Computational Tools in Cosmology

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1 Introduction

Following the line started in [1], some interesting numerical and computational techniques used in part of our research in Cosmology are now presented. As remarked in [1] (with other algorithms we built), we also think the tools described in this conference presentation may work in different tasks concerning our research fields. Moreover, they may even be extended in Science and Technology in general. The approaches showed here have been developed in N-body algorithms applied when describing the CMB (Cosmic Miicrowave Background) anisotropies (see [2, 3, 4, 5, 6]). In our methods, innovations in CMB maps treatments are performed. Such novelties may be extended to other studies.

2 Methods

Along more than a decade, we have presented the advance of our CMB anisotropy computations using N-body codes. The codes with more resolution and precision we used were the N-body Hydra ones (see [3]). All versions were designed by members of the Hydra Consortium. We used 1) Codes without baryons. 1.a) Sequential versions. 1.b) Parallel ones. With both of them we computed the weak lensing (WL) and the Rees-Sciama (RS) contributions to the CMB angular power spectrum.

Using our numerical techniques, we reported a higher contribution –to lensing– than previous approaches. Our CMB anisotropies computations on every step of the run allowed less interpolations and approximations. This could be the explanation of our excess of power in lensing computations. Our higher resolution could also contribute to this excess.

Afterwards, we also performed computations with baryons (see [6]). This version allowed us to compute Sunyaev-Zel'dovich (SZ) contribution to the CMB angular power spectrum too.

An appropriate ray-tracing procedure through N-body simulations was proposed in the following basic references: [7, 8]. In these papers it was explained how to chose a preferred direction (PD) to cross the N-body simulated boxes. Such directions wee chosen to reach the initial position after passing through 16 boxes. For a box size $L = 512h^{-1}Mpc$ the distance between points entering and leaving each box was $\sim 104h^{-1}Mpc$. So, one had independent regions from redshift $z \sim 6$ ($\sim 5900h^{-1}Mpc$). For our computations, starting in $z \sim 6$ was sufficient. There was no need to start computations at higher redshifts. Applications based on our ray-tracing methods through PM simulations can be seen, for instance, in: [9, 2].

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Some important computational details of the map construction are now detailed.

For weak lensing (see [3]), small, unlensed maps of CMB temperature contrasts ($\Delta = \delta T/T$) were constructed to be subsequently deformed by lensing. In order to deform the unlensed maps, the lens deviations corresponding to a set of directions, covering an appropriate region of the sky, were calculated. These deviations corresponded to the quantities:

$$\vec{\delta} = -2 \int_{\lambda_e}^{\lambda_0} W(\lambda) \vec{\nabla}_{\perp} \phi \; d\lambda \; , \tag{1.1}$$

where $\vec{\nabla}_{\perp}\phi = -\vec{n} \wedge \vec{n} \wedge \vec{\nabla}\phi$ is the transverse gradient of the peculiar gravitational potential ϕ, and $W(\lambda) = (\lambda_e - \lambda)/\lambda_e$. The variable λ is:

$$\lambda(a) = H_0^{-1} \int_a^1 \frac{db}{(\Omega_{m0} b + \Omega_{\Lambda} b^4)^{1/2}} \; . \tag{1.2}$$

Once the deviations were calculated, they could be easily used to get the lensed maps from the unlensed ones. This was achieved using the relation:

$$\Delta_{_L}(\vec{n}) = \Delta_{_U}(\vec{n} + \vec{\delta}) \; , \tag{1.3}$$

where $\Delta_{_L}$ and $\Delta_{_U}$ are the temperature contrasts of the lensed and unlensed maps, respectively. The unit vector $\vec{n}$ defines the observation direction (line of sight).

Given the unlensed map $\Delta_{_U}$, and the map $\Delta_{_L}$ obtained from it after deformation by lensing (the lensed map), the chosen power spectrum estimator could be used to get the quantities $C_\ell(U)$ and $C_\ell(L)$, whose differences $C_\ell(LU) = C_\ell(L) - C_\ell(U)$ could be considered as an appropriate measure of the weak lensing effect on the CMB.

For the Rees-Sciama contribution we computed the integral (see [2, 4]):

$$\frac{\Delta T}{T_B}(\vec{n}) = 2 \int_{\lambda_e}^{\lambda_0} W(\lambda) \; \frac{\partial \phi}{\partial \lambda} \; d\lambda \; , \tag{1.4}$$

where ϕ is the peculiar gravitational potential ϕ, $W(\lambda) = (\lambda_e - \lambda)/\lambda_e$ and λ is given in eq(1.2).

For the Sunyaev-Zel'dovich thermal contribution in the long wave regimes we computed the integral (see [6]):

$$\frac{\Delta T}{T_B}(\vec{n}) = -2 \; \frac{\sigma_T}{m_e c^2} \int_{\lambda_e}^{\lambda_0} n_e \; k \; T_e \; d\lambda \; , \tag{1.5}$$

where the subscript e refers to electrons.

Notice that we had to define different weak lensing regimes. Basically, this is the way to proceed (see [3]):

- *AWL* (A weak lensing), namely the effect due to scales $k > 2\pi/L_{max}$ (where $L_{max} = 42h^{-1}$ Mpc) at redshifts $z < 6$. This signal is dominated by strongly nonlinear scales (namely structures).
- *BWL*, the lensing signal due to scales $k < 2\pi/L_{max}$ which corresponds to modes that are always in the linear regime down to $z = 0$.
- *CWL*, the lensing signal due to scales $k \geq 2\pi/L_{max}$ but at redshifs $z > 6$.
- *RS*, the same regimes that one has for WL apply for RS.
- *SZ*, this distinction does not apply.

We now describe the main features of the algorithm designed to compute the physical quantities described above and that allowed to study the CMB anisotropies using the Hydra N-body codes (see [3]):

1. Decide upon the **direction** of the normal rays representing the geodesics.
2. Assuming the Born approximation and using the **photon step distance** Δ_{ps}, determine all the evaluation positions and times on the geodesics within the simulation volume from $z = 6$ down to the final redshift.
3. Associate **test particles** with each of these positions and times.
4. At each time-step of the N-body simulation (while it is running) determine which test particles require **force evaluations** (or other physical quantities depending on the CMB anisotropy effect to be computed).
5. At each test particle position evaluate the **force** (or corresponding physical quantity) on the test particle using the long-range **FFT** component and short-range **PP correction** as in the HYDRA algorithm.
6. During the FFT convolution for the test particles **eliminate** contributions from **scales larger than** $42h^{-1}$ **Mpc** by removing the signal from wavenumbers satisfying $k \leq 0.15h$ Mpc^{-1}.
7. If the evaluation time for a point on the geodesic lies between two time-steps calculate a **linear interpolation** of the two forces from the time-steps that straddle **the correct time**.
8. Resolve the **force** into its **transverse component** and hence recover the transverse component of the potential gradient. This applies for WL. For RS and SZ, see [2, 4] and [6], respectively, for the physical quantities to be computed (potential in Eq. (1.4) and electrons temperatures in Eq. (1.5), respectively).

3 Results

Simulations were performed in the framework of the concordance model with the following parameters: $h = 0.7$, $\Omega_b = 0.046$, $\Omega_d = 0.233$, $\Omega_\Lambda = 0.721$, optical depth $\tau = 0.084$ and $\sigma_8 = 0.817$. The power spectrum of the scalar (adiabatic) energy density perturbations was obtained with the CMBFAST code. No tensor modes were considered at all.

One of the numerical advances of our work was that the correlation function $\xi(r)$ extracted from one simulation (instead of 30 which was the usual method) was sufficient. This showed that our nonlinear algorithm was very robust. Besides, its form was that expected for the softening length and the box size. The code worked very well in spite of the modifications required by our CMB calculations.

For lens deformations (see [3]), the angular power spectra for one simulation was compared to the same simulation but where deflections were calculated by including an average over the 8 nearest geodesics. This reduced the resolution of the geodesic method, but maintained the same resolution in the gravitational solver. The resulting power spectrum was plotted in Figure 10 of [3] and showed a decaying signal at high ℓ which was similar to that found in earlier works (e.g. [10]). This showed that as we degraded the resolution of our ray-tracing method we indeed had close results to those of previous works with less resolution than ours. Therefore, the local averages, used in methods of other authors, might hide the highly nonlinear structures effects. Notice that these structures had a relative small size.

For RS (see [2, 4]) and SZ (see [6]), the results we obtained were of the order of magnitude or slightly greater than those obtained by other authors.

4 Conclusions

Our AP3M codes adapted to CMB calculations could be run for different values of the parameters defining the simulations; hence, this code allowed us to see how the resulting angular power spectra depended on the parameters defining both the N-body simulation and the ray-tracing procedure.

Simulations in boxes of $512h^{-1}$ Mpc led to good $C_\ell(LU)$ spectra for $1000 < \ell < 7000$. For $2000 < \ell < 7000$, all the simulations lied in a region of width $\sim$0.5 μK, indicating that the simulations gave consistent estimates of the signal in this range (see [3]). The signal in the range $4000 < \ell < 7000$ is 2.0 ± 0.4 μK, which is ~ 1.4 μK higher than that found elsewhere [10].

The values we obtained were compatible with studies based on the Millennium simulation (see [11]), where the authors reported a small contribution from nonlinearity at $\ell \simeq 4100$. However, the methods of [11] were designed to build all-sky lensed maps, and therefore did not have the necessary resolution to perform an accurate estimate of the weak lensing by strongly nonlinear structures in the ℓ-interval where we found our main effect.

Now we are working on the analysis and description of the numerical advances we have made in all the research described in the present paper. Such as the improvements we made on the resolution of N-body algorithms, on our FFT subroutines and on numerical parallelisation technics. Also our ameliorations on the numerical treatment of images necessary to extract the power spectrum of CMB. This work will be presented in the near future.

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