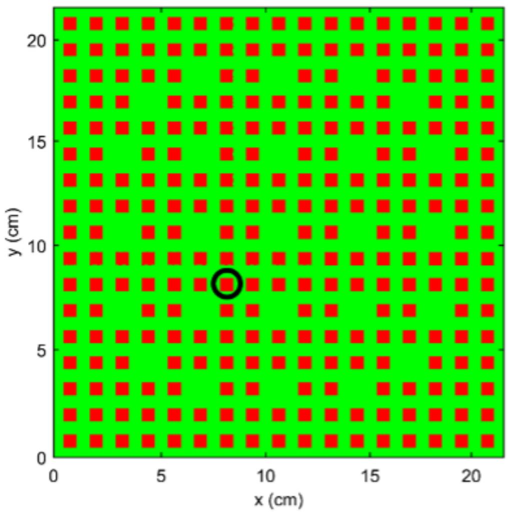
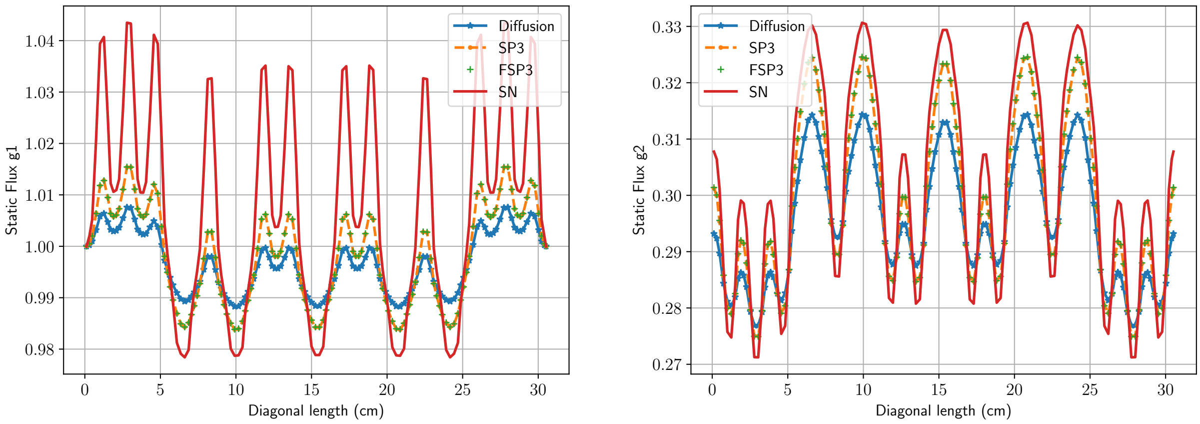


Figure 1: Fuel assembly problem.

A perturbation located in the pin marked with a circle in Figure **??** is introduced at $t = 0$:

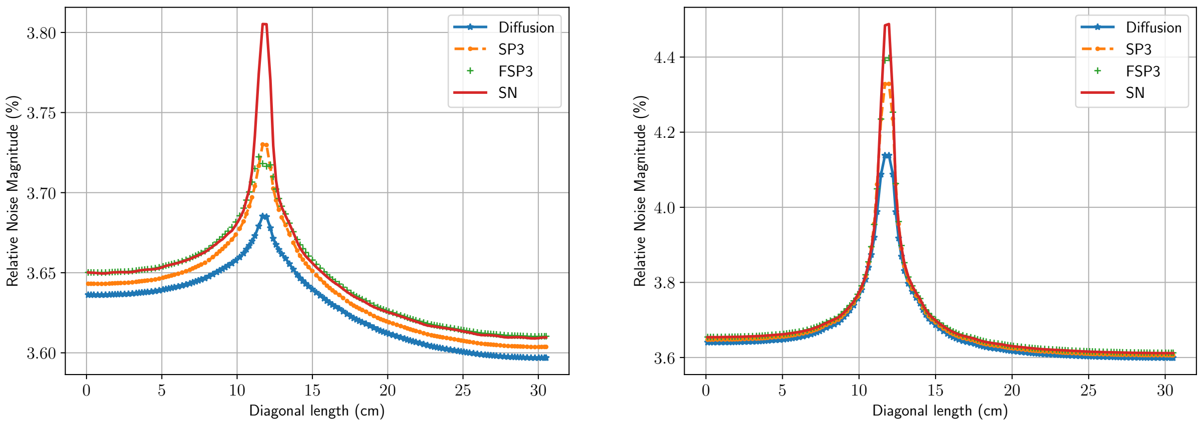
$$\delta\Sigma_t(t) = 0.041\,\Sigma_t^0 \sin(\omega_0 t)\,,$$
$$\delta\Sigma_s(t) = 0.034\,\Sigma_s^0 \sin(\omega_0 t)\,,$$
$$\delta\Sigma_f(t) = 0.021\,\Sigma_f^0 \sin(\omega_0 t)\,.$$

The angular frequency of the perturbation is set to $\omega_0 = 2\pi$, in other words, a frequency of 1 Hz. This type of perturbation over the macroscopic cross-sections is a generic kind that can construct all other types of perturbations.
Figure 2 shows the static results for the diffusion approximation (DSP_1), the Diffusive SP3 approximation (DSP_3), the Full SP3 approximation (FSP_3) and a reference SN calculations (S32) from [7,8]. It must be taken into account that the steady-state FSP_3 and DSP_3 formulations are mathematically equivalent. SP_3 approximations represent more accurately the neutron flux, specially when there is a strong change in the gradient of the neutron flux. This difference can be seen in the peaks in the centre of the water blades and the fuel pins.
Figure 3 shows the relative neutron noise amplitude (%) and Figure 4 shows the neutron noise phase in the two-dimensional benchmark for DSP_1 , DSP_3 and FSP_3 . On the one hand, an improvement over the diffusion approximation can be seen for DSP_3 and FSP_3 formulations. On the other hand, FSP_3 does not show remarkable difference with respect to its diffusive approximation, DSP_3 . It must be noted that the linear system associated with FSP_3 eqautions is much bigger than the DSP_3 system, and its associated matrix is worse conditioned [2]. Due to the difficulties to solve the Full SP_3 approximation and the no significant differences found with respect the DSP_3 , the full formulation is not recommended for this type of problems. Similar results can be observed for the neutron noise phase. However, the global change in the phase is not relevant as it is less than 1 degree.



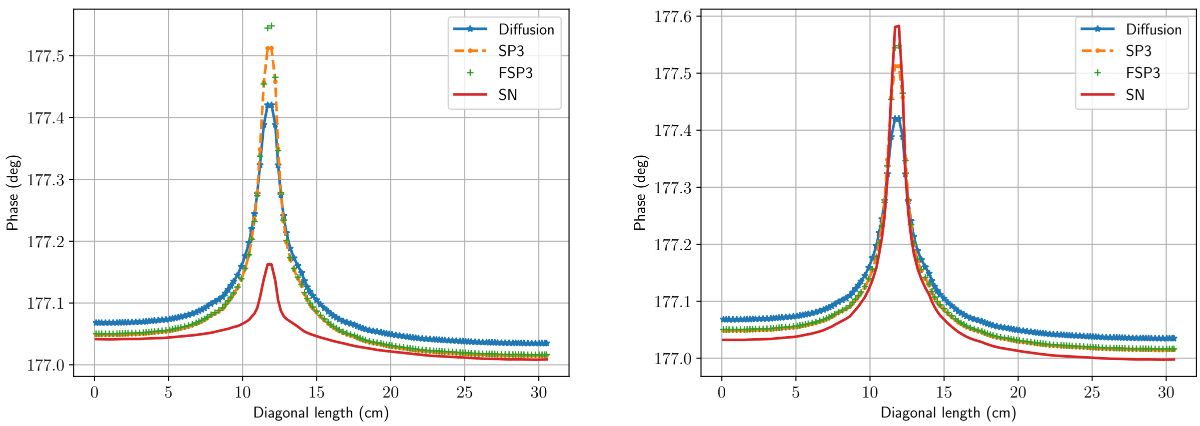
(a) Fast Flux (b) Thermal Flux

Figure 2: Steady-state solution.



(a) Fast Flux Noise Amplitude (b) Thermal Flux Noise Amplitude

Figure 3: Relative noise amplitudes in time-domain and frequency-domain calculation.



(a) Fast Flux Noise Phase (b) Thermal Flux Noise Phase

Figure 4: Noise phase.

4 Conclusions

This work presents a rigorous formulation of the neutron noise SP_N equations in the frequency domain. A comparison between this formulation and the diffusive approximation is provided studying a two-dimensional benchmark. A continuous Galerkin finite element method is implemented to solve these equations.
Approximations of order $N = 3$ obtain more accurate results than the diffusion approximation ($N = 1$) in neutron noise problems without excessive computational demands. Between the diffusive and full formulations, the differences are negligible. Therefore, to provide an accurate solution, with reasonable computational demands, the diffusive $\mathrm{SP_N}$ approximation is recommended.

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