MODELLING FOR ENGINEERING & HUMAN BEHAVIOUR 2022

PROCEEDINGS

Edited by

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Modelling for Engineering & Human Behaviour 2022

València, July 14th-16th, 2022

This book includes the extended abstracts of papers presented at XXIV Edition of the Mathematical Modelling Conference Series at the Institute for Multidisciplinary Mathematics *Mathematical Modelling in Engineering & Human Behaviour*.

I.S.B.N.: 978-84-09-47037-2

November 30th, 2022
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Edited by: I.U. de Matemàtica Multidisciplinar, Universitat Politècnica de València.
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Probabilistic analysis of a cantilever beam with load modelled via Brownian motion

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1 Introduction

In this contribution we deal with the stochastic analysis of the deflection of a cantilever beam with its load modelled through Brownian motion. This phenomenon can be mathematically described by means of the following fourth-order differential equation [1]

$$\frac{\mathrm{d}^4 Y(x)}{\mathrm{d}x^4} = \frac{W(x)}{EI}, \quad 0 < x < l, \tag{1}$$

with boundary conditions

$$Y(0) = 0,\ Y'(0) = 0,\ Y''(l) = 0,\ Y'''(l) = 0, \tag{2}$$

where $Y(x)$ represents the deflection of the beam, E is the Young's modulus of elasticity of the material of the beam, I denotes the moment of inertia of the cross section of the beam around a horizontal line through the centroid of the cross section (hereinafter, we will denote it as i, indicating that it will be a deterministic value), l is the length of the beam and $W(x)$ represents the density of downward force acting vertically on the beam at the space point x. We will assume that E and $W(x)$ are aleatoric factors due to the heterogeneity of the material of the beam and the load. For the latter, we will assume that $W(x)$ is given by the sum of a deterministic value, w_0 and a certain random quantity given by Brownian motion, $B(x)$

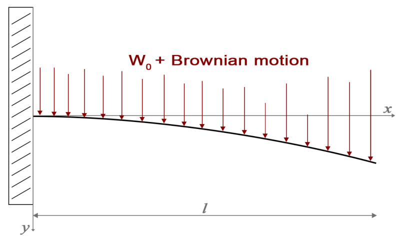

Figure 1: Cantilever beam with load modelled via Brownian motion.

$$W(x) = w_0 + B(x), \quad 0 < x \le l. \tag{3}$$

In Figure 1 we can observe a scheme of this model.

The aim of this contribution is to obtain the first probability density (1-PDF) function of the stochastic solution, and other quantities of interest, as the maximum deflection and slope, using the Random Variable Transformation method (RVT) [2], which we will develop in Section 2. All these theoretical findings are illustrated via a numerical example in Section 2.

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2 Computing the first probability density function

This section is devoted to obtain the 1-PDF of the stochastic solution that represent the deflection of the cantilever beam, and other quantities of interest. In order to do this, we will take advantage of the Karhunen–Loève expansion of the Brownian motion [3]

$$B(x) = \mu_B(x) + \sum_{j=1}^{\infty} \sqrt{\nu_j}\phi_j(x)\xi_j(\omega), \;\; \omega \in \Omega, \; 0 < x \leq l, \tag{4}$$

where $\mu_B(x) = 0$, $\xi_j(\omega)$ are independent and identically distributed random variables, $\xi_j(\omega) \sim \mathrm{N}(0,1)$, $j = 1, 2, \ldots$, and

$$\nu_j = \frac{4l^2}{\pi^2(2j-1)^2}, \quad \phi_j(x) = \sqrt{\frac{2}{l}} \sin\left(\frac{(2j-1)\pi}{2l}x\right), \quad j = 1, 2, \ldots$$

are the eigenvalues and eigenfunctions obtained when solving the homogeneous Fredholm integral equation of second kind [4]. Now, we will consider the approximation of $B(x)$ obtained by truncating its Karhunen–Loève expansion at N. So, the model is approximated via the following stochastic differential equation

$$\frac{\mathrm{d}^4 Y(x)}{\mathrm{d}x^4} = \frac{1}{Ei}\left(w_0 + \sum_{j=1}^{N} \sqrt{\nu_j}\phi_j(x)\xi_j(\omega)\right), \quad 0 < x < l. \tag{5}$$

To obtain the 1-PDF of the deflection we first need to explicitly calculate the stochastic solution of model (5)-(2). We can use, for example, the Laplace transform to obtain the solution which is given by the following parametric stochastic process, $0 < x \leq l$,

$$\begin{aligned} Y(x) = \frac{1}{Ei}\Bigg(& \frac{x^2}{2}\left(\frac{w_0}{2}l^2 + l\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{1-\cos(b_j l)}{b_j^2} - \sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{b_j l - \sin(b_j l)}{b_j^3}\right) \\ & + \frac{x^3}{6}\left(-w_0 l - \sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{1-\cos(b_j l)}{b_j^2}\right) + \frac{w_0}{24}x^4 + \sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{-6b_j x + b_j^3 x^3 + 6\sin(b_j x)}{6b_j^5}\Bigg). \end{aligned} \tag{6}$$

Then, we are going to use RVT method to obtain the expression of the 1-PDF of (6). In short, RVT is a method to obtain the PDF of a random vector V that results from mapping of another random vector U whose PDF is known. We have considered that E and ξ_j, $j = 1, \ldots, N$ are independent whose PDF's are known. In order to make this abstract a light read, we suppress the calculations and give the final result of the 1-PDF, that is given by

$$\begin{aligned} f_{Y(x)}(y) = \mathbb{E}_{\xi_1,\ldots,\xi_N}\Bigg[f_E\Bigg(& \frac{1}{Ei}\Bigg(\frac{x^2}{2}\left(\frac{w_0}{2}l^2 + l\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{1-\cos(b_j l)}{b_j^2} - \sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{b_j l - \sin(b_j l)}{b_j^3}\right) \\ & + \frac{x^3}{6}\left(-w_0 l - \sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{1-\cos(b_j l)}{b_j^2}\right) + \frac{w_0}{24}x^4 + \sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{-6b_j x + b_j^3 x^3 + 6\sin(b_j x)}{6b_j^5}\Bigg)\Bigg) \\ & \cdot \Bigg| -\frac{1}{E^2 i}\Bigg(\frac{x^2}{2}\left(\frac{w_0}{2}l^2 + l\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{1-\cos(b_j l)}{b_j^2} - \sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{b_j l - \sin(b_j l)}{b_j^3}\right) \\ & + \frac{x^3}{6}\left(-w_0 l - \sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{1-\cos(b_j l)}{b_j^2}\right) + \frac{w_0}{24}x^4 + \sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{-6b_j x + b_j^3 x^3 + 6\sin(b_j x)}{6b_j^5}\Bigg)\Bigg|\Bigg], \quad 0 < x \leq l. \end{aligned} \tag{7}$$

Other important characteristics that we can obtain are the maximum deflection, D, and the maximum slope, S, these being a cantilever beam, are obtained at the free end of the beam. The maximum deflection is obtained by evaluating the solution at l and the maximum slope by evaluating the derivative at l and are given by the following expressions

$$\begin{aligned} D = Y(l) =& \frac{1}{Ei}\left(\frac{w_0}{8}l^4 + \frac{1}{3}l^3\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{1-\cos(b_jl)}{b_j^2} - \frac{1}{2}l^2\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{b_jl-\sin(b_jl)}{b_j^3}\right.\\ &\left.+\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{-6b_jl+b_j^3l^3+6\sin(b_jl)}{6b_j^5}\right), \end{aligned} \tag{8}$$

and

$$\begin{aligned} S = Y'(l) =& \frac{1}{Ei}\left(\frac{w_0}{8}l^3 + \frac{1}{2}l^2\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{1-\cos(b_jl)}{b_j^2} - l\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{b_jl-\sin(b_jl)}{b_j^3}\right.\\ &\left.+\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{-2+b_j^2l^2+2\cos(b_jl)}{2b_j^4}\right). \end{aligned} \tag{9}$$

Then, applying again RVT method we can obtain the PDF of the maximum deflection,

$$\begin{aligned} f_D(d) =& \mathbb{E}_{\xi_1,\ldots,\xi_N}\left[f_E\left(\frac{1}{di}\left(\frac{w_0}{8}l^4 + \frac{1}{3}l^3\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{1-\cos(b_jl)}{b_j^2} - \frac{1}{2}l^2\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{b_jl-\sin(b_jl)}{b_j^3}\right.\right.\right.\\ &\left.\left.+\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{-6b_jl+b_j^3l^3+6\sin(b_jl)}{6b_j^5}\right)\right)\left|-\frac{1}{\delta^2 i}\left(\frac{w_0}{8}l^4 + \frac{1}{3}l^3\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{1-\cos(b_jl)}{b_j^2}\right.\right.\\ &\left.\left.\left.-\frac{1}{2}l^2\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{b_jl-\sin(b_jl)}{b_j^3} + \sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{-6b_jl+b_j^3l^3+6\sin(b_jl)}{6b_j^5}\right)\right|\right], \end{aligned} \tag{10}$$

and the PDF of the maximum slope

$$\begin{aligned} f_S(s) =& \mathbb{E}_{\xi_1,\ldots,\xi_N}\left[f_E\left(\frac{1}{si}\left(\frac{w_0}{8}l^3 + \frac{1}{2}l^2\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{1-\cos(b_jl)}{b_j^2} - l\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{b_jl-\sin(b_jl)}{b_j^3}\right.\right.\right.\\ &\left.\left.+\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{-2+b_j^2l^2+2\cos(b_jl)}{2b_j^4}\right)\right)\left|-\frac{1}{\theta^2 i}\left(\frac{w_0}{8}l^3 + \frac{1}{2}l^2\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{1-\cos(b_jl)}{b_j^2}\right.\right.\\ &\left.\left.\left.-l\sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{b_jl-\sin(b_jl)}{b_j^3} + \sqrt{\frac{2}{l}}\sum_{j=1}^{N}\xi_j\frac{-2+b_j^2l^2+2\cos(b_jl)}{2b_j^4}\right)\right|\right]. \end{aligned} \tag{11}$$

3 Numerical example

This section is dedicated to illustrate the above theoretical conclusions. We take the following data for the deterministic parameters of the model (5): the length of the beam, $l = 10$ m, the moment of inertia, $i = 722 \cdot 10^{-8}$ m^4, and $w_0 = 20$. For the random parameters, we will assume that the Young's modulus of elasticity, E, has a Gaussian distribution, $E \sim \mathrm{N}(210 \cdot 10^9; 420 \cdot 10^7)$. We will consider a Karhunen-Loève expansion truncated at order $N = 1$ to approximate the Brownian motion, $B(x)$.
In Figure 2, we show the graphical representation of the 1-PDF at different spatial points. We can observe, that the variance increases as the position does.

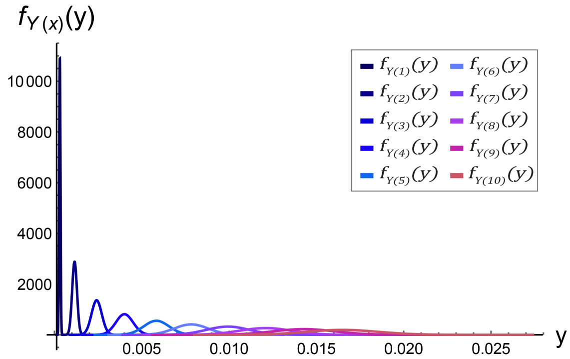

Figure 2: 1-PDF, $f_{Y(x)}(y)$, of the solution stochastic process (6), computed by (7), at different spatial position $x \in \{1, ..., 10\}$ of the cantilever beam considering an approximation of the Brownian motion, $B(x)$, via a Karhunen-Loève expansion truncated at order $N = 1$.

In Figure 3, we show on the left hand side the plot of the PDF of the maximum deflection and on the right hand side the PDF of the maximum slope.

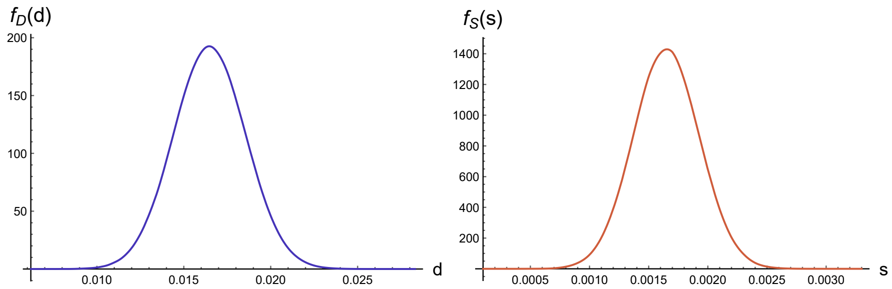

Figure 3: Left: PDF, $f_D(d)$, of the maximum deflection at the free end. Right: PDF, $f_S(s)$, of the maximum slope at free end.

Finally, in Figure 4 we show the graphical representation of the maximum deflection for different values of the truncation order, N. We can observe in this zoom, that the approximations are very similar with $N = 1$.

4 Conclusions

In this work, we have obtained a probabilistic description via the first probability density function of the stochastic solution of the fourth-order random differential equation, that describes the deflection of a cantilever beam whose load is modelled through Brownian motion. In addition, we have obtained other densities of quantities of interest such as the maximum deflection and the maximum slope at the free end of the beam. In order to obtain the probability density function we have taken advantage of the Random Variable Transformation method. Finally, we have illustrated these theoretical findings with a numerical example.

Acknowledgments

This work has been supported by the grant PID2020-115270GB-I00 funded by MCIN/AEI/10.13039/501100011033 and the grant AICO/2021/302 (Generalitat Valenciana).

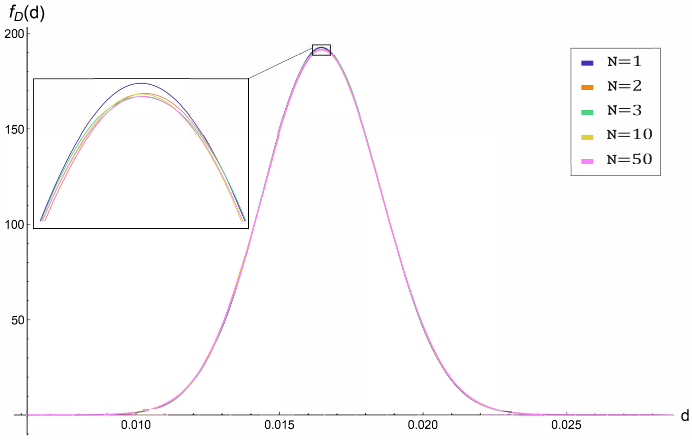

Figure 4: PDF of the maximum deflection at free end, $f_D(d)$, for different values of the truncation order, N, to approximate the Brownian motion by its Karhunen-Loève expansion.

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