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Analytical Models for Randomized Neighbor Discovery Protocols based on Collision Detection in Wireless Ad Hoc Networks

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Abstract

Neighbor discovery is a crucial first step after the deployment of wireless ad hoc networks, which do not have a communications infrastructure. In this paper we present analytical models of randomized neighbor discovery protocols for static one-hop environments: CDPRR (Collision Detection Probabilistic Round Robin) and CDH (Collision Detection Hello). For CDPRR we assumed a geometric distribution and a uniform distribution for CDH. For comparison purposes, we chose two protocols from the literature, i.e., Hello and PRR, to be used as reference. To assess the performance of the proposals we carried out a mathematical study regarding six metrics, i.e., neighbor discovery time, energy consumption, overhead (number of packets sent), packet delivery ratio, the CDF of discoveries, and percentage of idle slots, and presented graphical results obtained from the equations. According to the analytical results, CDH protocol outperforms the other solutions regarding the neighbor discovery time, energy consumption, number of packets sent, packet delivery ratio and CDF of discoveries, while CDPRR achieves good results and it is better than Hello and PRR in terms of neighbor discovery time, energy consumption, CDF of discoveries and the overhead (number of packets sent). Moreover, we found that CDPRR presents more percentage of idle slots than PRR, which is a clear advantage in terms of energy consumed and number of packets sent. In addition, as novelty compared to the reference protocols, we found that both CDH and CDPRR protocols manage to discover all the neighbors, know when to terminate the discovery process, and achieve to operate under more realistic assumptions. We also focused the study on the CDH protocol varying the slot width, and demonstrated that the number of nodes in the network can be unknown, still providing reasonable results.

Index Terms

wireless ad hoc networks, neighbor discovery, randomized protocol, analytical model, one-hop, collisions.

I. INTRODUCTION

Wireless ad hoc networks are a special type of networks which do not provide a communications infrastructure right after their deployment, and neighbor discovery must be carried out as a first step [1][2] since the neighbors are a-priori unknown, so that the nodes are able to establish a communications infrastructure and discover the nodes within transmission range. Therefore, the scenario is that a few nodes can send messages directly to the nodes within transmission range, also known as one-hop neighbors, while other nodes require multiple intermediate nodes to forward the information in a multi-hop manner. To achieve this behavior, each node must be able to act as a router [3][4].

On one side, wireless ad hoc networks can be classifed as (1) static, in which the nodes can not move in the deployment area (e.g., sensor networks, whose nodes are placed in a forest [5] so that they can monitor several parameters such as fire, or humidity) or (2) mobile, in which the node can move throughout the deployment area (e.g., mobile robots which aim at exchanging data [6]).

On the other hand, the algorithms that overcome the Neighbor Discovery problem can be (1) randomized, each node transmitting in a randomly chosen time or state (i.e., transmitting or listening), time is slotted in rounds, and all the neighbors are discovered with high probability, and (2) deterministic, whose nodes transmit according to a schedule and often introduce unrealistic assumptions as synchronization and knowledge of the number of neighbors, and manage to discover all the neighbors with probability 1.

As for the energy efficiency, it is an important issue to consider, since the devices are powered by batteries that may deplete in a given amount of time.

Among the applications of wireless ad hoc networks [7], we can quote the following: industrial (e.g., mesh networks), medical (e.g., monitor patient), military (e.g., hostile environments) or teaching.

As motivations for our work, many deterministic protocols from the literature need a transmission schedule for neighbor discovery, whereas some randomized protocols require non-realistic assumptions such as ignoring the termination condition of the discovery process, lack of knowledge of the number of neighbors and they do not manage to discover all the neighbors with probability 1. In addition, little theoretical analysis is available in the literature, especially regarding metrics such as throughput, overhead, packet loss ratio, percentage of discoveries per round, or percentage of idle slots.

Therefore, the main objective of our work is to propose an analytical model of randomized protocols which do not rely on a transmission schedule, take into account the existence of channel collisions, follow more realistic assumptions and aim at presenting better performance than existing solutions.

The problem statement we must cope with for neighbor discovery protocols includes: the protocols must be suitable for its use in static environments, the nodes include limited range radio transceivers thus half-duplex mode is available, the nodes are randomly deployed in a given area, the nodes should operate asynchronously, collisions may exist, the nodes must be able to detect collisions, the number of nodes should be unknown and the nodes should be able to start transmission at different time instants, the nodes must be able to discover all their neighbors with probability 1 and know when to terminate the discovery process, i.e., when all the neighbors have been discovered.

In this paper we focus our work on the presentation of analytical models for two randomized neighbor discovery protocols based on collision detection in static one-hop wireless ad hoc networks in the presence of collisions, that take place when two or more nodes try to transmit at the same time.

The CDPRR (Collision Detection Probabilistic Round Robin) proposal knows when to terminate the discovery process, achieves to discover all the neighbors with probability 1 although it requires to know the number of nodes. As for the CDH (Collision Detection Hello) proposal, it includes a termination detection mechanism, also achieves to discover all the neighbors with probability 1 and it does not require to know the number of nodes.

Notice that for both proposals we aim at discovering all the neighbors with probability 1, thus improving existing randomized protocols. We also focus on improving several metrics, such as the neighbor discovery time, the energy consumption, the overhead (number of packets sent), the packet delivery ratio, the CDF of discoveries, and the percentage of idle slots, following more realistic assumptions.

The main contributions of this work are: (i) CDPRR (a randomized proposal based on collision detection and PRR) with fixed transmission probability $\frac{1}{N}$ during all the neighbor discovery process that can be used both in one-hop and multi-hop environments, (ii) CDH (a randomized proposal based on collision detection and Hello) that can be used both in one-hop and multi-hop environments, (iii) an analytical model of both proposals CDPRR and CDH in terms of six metrics, i.e., neighbor discovery time, energy consumption, overhead (number of packets sent), packet delivery ratio, CDF of discoveries, and percentage of idle slots, in a one-hop scenario, (iv) an analytical model of two reference protocols (i.e., Hello and PRR) in terms of the same six metrics also in a one-hop scenario.

The outline of this paper is as follows: A brief related work can be found in section II, a description of both proposals is carried out in section III, an analytical model of our proposals is presented in section IV, an analysis of reference protocols is included in section V, graphical results obtained from the equations are shown in section VI, a brief discussion takes place in section VII, and finally in section VIII some concluding remarks are made.

II. RELATED WORK

In the literature, there are many works available which deal with the neighbor discovery, some of which have been chosen to be discussed.

A. Probabilistic protocols

In [5] an energy-efficient protocol for static wireless ad hoc networks is presented, that belongs to the Birthday protocols, a family of probabilistic protocols. [5] also presents the PRR (Probabilistic Round Robin), i.e., an analog of the deterministic round robin scheduling algorithm, which can maximize the probability of neighbor discovery, although it does not present good energy efficiency, and it may fail to discover some of the neighbors in dense networks.

In [8], authors focus on the impact of collisions on neighbor discovery in static multi-hop wireless networks. Basic Hello protocol and Energy-aware Hello protocol are presented, the latter one managing to reduce the energy consumption.

Authors present in [2] several randomized protocols for static networks, discuss their resulting performance which depend on the assumptions taken into account. ALOHA-like algorithm for one-hop networks of N nodes, that manages to discover all the neighbors in $O(N \ln N)$, is presented. An order-optimal protocol in one-hop networks, which allows to discover all the neighbors in $O(N)$, a reasonable result achieved even when nodes can not detect collisions. Authors found that the absence of an estimate of the number of neighbors N or the lack of synchronization results at most in a slowdown of no more than a factor of two in the performance, in comparison to when nodes know N or when nodes are synchronized. In conclusion, some of the proposals allow nodes to begin execution at different time instants, and know when to terminate the neighbor discovery. Finally, an extension to a more general multi-hop wireless environment, which achieves better performance than the ALOHA-line algorithm is also available.

FRIEND, a synchronous full-duplex randomized pre-handshaking protocol for static networks can be found in [9] and [1]. Half-duplex operation, multi-hop scenarios, and duty cycled networks are also presented. According to analytical and simulation results the proposals in [9] and [1] manage to improve the time consumption of neighbor discovery by up to 68% in comparison to the ALOHA-like protocols presented earlier in [2]. In addition, the protocols achieve to reduce the probability of generating collisions.

Direct Algorithm and Group Testing with Binning, two protocols developed from the group testing viewpoint for static networks, are presented in [10]. The complexity of the Direct Algorithm is $O(k(\log k)^2 \log\log k)$. It performs well as the total number of nodes increases, however its complexity can be further improved. For this reason, Group Testing with Binning is proposed, providing a resulting complexity of $O(\lceil\frac{1}{\beta}\rceil max\{k^{\beta}(\log k^{\beta})^2 \log\log k^{\beta}, \lceil k^{1-\beta}\rceil\})$. although a system can be designed with complexity $O(k \log k)$. Both proposals achieve high discovery accuracy, and present a lower time consumption than random access discovery schemes similar to the Birthday-listen-and-transmit algorithm [5].

PSBA [11], is a prime-set based probabilistic algorithm tailored that works well in low duty cycle mobile Wireless Sensor Networks (WSN). Every node randomly chooses a prime p from a prime set (related to the duty cycle). The nodes wake up every p slots in a cycle. PSBA improves the long tail of the probabilistic algorithms. Authors found that PSBA outperforms the Birthday protocol [5], Disco [12] and SearchLight [13] protocols, in terms of average latency when the duty cycle is from 1-5%. Furthermore, the authors found that the performance is better regarding the average latency and energy consumption as the duty is reduced, in comparison to existing algorithms.

Panda [14], Power Aware Neighbor Discovery Asynchronously protocol, is a probabilistic protocol, and it represents the first neighbor discovery protocol available for Energy Harvesting (EH) nodes. Panda-D, is also available in [14], a version that extends the protocol to work well in non-homogeneous power harvesting. The authors found that for a higher power budget, the discovery latency is improved. Panda outperforms the low-power SearchLight-E (SearchLight [13] for power budget) and low-power BD-E (Birthday [5] for power budget) protocols regarding the average discovery rate. Furthermore, Panda outperforms SearchLight-E protocol in terms of worst case discovery latency. Panda and Panda-D have similar energy consumption and discovery rates in a one-hop scenario with homogeneous power budgets. An implementation of Panda in a unique EH ultra-low-power node prototype is also available. An important result is that Panda is highly practical and can be used when nodes are powered by a non-rechargeable battery.

Nihao [15] is an energy efficient asynchronous protocol. Simplified Nihao (S-Nihao) is a version that uses only one wake-up slot in a schedule cycle, and guarantees the discovery. According to analytical results authors found that S-Nihao is better than the LL-Optimal (Combinatoric) [16] schedule, given a duty-cycle, regarding the latency bound. S-Nihao outperforms existing solutions when only duty-cycle and latency are considered. Generic Nihao (G-Nihao) is another version that guarantees discovery and provides a good duty-cycle granularity. Balanced Nihao (B-Nihao) is most appropriate for practical applications with the best-balanced performance. An implementation is available for Nihao on TinyOS 2.1.2. According to real-world results, B-Nihao is faster than Birthday [5], Disco [12], U-Connect [17] and SearchLight [13], for duty-cycles of 1% and 5% and achieves the lowest latency bound. As for G-Nihao it presents better latency than Disco, U-Connect, SearchLight and BlindDate [18], for duty-cycles of 1% and 5%.

Centron [19] is a solution that improves the successful discovery probability, achieves to minimize the collisions in crowded regions, and focuses on improving the energy consumption. According to the mathematical results through Matlab and simulation results through NS-3, authors conclude that Centron behaves better than existing solutions regarding energy consumption and average discovery latency.

B. Deterministic protocols

Hedis and Todis [20] are asynchronous neighbor discovery protocols. Through simulations, authors conclude that Hedis and Todis greatly outperform all the other solutions, while they improve the energy consumption. An implementation for Hedis and Todis over smartphones (Xiaomi Mi-Note) including Bluetooth Low Energy (BLE) is available. Authors conclude that Hedis is the most appropriate protocol to be used in WSNs, prolonging the battery lifetime and providing a reasonable discovery latency bound.

In [21] a quorum-based asynchronous deterministic multi-channel handshake-based neighbor discovery protocol to be used in cognitive radio MANETs is presented. A successful discovery takes place when two neighbors tune their transceivers on the same frequency channel during an advertisement interval. Authors conclude that a faster discovery process with lower energy consumption can be achieved. Grid techniques and Sync Grid techniques are proposed in [21], the latter achieving lower energy consumption. Through simulations, authors found that both techniques achieve almost the same results. However, this proposal requires a dedicated circuitry, and the transceivers must be able to receive on two independent channels at the same time, and cannot cope with the asymmetric cases.

ND_HC [29], a cross-layer neighbor discovery algorithm tailored for large wireless networks, makes use of TDMA, regular hexagonal network clustering and GPS. The Hello messages are generated in the MAC layer and then they are transmitted following a TDMA manner with a random backoff. ND_HC achieves to reduce the collisions and improve the throughput, while it is collision-free. According to the simulation results through NS-2, ND_HC performs better than ND_802.11 in terms of discovery efficiency.

C. Wake-up based

In [34] a neighbor discovery protocol for MANETs with social information recognition is presented. The nodes include both a wake-up radio and a radio transceiver which allows half-duplex operation. The wake-up radio signal and the hello messages are broadcasted and then the receivers will change from idle to active mode. The passive discovery framework allows the use in mobile social applications. According to simulations through NS-2, the proposal outperforms Disco [12] U-Connect [17] and SearchLight [13] regarding latency and energy consumption. In addition, an implementation in a smartphone is also available.

PWEND [35] is a neighbor discovery protocol for MSNs (Mobile Sensor Networks), which can provide better latency, achieves reduced energy consumption through a wake-up based mechanism, and the worst-case discovery latency can be reduced. According to simulations through Matlab, PWEND outperforms existing solutions, such as G-Nihao [15], $Q - Connect_A$ [36], Disco [12]and SearchLight (stripe) [13] regarding latency and energy consumption.

D. Highly dynamic MANETs

KPND (Kalman Prediction-based Neighbor Discovery) [23] is a protocol which aims at improving latency and efficiency and it is tailored for highly dynamic MANETs. KPND is based on a mobility prediction model using Kalman filter theory and hello messaging, and GPS allows to detect when neighbors join and leave the network. According to simulation results through NS3.28 and Mobisim, the proposal outperforms HP-AODV, ARH [24] and ROMSG [25].

In [28] a neighbor discovery protocol suitable for highly dynamic MANETs where nodes are resource constrained, and combines routing, scheduling and neighbor discovery is presented. The proposal uses versions of AODV and CSMA to achieve blind route discovery and forward packets at the same time. According to simulation results, the proposal presents a proper behavior. Moreover, the protocol is robust in terms of mobility, failure or in the case of nodes joining the network.

E. Antenna and radar

In [26] a neighbor discovery protocol to be used in MTC (Machine-type communication) wireless ad hoc networks which uses radar capabilities, is presented. According to numerical results, the latency of the protocol is better with prior information from the radar. It is found that the process can be speeded up when a stop-discovery mechanism and non-response mechanism are used. Moreover, the proposal outperforms the CRA (Completely random algorithm) [27] regarding the latency. However, radar and communication must be integrated, while synchronization and half-duplex operation must be among its assumptions.

In [30], a neighbor discovery algorithm modeled as a learning automaton is presented, in which the nodes can learn about its environment and from prior observations and achieve a faster discovery in dense networks. The intelligent learning-based proposal is based on finite-state learning automata (FLA) and achieves the discovery with a high probability. The nodes include a steerable directional antenna and use an ALOHA-like manner to transmit. According to simulation results, the proposal presents better latency and a clear improvement against a 2-way random handshaking protocol [31] and a scan based algorithm [32].

RCI-SBA [33], an energy efficient two-way handshaking neighbor discovery scan based algorithm for ad hoc networks, integrates radar and communications. According to RCI-SBA, the nodes include directional antennas, and makes use of radar and communication signals and GPS. Mathematical analysis results prove that the proposal achieves better energy consumption. According the the simulation results, the energy consumption of CRA [27] is worse than that of RCI-SBA, while RCI-SBA outperforms the scan based algorithm (SBA) [32] regarding the energy consumption.

F. Secure protocols

In [22], a complete self-configured secure protocol tailored for spontaneous wireless ad hoc networks composed of resource constrained devices, is presented. It makes use of symmetric and asymmetric schemes and the trust between users to exchange the data, the network creation is allowed, resources are shared and new services are offered.

Authors in [37] focus on the security of finitely many sessions of a protocol that tosses coins in addition to standard cryptographic primitives against a DolevYao adversary. Authors focus on secrecy (to determine if an adversary can determine a secret) and indistinguishability (to determine if the probability of observing is the same from different observers under the same adversary) Both metrics are coNP-complete for non-randomized protocols. However, authors demonstrate that, for randomized protocols, secrecy and indistinguishability are both decidable in coNEXPTIME, and also that exists a lower bound for the secrecy problem achieved by reducing the non-satisfiability problem of monadic first order logic without equality.

G. Access Control

Authors in [38] include a survey for access control in IoTs, which addresses different applications and need a huge amount of private information from the user thus security problems may arise. In this context, access control is used to ensure authorized users access information resources under legitimate conditions. The survey analyses the main problems and challenges of access control in real-life large dynamic heterogeneous environments. For this purpose, [38] provides theoretical, and technical guidance for IoT search access control, and analyze future directions of access control in IoTs.

Our novel randomized proposals differ from previous solutions as the proposals manage to discover all the neighbors with probability 1, even in dense networks, overcoming the problem of previous randomized protocols, which discover all the neighbors with high probability. Furthermore, CDH proposal does not need to know the number of nodes in the network, and both proposals know when to terminate the discovery process and are handshake-based. We also aim at improving the neighbor discovery time, energy consumption, number of packets sent, packet delivery ratio, percentage of discoveries per round and percentage of idle slots. Moreover, the protocols are suitable for static multi-hop network scenarios.

Furthermore, our proposals are probabilistic, meaning that they are not deterministic, no wake-up based mechanism is included, they can not be used in MANETs, neither antenna nor radar is used, they do not face the security problem, and an access control related to the use of the channel to achieve neighbor discovery is used.

III. RANDOMIZED NEIGHBOR DISCOVERY PROTOCOLS BASED ON COLLISION DETECTION

A. CDPRR

According to Figure 1, the CDPRR protocol consists of several rounds and finishes when all the neighbors have been discovered.

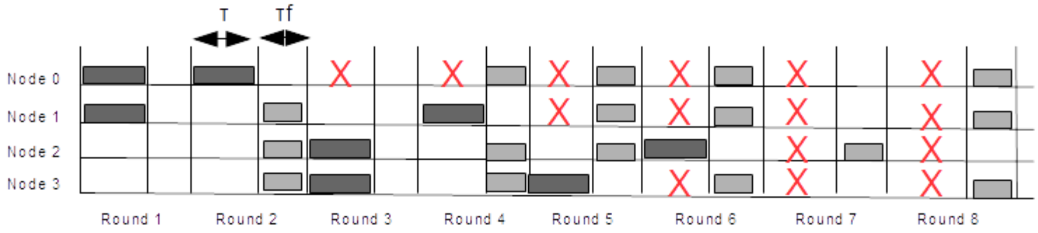

Fig. 1. CDPRR protocol.

This protocol requires synchronization in slot boundaries, and uses slotted transmissions of slot width τ. We assume that the nodes can detect collisions when they are listening. It is also assumed for CDPRR that the number of nodes in the network is known by every node. In the protocol, there are two sub-slots, the first one to send BROADCAST packets and the second one is used to send feedback packets.

As shown in the flow diagram in Figure 2, in the first sub-slot at the beginning of a round each node randomly chooses a state either transmitting T with probability $\frac{1}{N}$ or listening L with probability $1 - \frac{1}{N}$. If the state is T, the node sends a BROADCAST message containing its identifier towards the nodes within transmission range, otherwise it keeps listening. At the end of the round, collision detection has been performed by the receivers since half-duplex makes the sender unable to do so. Then, a sub-slot of width τ_f is opened to send the feedback that consists of a single feedback packet. When a single node manages to transmit successfully in a round, the receivers of the BROADCAST do not detect collision nor idle channel and proceed to update the neighbor table with the identifier in the BROADCAST and send a feedback packet towards the other nodes. Otherwise, a collision is detected or the channel is idle and they do not send the feedback packet. At the same time, the nodes that transmitted, listen to the channel and when they detect energy in the channel, the state will change to S, meaning that it transmitted successfully, starts a new round, and remains in this state until the end of the algorithm, this node will not keep contending in the following rounds (a red X mark in Figure 1) and it will remain listening until the end of the algorithm, although it will keep sending the feedback packets when necessary. Otherwise, i.e., no energy is detected, the node that transmitted starts a new round choosing a new state. The nodes that received the BROADCAST simply start a new round. In the case that the node is in state L, the node starts a new round and chooses a new state. Otherwise, i.e., if the node is in state S, the node starts a new round but it does not choose a new state. The feedback packet length is much smaller than the BROADCAST, and it indicates that a successful transmission took place. The protocol finishes when all the neighbors have been discovered. Notice that in the feedback process, the receivers will only send a feedback packet, and they do not provoke collisions since the transmissions of the feedback packets are perfectly synchronized, and the transmitters only need to detect energy.

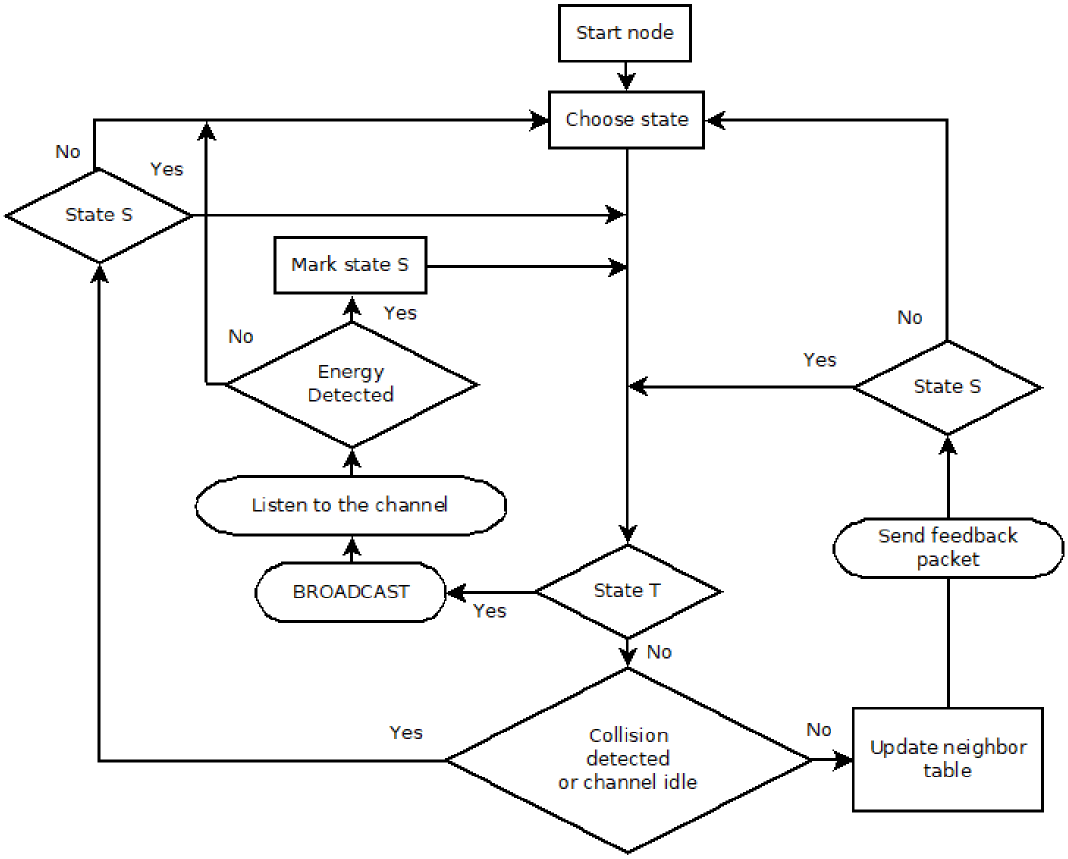

Fig. 2. CDPRR flow diagram.

If there is a collision in the first sub-slot, this means that the nodes that provoked it have not managed to transmit successfully thus in the second sub-slot the nodes in state L or S do not send a feedback packet.

Notice that a collision can not take place in the second sub-slot since only energy is detected.

A termination detection mechanism is available for CDPRR. In one-hop networks, when the number of nodes is known, the protocol knows when to terminate if all the nodes have discovered all their neighbors ($N-1$). When a node has discovered $N-1$ nodes (the node knows that $N-1$ nodes have been discovered by checking the number of nodes in its neighbor table) it will continue contending. As soon as the node that discovered the $N-1$ nodes chooses to transmit the BROADCAST the remaining nodes have discovered $N-1$ nodes and send the feedback packet. Then the node detects energy, changes its state to S and waits a single round. Then, in the following round, the node must only send the feedback packet and stop. The remaining nodes that have discovered their $N-1$ neighbors detect energy and wait a single round. In the next round for these $N-1$ neighbors they must only send the feedback packet each one and stop. Therefore, two additional rounds are necessary for the termination handshake. According to the example in Fig. 1, at the beginning of round 6 the nodes 0,1 and 3 have discovered $N-2$ neighbors, while node 2 has discovered $N-1$ neighbors and continues contending. In round 6, node 2 transmits the BROADCAST, nodes 0,1 and 3 send the feedback packet, thus node 2 detects energy and waits for round 7. In round 7, nodes 0,1 and 3 have discovered $N-1$ neighbors and node 2 must only send the feedback packet and stops, and energy is detected by nodes 0,1 and 3 thus nodes 0,1 and 3 wait for round 8. In round 8, nodes 0,1 and 3 must only send the feedback packets and stop. Then, the protocol has finished for all the nodes. However, in the multi-hop case, the protocol finishes when in a number of consecutive rounds (which must be properly fixed), the nodes do not receive any BROADCAST (all of them are listening), since the probability that no node send a BROADCAST and they are in state L in several consecutive rounds is very low, thus we conclude that all the nodes are in state S. Of course, in the multi-hop case, the probability of successful discovery is not 1.

Notice that τ matches the duration of the BROADCAST packet.

If a BROADCAST packet is lost, the node will continue in the next round. Moreover, if a feedback packet is lost and the feedbacks of the other nodes are not lost, the protocol works well. However, if all the feedback packets are lost, then the nodes will continue in the next round.

CDPRR copes with the hidden node problem as follows (allowing scalability in multi-hop scenarios). Say 3 nodes A, B, C. A and B are within transmission range, B and C are within transmission range, while A and C are out of each other's transmission range. The protocol behaves properly, coping with the hidden node problem, except in the following case. If A and B send a BROADCAST then a collision occurs, while C receives the BROADCAST of B and C sends feedback thus B is discovered by node C and stops contending (therefore A did not discover node B and B is not neighbor of A but it is of C when this situation takes place) and both A and C continue contending. To conclude, due to the hidden node problem some nodes may not be discovered.

CDPRR can be extended to the multi-hop case in which one or more nodes may belong to several subnets. The main problem that occurs in the multi-hop case is the hidden node problem which we addressed above. In the other cases, the protocol operates properly in the same way as the one-hop case. In the multi-hop case the termination condition changes as stated above, i.e., the protocol finishes when all the nodes are listening in several consecutive rounds.

A typical case that may occur in a network composed of nodes A,B,C, in which A and C are out of each other's transmission range, A is within transmission range of B and C is within transmission range of B, is the following. If both B and C send BROADCAST simultaneously, assuming a half-duplex operation, they are both transmitting thus a collision takes place. However, if we consider the RF capture effect that may occur in practical scenarios, B might be able to receive C's BROADCAST correctly instead of detecting a collision. In addition, the threshold for detection of transmission energy is normally lower than for correct reception of packets. Thus B's feedback can be detected by both A and C. In this case, C will be discovered by B, while A will not be discovered by B, then A stops contending and C stops contending, thus A will not be discovered by any node in the network.

In a realistic scenario in which the number of currently actual working nodes (N') is less than the number of nodes known (N), first, the termination condition stated is not valid since when $N' < N$ the nodes will not know when to terminate. Therefore, the termination condition should be changed so that the protocol finishes when in a number of consecutive rounds all the nodes are listening. The protocol will operate properly even when $N' < N$, since the probability of transmission is still $\frac{1}{N}$ and the probability of listening is still $1-\frac{1}{N}$, and both the BROADCASTs and the feedbacks will be sent and received only by the N' nodes.

B. CDH

In the CDH protocol, as shown in Figure 3, there are two sub-slots in a round. The duration in seconds of the first sub-slot is ω while the width of the second sub-slot (feedback) is ω_f in seconds. The times ω and ω_f are fixed (the same for all the rounds) and do not depend on the number of nodes.

First, as shown in Figure 3 and the flow diagram in Figure 4, the CDH protocol consists of several rounds and finishes when all the neighbors have been discovered.

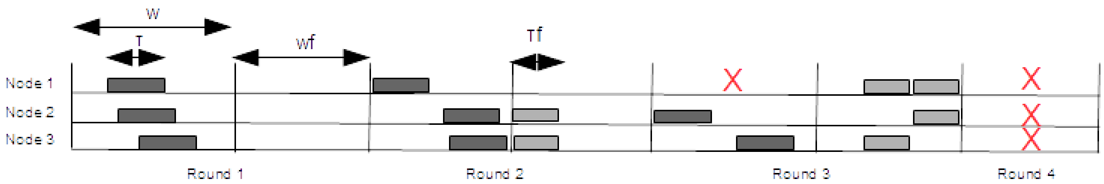

Fig. 3. CDH protocol.

According to Figure 3 and Figure 4, in the first sub-slot of a round, each node transmits a single BROADCAST packet containing the identifier beginning in a randomly chosen time t_i ($t_i \in [0, \omega - \tau]$ being ω the duration of the first sub-slot) of duration τ, and listens for incoming messages during the rest of the slot, i.e., $\omega - \tau$. During the listening periods in the first sub-slot of each node, a collision detection process is performed by that node. A second sub-slot is used by all the nodes to tell the nodes which nodes have transmitted successfully sending a serial of feedback packets one after another. The number of feedback packets fixed must be enough to consider the identifier of all the nodes, and the identifiers could be no consecutive numbers. The order of transmission of feedback packets is from ident_min to ident_max (the minimum and maximum identifiers of the possible nodes in the network). The IDs can be non consecutive numbers, but they are transmitted in order (from ident_min to ident_max). Furthermore, the nodes are not assumed to know the list of IDs.

When the jth feedback is scheduled to be sent the nodes with $identifier$ different from j will send the feedback packet if node j transmitted successfully, while the node with $identifier$ equal to j will listen to the channel. Otherwise, node $identifier$ will not send the feedback packet since node j provoked a collision. If a collision was not detected for node j, the rest of the nodes update their neighbor tables with the identifier of j in the BROADCAST that did not collide, and send the feedback packet. Otherwise, they do not send a feedback packet. The nodes

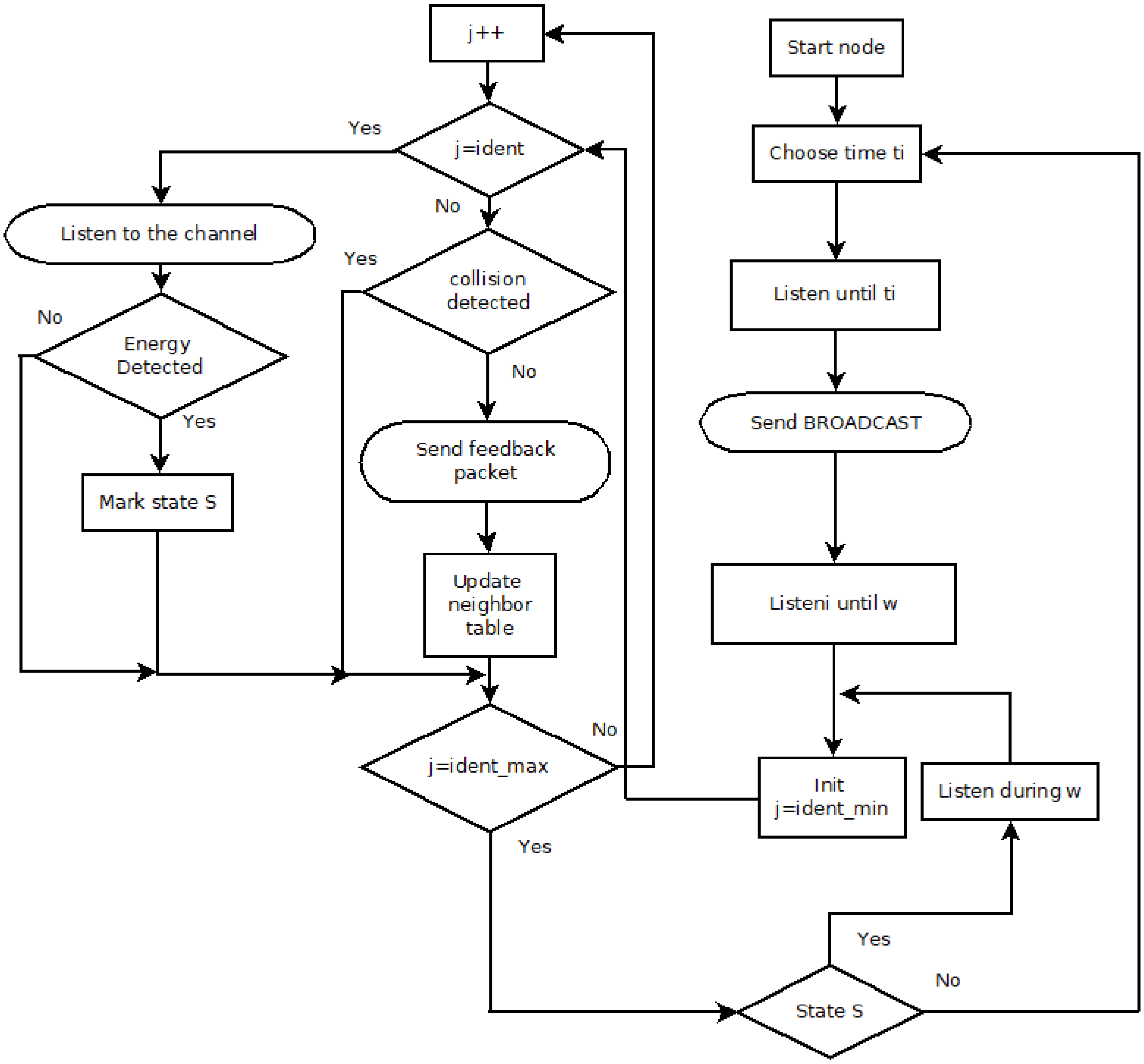

Fig. 4. CDH flow diagram.

with $identifier$ equal to j now listen to the channel and if energy is detected, i.e., a feedback packet in the channel, a red X mark in Figure 3, it will change the state to S, it will not contend in the following rounds and it will remain listening, although it will keep sending the feedback packets when necessary. Otherwise, it will keep contending in the following rounds. When j reaches the ident_max (maximum identifier), if the state of the node is S it will not send the BROADCAST in the following rounds, otherwise a new round begins and the node will choose a new t_i.

The feedback packets do not collide since the nodes must only detect energy. The feedback packets are much smaller than the BROADCASTs.

If a node that is listening detects another transmission from a node j, the behavior will depend on the collision detection. If a collision did not occur, the node will send jth feedback packet in the second sub-slot, otherwise it will not send a feedback packet in the second sub-slot.

This protocol includes a termination detection mechanism, in which the protocol finishes when all the nodes have managed to transmit successfully in previous rounds, when in a round there is no signal in the channel in the first sub-slot, i.e., the nodes did not send their BROADCAST packets, since they are all in state S.

If there are collisions in the first sub-slot, this means that the nodes that provoked it have not managed to transmit successfully thus in the second sub-slot the nodes do not send a feedback packet for this colliding nodes. Notice that a collision can not take place in the second sub-slot since only energy is detected and the nodes are perfectly synchronized.

We assumed that the nodes can detect collisions when they are listening.

This protocol requires synchronization in slot boundaries.

If a BROADCAST packet is lost, the node continues in the next round. However, when this is the only node that is contending and it is lost, the protocol assumes that all the nodes have transmitted successfully, the protocol ends and this node is not discovered. Moreover, if a feedback packet is lost and the other feedback packets for the same node are not lost, the protocol works well. If all the feedbacks for a given node are lost, then this node will continue in the next round.

The feedback packet is much smaller than the BROADCAST, which means an advantage over the acknowledgements exchanged (containing source and destination) when sending BROADCAST deterministically announcing each node (according to a schedule). Moreover, the feedback packets do not provoke collisions since only energy must be detected.

CDH copes with the hidden node problem as follows. Say 3 nodes A, B, C. A and B are within transmission range, B and C are within transmission range, while A and C are out of each other's transmission range. Similarly to CDPRR, if A and B send a BROADCAST then B will not be discovered by A and both A and C continue contending. To conclude, due to the hidden node problem some neighbors may not be discovered.

CDH can be extended to the multi-hop scenario in the same way as the one-hop scenario. The main problem that occurs is the hidden node problem which we addressed above. In the other cases, the protocol operates properly in the same way as the one-hop scenario. In the multi-hop scenario the termination condition is the same as in the one-hop scenario, i.e., the protocol finishes for a node when the channel is idle during a round, i.e. all the nodes within transmission range of this node have managed to transmit successfully before.

In the same way as in CDPRR, a typical case that may occur in a network composed of nodes A,B,C, in which A and C are out of each other's transmission range, A is within transmission range of B and C is within transmission range of B, is the following. Again, if both B and C send BROADCAST at the same time, considering the RF capture effect B might be able to receive C's BROADCAST correctly instead of detecting a collision. Therefore B's feedback can be detected by both A and C. In this case, C will be discovered by B, while A will not be discovered by B, then A stops contending thus A will not be discovered by any node in the network.

In conclusion, the listening nodes that received the BROADCAST must send feedback packets in the second sub-slot corresponding to the successful senders. For this purpose, the feedback needs to be sent with precise timing and in the correct order since the sender will only check for transmission energy. The protocol must only need to know the minimum identifier (ident_min) and the maximum identifier (ident_max) of the possible nodes in the network, it does not need to know the number of neighbors involved. Furthermore, the identifiers can be no consecutive numbers (the nodes that are not deployed in the network will not be considered when sending BROADCASTs and feedback packets). However, the feedbacks must be sent in a deterministic way. The protocol does not need to know the order of each neighbor's ID. So the order will be determined by the identifier.

IV. ANALYTICAL MODELS

In this Section, some equations will be presented to assess the performance of both randomized neighbor discovery protocols in static one-hop scenarios.

A. CDPRR

According to the protocol, the variables used in the analysis are defined in Table I.

For our analysis, we assume that the protocol consists of different phases, each one finishing when a single node manages to transmit successfully. Therefore there are N phases, each phase i being $1 \leq i \leq N$ consisting of a number of rounds. The number of rounds necessary to discover all the neighbors is the total number of rounds.

TABLE I

Variable definitions for CDPRR.

Variable	***Definition***
τ	The time a node is transmitting in seconds, and it matches the round duration.
τ_f	The time a node is transmitting a feedback in second, and it matches the duration of the second sub-slot (feedback)
N	The number of nodes in the network.
$\frac{1}{N}$	The probability than a node transmits.
$1 - \frac{1}{N}$	The probability that a node listens.
p_i	The probability that a node successfully transmits
W_i	The number of rounds until one node transmits successfully in phase i.
W	The total number of rounds until all the neighbors have been discovered.
X_i	The expected value of the number of rounds until one node transmits successfully in phase i.
X	The expected value of the total number of rounds when all the neighbors have been discovered.
Tt	The neighbor discovery time in seconds.
T_{hr}	The throughput in packet/s.
NT_i	The total number of nodes transmitting in phase i, taking into account only the nodes that are contending.
NL_i	The total number of nodes listening in phase i, taking into account only the nodes that are contending.
$E(NT_i)$	The expected value of NT_i.
$E(NL_i)$	The expected value of NL_i.
n_i	The number of experiments of the nodes that are still contending in phase i, i.e., the nodes that have not transmitted successfully yet.
$E(P_i)$	The expected energy consumption in phase i.
$E(P)$	The average energy consumption by each node of the N phases.
$E(P_t)$	The average energy consumption by each node of the N phases plus the average energy consumption of the 2 termination rounds.
$E(P_f)$	The average energy consumption of the feedbacks.
E_T	The total average energy consumption.
E_{tx}	The amount of energy consumed by a node transmitting per second.
E_l	The amount of energy consumed by a node listening per second.
P_{sent}	The total number of packets sent in the N phases.
P_{rec}	The total number of packets received by each node in the N phases.
P_{sentf}	The total number of packets sent in the feedbacks.
P_{recf}	The total number of packets received by each node in the N phases in the feedbacks.
s_b	The size of the BROADCAST packet in bytes.
s_f	The size of the feedback packets in bytes.
PDR	The packet delivery ratio.
PDR_f	The packet delivery ratio for the feedbacks.
$Fx(k)$	CDF (Cumulative Distribution Function) of the discoveries.
$E(NIS_i)$	Total number of rounds in phase i in which all the remaining nodes are listening.
$E(NIS)$	Total number of idle slots.
PIS	Percentage of idle slots vs the total number of rounds.

1) Neighbor discovery time: When we talk about the Neighbor discovery time, we refer to the time it takes the algorithm to discover all the neighbors.

The probability that a single node transmits successfully in a round in phase 1 (p_1) is:

$$p_1 = \frac{1}{N} \cdot \left(1 - \frac{1}{N}\right)^{N-1} \tag{1}$$

being N the number of nodes in the network, $\frac{1}{N}$ the probability that a node transmits, and $1 - \frac{1}{N}$ the probability that a node listens.

In phase i, $i-1$ nodes have already transmitted successfully, thus they are not contending in phase i, and remain listening with probability 1. In phase i only $N-i+1$ are contending. Thus, for the nodes that are contending, the probability that a single node transmits in round i while the rest of the nodes $(N-i)$ listen is p_i.

$$p_i = \frac{1}{N} \cdot \left(1 - \frac{1}{N}\right)^{N-i} \tag{2}$$

We assume that W_i (the number of rounds in phase i until a node successfully transmits) follows a geometric distribution, $W_i \sim Geo((N-i+1) \cdot p_i)$, being p_i obtained from equation 2. We obtain X_i as the expected number of rounds in phase i until a node successfully transmits. A geometric distribution is assumed since this type of probability distribution models the number of Bernoulli trials needed to get one success. In our case, the number of rounds until success, i.e., one node transmits successfully.

The expected number of rounds in phase i is given in equation 3 since in phase i there are $N-i+1$ nodes that have not successfully transmitted in previous rounds.

$$X_i = E(W_i) = \frac{1}{(N-i+1) \cdot p_i} = \frac{N}{(N-i+1) \cdot (1-\frac{1}{N})^{N-i}} \tag{3}$$

The resulting total number of rounds is $W = W_1 + W_2 + \cdots W_N$, and the expected value is given in equation 4.

$$X = E(W) = \sum_{i=1}^{N} X_i = \sum_{j=1}^{N} \frac{N}{j \cdot (1-\frac{1}{N})^{j-1}} \tag{4}$$

We must add to the total number of rounds, 2 more rounds to include the final handshaking to terminate the discovery as shown in Section III-A.

In conclusion, the average neighbor discovery time T_t is the expected number of total rounds $X+2$ multiplied by $\tau + \tau_f$, i.e., the total round duration and is given in equation 5.

$$T_t = (X+2) \cdot (\tau + \tau_f) \tag{5}$$

2) Energy consumption: Next, we calculate the energy consumption P_i in each phase i and then the average energy consumption for all the phases $E(P)$ in Joules.

We assume a binomial distribution $NT_i \sim B(n_i, \frac{1}{N})$ for the total number of transmissions of all the nodes that are contending (i.e., n_i) in phase i with a fixed transmission probability

$\frac{1}{N}$ and two possible results (i.e., transmit or listen). We also assume a binomial distribution $NL_i \sim B(n_i, 1 - \frac{1}{N})$ for the total number of listenings of all the nodes that are contending (i.e., n_i) in phase i, with a fixed listen probability $1 - \frac{1}{N}$ and two possible results, either transmit or listen.

We will show later that $n_i = (N - i + 1) \cdot X_i$ for phase i.

The total number of times that the nodes transmit in phase 1 is NT_1, taking into account that phase 1 consists of X_1 rounds. In addition, in phase 1 there are N nodes that are contending and can either transmit or listen, during X_1 rounds, thus there are $n_1 = N \cdot X_1$ experiments. Similarly, the total number of times that the nodes listen in phase 1 is given by NL_1.

As for phase i, $i - 1$ nodes transmitted successfully in previous phases, therefore these nodes will not take state neither T nor L.

The total number of times that the nodes transmit in X_i rounds (i.e., in phase i) is given in equation 6, being $n_i = (N - i + 1) \cdot X_i$, since in phase i there are still $N - i + 1$ nodes that are contending and there are X_i rounds. And the number of times that the nodes listen in X_i rounds (i.e., in phase i) is given in equation 7.

$$E(NT_i) = \sum_{k=1}^{n_i} k \cdot \binom{n_i}{k} \cdot \left(\frac{1}{N}\right)^k \cdot \left(1 - \frac{1}{N}\right)^{n_i - k} \tag{6}$$

$$E(NL_i) = \sum_{k=1}^{n_i} k \cdot \binom{n_i}{k} \cdot \left(\frac{1}{N}\right)^{n_i - k} \cdot \left(1 - \frac{1}{N}\right)^{k} \tag{7}$$

$E(P_1)$, is the expected energy consumed during phase 1 in Joules is given in equation 8.

$$E(P_1) = \tau[E_{tx} \cdot E(NT_1) + E_l \cdot E(NL_1)] \tag{8}$$

Considering that only a node transmitted successfully in phase 1, i.e., it keeps listening, in phase 2 a term $\tau \cdot X_2 \cdot E_l$ must be added to $E(P_2)$ for this node. In phase 3, two nodes transmitted successfully thus X_3 rounds listening, thus a total energy consumed of $\tau \cdot 2 \cdot X_3 \cdot E_l$ must be added to $E(P_3)$. In general, for phase i, the energy consumed by the nodes that transmitted successfully in the previous $i - 1$ phases is $\tau \cdot (i - 1) \cdot X_i \cdot E_l$ and must be added to $E(P_i)$ obtaining the following equation for $E(P_i)$.

$$E(P_i) = \tau \cdot [E_{tx} \cdot E(NT_i) + E_l \cdot E(NL_i) + E_l \cdot (i - 1) \cdot X_i] \tag{9}$$

being $\tau \cdot E_l \cdot (i-1) \cdot X_i$ the energy consumed in phase i by the nodes that successfully transmitted in previous phases.

The average energy consumption by each node ($E(P)$) in Joules is given in equation 10, using the summation from equation 11. Remember that we must add 2 more rounds for the termination handshake, thus the total average consumption is given in equation 12.

$$E(P) = \frac{1}{N} \cdot \sum_{i=1}^{N} E(P_i) = \frac{\tau}{N} \cdot \left[E_{tx} + E_l \cdot N \cdot \left(1 - \frac{1}{N}\right)\right] \cdot X + \frac{\tau}{N^2} \cdot [E_l - E_{tx}] \cdot \sum_{k=1}^{N} (k-1) \cdot X_k \tag{10}$$

$$\sum_{k=1}^{N} (k-1) \cdot X_k = \sum_{k=1}^{N} \frac{N \cdot (k-1)}{(N-k+1) \cdot (1-\frac{1}{N})^{N-k}} \tag{11}$$

$$E(P_t) = E(P) + \tau \cdot \frac{2 \cdot N - 1}{N} \cdot E_l \tag{12}$$

Now, the feedbacks must be added. We must take into account that in phase i, a number of $\tau_f \cdot E_l \cdot N \cdot (X_i - 1)$ are listening when no neighbor is discovered, but $\tau_f \cdot E_{tx} \cdot (N-1)$ are sending the feedback packets in the last round of phase i, i.e., when the neighbor is discovered, while $\tau_f \cdot E_l$ are listening in the last round of phase i when the neighbor is discovered. However, in the last two rounds (termination checking), 1 node is transmitting feedback while the other nodes listen ($\tau_f \cdot E_{tx} \cdot 1 + \tau_f \cdot E_l \cdot (N-1)$) in the first round for termination checking and $\tau_f \cdot E_{tx} \cdot (N-1)$ in the second round for termination checking. Therefore, the average energy consumption for the feedbacks is:

$$E(P_f) = \frac{1}{N} \sum_{i=1}^{N} [\tau_f \cdot E_l \cdot N \cdot (X_i - 1) + \tau_f \cdot E_l + \tau_f \cdot (N-1) \cdot E_{tx}] \tag{13}$$

$$+ \frac{1}{N} \cdot [\tau_f \cdot (N-1) \cdot E_l + \tau_f \cdot E_{tx} + \tau_f \cdot (N-1) \cdot E_{tx}] \tag{14}$$

$$E(P_f) = \tau_f \cdot E_l \cdot X + \frac{2 \cdot N - 1}{N} \cdot \tau_f \cdot E_l + N \cdot \tau_f \cdot E_{tx} \tag{15}$$

Finally, the total average energy consumption is given in equation 16.

$$E_T = E(P_t) + E(P_f) \tag{16}$$

3) Overhead: From equation 6 we obtain the number of nodes that transmit in a phase i of length X_i rounds.

$$E(NT_i) = (N \cdot X_i - (i-1) \cdot X_i) \cdot \frac{1}{N} \tag{17}$$

As for the total of packets sent (P_{sent}) is given in equation 19, using the summation from equation 11.

$$P_{sent} = \sum_{i=1}^{N} \frac{1}{N} \cdot (N \cdot X_i - (i-1) \cdot X_i) = \sum_{i=1}^{N} X_i - \frac{1}{N} \cdot \sum_{i=1}^{N} (i-1) \cdot X_i \tag{18}$$

$$P_{sent} = X - \frac{1}{N} \cdot \sum_{i=1}^{N} (i-1) \cdot X_i = \frac{1-(1-\frac{1}{N})^{-N}}{1-(1-\frac{1}{N})^{-1}} \tag{19}$$

We must add the total number of packets sent in the 2 termination rounds. In the first and second termination round, none of the nodes send (they are listening) thus 0 packets are sent. Therefore, P_{sent} remains as in equation 19.

As for the packets sent in the feedbacks, $(N-1)$ packets are sent in the last round of phase i, while no feedbacks are sent in the $X_i - 1$ first rounds of phase i. Thus, a total of $N \cdot (N-1)$ packets are sent in the N rounds. As for the first termination round, 1 packet is sent, while in the second termination round, $(N-1)$ packets are sent. Therefore, for the feedbacks we obtain equation 21.

$$P_{sentf} = N \cdot (N-1) + 1 + (N-1) \tag{20}$$

$$P_{sentf} = N^2 \tag{21}$$

4) Packet delivery ratio: As for the packet delivery ratio, we calculate it by taking into account the P_{rec} and P_{sent}. The average number of packets received per node, is $P_{rec} = N-1$, since there are N phases to discover all the N neighbors and in each phase 1 a packet is successfully received thus a total of $N-1$ packets are received, and the neighbor discovery time T_t is obtained from equation 5.

$$P_{rec} = N - 1 \tag{22}$$

We must add the total number of packets sent in the 2 termination rounds. In the first and second termination round, none of the nodes send (they are listening) thus 0 packets are sent. Therefore, P_{rec} remains as in equation 22.

As for the packets received in the feedbacks, $N-1$ packets are received in the last round of phase i, while no feedbacks are received in the $X_i - 1$ first rounds of phase i. Thus, a total of $N \cdot (N-1)$ packets are received in the N rounds. As for the first termination round, $(N-1)$ packets are received, while in the second termination round, 0 packets are received. Therefore, for the feedbacks we obtain equation 24.

$$P_{recf} = N \cdot (N-1) + (N-1) \tag{23}$$

$$P_{recf} = (N+1) \cdot (N-1) = N^2 - 1 \tag{24}$$

To obtain the packet delivery ratio in equation 25 and the packet delivery ratio for feedbacks in equation 26, we use equation 19 for P_{sent}, equation 22 for P_{rec}, equation 21 for P_{sentf} and equation 24 for P_{recf}.

$$PDR = \frac{P_{rec}}{P_{sent}} \tag{25}$$

$$PDR_f = \frac{P_{recf}}{P_{sentf}} \tag{26}$$

5) CDF of discoveries: We assumed that CDPRR follows a geometric distribution. Therefore, the CDF of discoveries for round k is obtained as follows:

$$Fx(k) = \begin{cases} 1-(1-p_1)^k \ if \ 1 \leq k \leq X_1 \\ Fx(X_1) + (1-p_2)^{X_1} - (1-p_2)^k \ if \ X_1 < k \leq X_1 + X_2 \\ Fx(X_1+X_2) + (1-p_3)^{X_1+X_2} - (1-p_3)^k \ if \ X_1 + X_2 < k \leq X_1 + X_2 + X_3 \\ \ldots \end{cases} \tag{27}$$

being $p_i = (N-i+1) \cdot \frac{1}{N} \cdot (1-\frac{1}{N})^{N-i}$.

6) Percentage of idle slots: According to equation 28, the total number of rounds in phase i in which all the remaining nodes are listening is shown, being $pl_i = (1-\frac{1}{N})^{N-i+1}$, and $X_i = \frac{N}{(N-i+1)\cdot(1-\frac{1}{N})^{N-i}}$. The total number of idle slots $E(NIS)$ is given in equation 29.

$$\begin{aligned} E(NIS_i) &= \sum_{k=0}^{X_i} k \cdot \binom{X_i}{k} \cdot pl_i^k \cdot (1-pl_i)^{X_i-k} = X_i \cdot pl_i \\ &= \frac{N}{(N-i+1)\cdot(1-\frac{1}{N})^{N-i}} \cdot (1-\frac{1}{N})^{N-i+1} \\ &= \frac{N\cdot(1-\frac{1}{N})}{N-i+1} \end{aligned} \tag{28}$$

$$\begin{aligned} E(NIS) &= \sum_{i=1}^{N} E(NIS_i) + 2 = \sum_{i=1}^{N}[\frac{N(1-\frac{1}{N})}{N-i+1}] + 2 \\ &= N\cdot(1-\frac{1}{N})\cdot[\sum_{j=1}^{N}\frac{1}{j}] + 2 = N\cdot(1-\frac{1}{N})\cdot H_N + 2 \end{aligned} \tag{29}$$

being H_N the harmonic number.

And the percentage of idle slots can be found in equation 30, being X given in equation 4.

$$PIS = \frac{E(NIS)}{X+2} = \frac{N\cdot(1-\frac{1}{N})\cdot[\sum_{j=1}^{N}\frac{1}{j}]+2}{X+2} == \frac{N\cdot(1-\frac{1}{N})\cdot H_N + 2}{X+2} \tag{30}$$

B. CDH

To summarize, the variables used in the analysis for CDH are defined in Table II.

For our analysis, we assume that the protocol consists of different rounds, and a number of 0 or more nodes can be discovered in a round. We call n_i the number of nodes that manage to transmit successfully in round i. After r rounds all the neighbors have been discovered in $r\cdot(\omega+\omega_f)$ seconds, that is, the time that the algorithm takes to discover all the neighbors.

We assume a uniform distribution $t_i \sim U(0,\omega-\tau)$, since for all the intervals of equal length (τ) in the distribution in their range ([0, $\omega-\tau$]) are equally probable. Notice that time is slotted, with a first sub-slot width ω, in which the nodes can transmit beginning in a randomly chosen time t_i according to a distribution $U(0,\omega-\tau)$ in this sub-slot, and a second sub-slot is used for the feedbacks.

TABLE II

VARIABLE DEFINITIONS FOR CDH.

Variable	***Definition***
τ	The time a node is transmitting in seconds
ω	The time a node is transmitting plus the time a node is listening in seconds, (i.e., the width of the first sub-slot).
ω_f	The duration of the second sub-slot.
τ_f	Duration of a feedback packet.
$\omega - \tau$	The time a node is listening in seconds in the first sub-slot.
r	The total number of rounds to discover all the neighbors.
N	The number of nodes in the network.
a	Probability that two nodes collide.
t_i	A random time in which the node will transmit a BROADCAST.
n_i	Expected number of nodes that transmit successfully in round i.
$P(C_i)$	The probability that a node i collides.
$P(S_i)$	The probability of successful transmission.
p_k	The probability that a node successfully transmits in round k.
Y_i	Number of successful transmissions in round i.
T_t	The average neighbor discovery time in seconds.
Pr	The total of packets received in the r rounds.
T_{hr}	The throughput in packet/s.
s_b	The size of the BROADCAST packet in bytes.
s_f	The size of the feedback packets in bytes.
$E(P_i)$	The expected energy consumed in round i.
$E(P_f)$	Average energy consumed by each node during the feedbacks.
$E(P)$	The average energy consumption by each node in Joules.
$E(P_t)$	The total average energy consumption in Joules, which includes the feedbacks.
P_{rec}	Total number of packets received by each node in the r rounds.
P_{recf}	Total number of feedback packets received.
P_{sent}	The total number of packets sent in the first sub-slot.
P_{sentf}	The total number of packets sent in the feedbacks.
P_{sentT}	The total number of packets sent.
E_{tx}	The amount of energy consumed by a node transmitting per second.
E_l	The amount of energy consumed by a node listening per second.
PDR	Packet delivery ratio.
PDR_f	Packet delivery ratio for the feedbacks.
$F(k)$	CDF (Cumulative Distribution Function) of the discoveries.
PIS	Percentage of idle slots.

1) Neighbor discovery time: When we talk about the Neighbor discovery time, we refer to the time it takes the algorithm to discover all the neighbors.

Assuming a uniform distribution, the following property can be applied, to obtain the probability that for two nodes i and j their BROADCAST messages overlap:

$$P((t_i \leq t_j \leq t_i + \tau) \cup (t_i - \tau \leq t_j \leq t_i)) = \frac{\tau}{\omega - \tau} \tag{31}$$

And $P(C_i)$, the probability that a node i collided is the union of collisions with the other nodes in round 1: In round 1, there are N nodes that did not successfully transmit, being $P(C_{i,j})$

the probability that a node i collided with node j.

$$P(C_i) = P(C_{i,1} \cup C_{i,2} \cup \cdots \cup C_{i,N-1}) \tag{32}$$

Applying the equation for the union of probabilities, and defining $a = \frac{\tau}{\omega-\tau}$ to simplify the equations, we obtain the following equation.

$$P(C_i) = 1 - (1-a)^{N-1} \tag{33}$$

As for the probability of successful transmission $P(S_i)$ in round 1, being S_i the event that a node i successfully transmits.

$$p_1 = P(S_i) = 1 - P(C_i) = (1-a)^{N-1} \tag{34}$$

For round 2, we obtaining the following probabilities, being n_1 the number of nodes that transmitted successfully in round 1.

$$P(C_i) = P(C_{i,1} \cup C_{i,2} \cup \cdots \cup C_{i,N-n_1-1}) \tag{35}$$

And the probability for round k, is given in the following equation.

$$P(C_i) = P(C_{i,1} \cup C_{i,2} \cup \cdots \cup C_{i,N-n_1-\cdots-n_{k-1}-1}) \tag{36}$$

Therefore, we obtain the general equation, for round k (p_k). All the nodes in round k have the same probability of success $P(S_i)$. From now on we will call p_k to the probability that a node transmits successfully in round k.

$$p_k = P(S_i) = 1 - P(C_i) = (1-a)^{N-n_i\cdots-n_{k-1}-1} \tag{37}$$

Then we will find out the number of nodes that transmit successfully in round 1 (n_1). In round 1 the following equation holds, since n_1 nodes transmit successfully, being $p_1 = (1-a)^{N-1}$, Y_1 the number of successful transmissions in round 1, and $Y_1 \sim B(N, p_1)$ follows a binomial distribution. Once we have calculated p_i using a uniform distribution, we now use a binomial distribution to count the number of nodes that transmit successfully.

$$n_1 = E(Y_1) = \sum_{x=0}^{N} x \cdot \binom{N}{x} p_1^x \cdot (1 - p_1)^{N-x} \tag{38}$$

$$n_1 = N \cdot (1 - a)^{N-1} \tag{39}$$

And for round k, the expected number of nodes that transmit successfully is n_k, given in equation 40, being $p_k = (1 - a)^{N - \sum_{i=1}^{k-1} n_i - 1}$.

$$n_k = (N - \sum_{i=1}^{k-1} n_i) \cdot p_k = (N - \sum_{i=1}^{k-1} n_i) \cdot (1 - a)^{N - \sum_{i=1}^{k-1} n_i - 1} \tag{40}$$

Next, we calculate the total number of rounds r after which the algorithm finishes, i.e., the N nodes have been discovered, from equation 41. Since the expression is difficult to derive, we only show the results obtained in the graphical section VI.

$$n_1 + n_2 + \cdots n_r = N \tag{41}$$

The rounds include a second sub-slot, i.e., a feedback mechanism, of width ω_f. Using the value total number of rounds r obtained from equation 41, taking into account that the round duration is $\omega + \omega_f$, being ω_f the duration of the second sub-slot (feedbacks), the neighbor discovery time in seconds can be obtained from equation 42. Notice that the protocol includes a termination round, thus the number of rounds must be $(r + 1)$.

$$T_t = (r + 1) \cdot (\omega + \omega_f) \tag{42}$$

2) Energy consumption: In round 1, N nodes transmit in a duration τ and N listen in a duration $\omega - \tau$, thus the expected energy consumed in round 1 ($E(P_1)$) in Joules is given in equation 43, being E_{tx} the amount of energy consumed by a node transmitting per second and E_l the amount of energy consumed by a node listening per second.

$$E(P_1) = \tau \cdot E_{tx} \cdot N + (\omega - \tau) \cdot E_l \cdot N \tag{43}$$

Assuming that n_1 nodes transmitted successfully in round 1, in round 2, the remaining nodes that have not been discovered in round 1, i.e., $N - n_1$ nodes transmit in a duration τ and the same $N - n_1$ nodes listen in a duration $\omega - \tau$, while the nodes that transmitted successfully in round 1, i.e., n_1 keep listening during all the slot ω. Therefore, the expected energy consumed in round 2 ($E(P_2)$) and the expected energy consumed in round 3 ($E(P_3)$) are given below. The average energy consumption $E(P)$ by each node in Joules is given in equation 47. We added the average energy consumed during the termination round, i.e., $\omega \cdot E_l$.

$$E(P_2) = \tau \cdot E_{tx} \cdot (N - n_1) + (\omega - \tau) \cdot E_l \cdot (N - n_1) + \omega \cdot E_l \cdot n_1 \tag{44}$$

$$E(P_3) = \tau \cdot E_{tx} \cdot (N - n_1 - n_2) + (\omega - \tau) \cdot E_l \cdot (N - n_1 - n_2) + \omega \cdot E_l \cdot (n_1 + n_2) \tag{45}$$

$$E(P) = \frac{1}{N} \cdot \sum_{k=1}^{r} E(P_k) + \omega \cdot E_l \tag{46}$$

$$\begin{aligned} E(P) = \frac{1}{N} \cdot [\tau E_{tx} \cdot N \cdot r + (\omega - \tau) \cdot E_l \cdot N \cdot r \\ - (\tau \cdot E_{tx} + (\omega - \tau) \cdot E_l - \omega \cdot E_l) \cdot \sum_{k=1}^{r-1} \sum_{i=1}^{k} n_i] + \omega \cdot E_l \end{aligned} \tag{47}$$

Next, we add the average energy consumed by each node during the feedbacks, i.e., $E(P_f)$. The number of rounds r is obtained from equation 41.

The average energy consumption by each node ($E(Pf)$) in Joules for the feedbacks is given in equation 48, adding the average energy in the termination round $\tau_f \cdot N \cdot E_l$. The total energy consumption is given in equation 49.

$$\begin{aligned} E(P_f) = \frac{\tau_f}{N} \cdot \sum_{i=1}^{r} [n_i \cdot [(n_i - 1) \cdot E_{tx} + (N - n_i + 1) \cdot E_l] \\ + [(N - n_i) \cdot [n_i \cdot E_{tx} + (N - n_i) \cdot E_l]] + \tau_f \cdot N \cdot E_l \end{aligned} \tag{48}$$

$$E(P_t) = E(P) + E(P_f) \tag{49}$$

3) Overhead: To obtain the total number of packets sent (P_{sent}) in the first sub-slot, we add the number of packets that are sent in the first sub-slot of each round. In round 1 all the N nodes transmit 1 packet, in round 2 only the remaining nodes that did not transmit successfully in round 1, i.e., $(N-n_1)$ nodes transmit 1 packet. We conclude that in round 3 only $N-n_1-n_2$ nodes have not managed to transmit successfully in previous rounds, thus the $N-n_1-n_2$ nodes transmit 1 packet.

$$P_{sent} = N + (N - n_1) + (N - n_1 - n_2) + \cdots + (N - n_1 - \cdots - n_{r-1}) \tag{50}$$

$$P_{sent} = N \cdot r - \sum_{k=1}^{r-1} \sum_{i=1}^{k} n_i \tag{51}$$

We can solve equation 51 using r from equation 41 and equation 40. To P_{sent} we must add the packets sent in the termination round, in which 0 nodes are sent, thus the P_{sent} is not modified.

We must now obtain the packets sent in the feedbacks (second sub-slot). In the feedback of each round i, $(n_i - 1) \cdot n_i + n_i \cdot (N - n_i)$ packets are sent. So the total number of packets sent in the feedbacks in r rounds is given in equation 52.

$$P_{sentf} = \sum_{k=1}^{r} [(n_i - 1) \cdot n_i + n_i \cdot (N - n_i)] = (N - 1) \cdot N \tag{52}$$

In the termination round, 0 feedback packets are sent, thus P_{sentf} is not modified. The total packets sent is given in equation 53 using equation 51 and equation 52.

$$P_{sentT} = P_{sent} + P_{sentf} \tag{53}$$

4) Packet delivery ratio: In round 1, $N-1$ nodes receive n_1 packets each one, whereas in round 2, $N-1$ nodes receive n_2 packets each one. So, we obtain P_{rec} as the total number of packets received in equation 54.

$$P_{rec} = \sum_{k=1}^{r} [(N - 1) \cdot n_k] = (N - 1) \cdot N \tag{54}$$

In the termination round, 0 packets are received, thus the P_{rec} is not modified. As for the feedbacks, in phase i, $(N-1) \cdot n_i$ total packets are received. P_{recf} is obtained from equation 55.

$$P_{recf} = \sum_{i=1}^{r} [n_i \cdot (N-1)] = N \cdot (N-1) \tag{55}$$

In the termination round, 0 feedback packets are received, thus P_{recf} is not modified.

The packet delivery ratio is obtained from equation 56, using P_{sent} from equation 51 and equation P_{rec} from equation 54.

$$PDR = \frac{P_{rec}}{P_{sent}} \tag{56}$$

As for the packet delivery ratio for the feedbacks, it is obtained from equation 57, using P_{sentf} from equation 52 and P_{recf} from equation 55.

$$PDR_f = \frac{P_{recf}}{P_{sentf}} \tag{57}$$

5) CDF of discoveries: The CDF of discoveries for round k represents a CDF to indicate how long it takes for the convergence to reach 100%.

Taking into account the probability of discovery in round k (p_k) given in equation 37 and the number of neighbors discovered in round k (n_k) given in equation 40, the CDF of the discoveries for round k can be obtained as follows:

$$Fx(k) = P(X \leq k) = \sum_{x=0}^{k} \binom{r}{x} \cdot p_x^x \cdot (1-p_x)^{r-x} \tag{58}$$

being $p_x = (1-a)^{N-\sum_{i=1}^{x-1} n_i - 1}$, $n_x = (N - \sum_{i=1}^{x-1} n_i) \cdot p_x$, and r obtained from equation 41.

6) Percentage of idle slots: CDH only generates 1 idle slot i.e., the termination round. Thus the probability of generating idle slots is given in equation 59

$$PIS = \frac{1}{r+1} \tag{59}$$

We do not include this percentage in the graphical results section VI since it is low.

V. ANALYSIS OF REFERENCE PROTOCOLS

For comparison purposes, we include the analysis of two randomized protocols chosen from the literature: Hello [8] and PRR [5].

CDPRR is similar to PRR while CDH is similar to Hello, but in the case of PRR and Hello they are not handshake-based and all the nodes contend during all the rounds, (i.e., no node stops contending when it successfully transmits). Thus, PRR and Hello are appropriate for comparison purposes and we have chosen those protocols as reference. Since there is no complete analytical model available for PRR and Hello, we have developed an analytical model for those reference protocols. Furthermore, the analytical model for PRR is based on that for CDPRR but no nodes stop contending and there is not collision detection mechanism, while the analytical model for Hello is based on the model obtained for CDH but no nodes stop contending and a collision detection mechanism is not used (analytical models for both CDPRR and CDH are included in Section IV).

A. PRR

In PRR, the time is slotted in rounds, and in every round each node chooses to transmit with probability $\frac{1}{N}$ or listen with probability $1 - \frac{1}{N}$. Moreover, the protocol is not handshake-based, no nodes stop contending, thus the number of rounds (Nr), after which the protocol finishes, must be carefully set.

The variables used in the analysis of PRR are defined in Table III.

The neighbor discovery time is the number of rounds multiplied by the round duration, in seconds. We obtain the neighbor discovery time in seconds from equation 60.

$$T_t = Nr \cdot \tau \tag{60}$$

First, to obtain the energy consumption, we calculate NT as the number of nodes transmitting in Nr rounds and NL as the number of nodes listening in Nr rounds. They follow a binomial distribution: $NT \sim B(n, p)$ to count the number of transmissions in several experiments n and $p = \frac{1}{N}$, while $NL \sim B(n, q)$ to count the number of listenings in several experiments n being $q = 1 - \frac{1}{N}$.

From equation 6, we obtain $E(NT)$ (i.e., the expected number of nodes transmitting), being $i = 1$, and $n = N \cdot Nr$ since there are N nodes that can transmit or receive and Nr rounds thus the total number of possible transmissions are counted in n experiments, whereas from equation 7 we obtain $E(NL)$ (i.e., the expected number of nodes listening) being $i = 1$ and $n = N \cdot Nr$ (for the same reason explained for the transmissions).

The energy consumption in Joules $E(P_1)$ is obtained from equation 8, using Nr instead of X. And the average energy consumption by each node ($E(P)$) in Joules is obtained from equation 10, taking into account that no nodes stop contending.

TABLE III

VARIABLE DEFINITIONS FOR PRR.

Variable	***Definition***
Nr	Number of rounds (fixed value).
τ	The time a node is transmitting in seconds, and it matches the round duration in seconds.
N	The number of nodes in the network.
$\frac{1}{N}$	Probability that a node transmits.
$1-\frac{1}{N}$	Probability that a node listens.
T_t	The neighbor discovery time in seconds.
NT	The total number of nodes transmitting in the Nr rounds.
NL	The total number of nodes listening in the Nr rounds.
p	the probability that a node transmits in a round ($\frac{1}{N}$).
q	The probability that a node listens in a round ($1-\frac{1}{N}$).
n	The number of experiments in which the nodes can transmit or listen, i.e., $N \cdot Nr$.
$E(NT)$	The expected number of transmissions in Nr rounds.
$E(NL)$	The expected number of listenings in Nr rounds.
$E(P_1)$	The expected energy consumption in Joules in round 1.
$E(P)$	The average energy consumption by each node in Joules in the Nr rounds.
Y	The number of rounds in which 1 single node transmits successfully (i.e., the total number of packets that do not collide) in Nr rounds.
p_s	The probability that a node transmits successfully in a round.
P_{rec}	The total number of packets successfully received (i.e., the packets that did not collide) by each node in Nr rounds.
P_{sent}	The total number of packets sent.
T_{hr}	The throughput in packet/s.
E_{tx}	The amount of energy consumed by a node transmitting per second.
E_l	The amount of energy consumed by a node listening per second.
PDR	The packet delivery ratio.
$F(k; Nr, p)$	CDF (Cumulative Distribution Function) of the discoveries.
$E(NIS)$	Total number of rounds in which all the nodes are listening.
PIS	Percentage of idle slots.

$$E(P) = \frac{\tau \cdot Nr}{N} \cdot \left[E_{tx} + E_l \cdot N \cdot \left(1 - \frac{1}{N}\right)\right] \tag{61}$$

And the total number of packets sent (P_{sent}) in Nr rounds, taking into account that NT follows a binomial distribution as stated above $NT \sim B(n, \frac{1}{N})$ and $n = N \cdot Nr$, i.e., N nodes in Nr rounds, thus $N \cdot Nr$ experiments is given below. The overhead, i.e., P_{sent} is obtained from equation 63.

$$P_{sent} = E(NT) = \sum_{k=1}^{n} \binom{n}{k} k \cdot \left(\frac{1}{N}\right)^{k} \cdot \left(1 - \frac{1}{N}\right)^{n-k} = N \cdot Nr \cdot \frac{1}{N} = Nr \tag{62}$$

$$P_{sent} = Nr \tag{63}$$

To obtain the packet delivery ratio, we must take into account that in Nr rounds the total number of nodes in which 1 single node transmits, thus P_{rec} total number of packets are received by each node in equation 65, being $p_s = \frac{1}{N} \cdot (1 - \frac{1}{N})^{N-1}$ and $Y \sim B(Nr, p_s)$ a binomial distribution.

$$P_{rec} = E(Y) = \sum_{k=0}^{Nr} \binom{Nr}{k} \cdot k \cdot p_s^k \cdot (1 - p_s)^{Nr-k} = Nr \cdot p_s \tag{64}$$

$$P_{rec} = \frac{Nr}{N} \cdot \left(1 - \frac{1}{N}\right)^{N-1} \tag{65}$$

The packet delivery ratio is obtained from equation 66 using P_{sent} from equation 63 and P_{rec} from equation 65.

$$PDR = \frac{P_{rec}}{P_{sent}} = \frac{1}{N} \cdot \left(1 - \frac{1}{N}\right)^{N-1} \tag{66}$$

The CDF of the discoveries for round k, assuming that PRR follows a binomial distribution $B(n, p)$, and taking into account that the probability of successful transmission in a round is $p = \frac{1}{N} \cdot (1 - \frac{1}{N})^{N-1}$ and $n = Nr$ the number of rounds fixed.

$$F(k; Nr, p) = P(X \leq k) = \sum_{x=0}^{k} \binom{Nr}{x} \cdot p^x \cdot (1 - p)^{Nr-x} \tag{67}$$

Therefore, we obtain the following equation:

$$F(k; Nr, p) = I_{1-p}(Nr - k, k + 1) \tag{68}$$

being $I_x(c, d)$ the regularized beta function.

We conclude that for PRR, the packet delivery ratio does not depend on the number of rounds Nr, although it depends on N. However, the CDF of discoveries for PRR depends on Nr, i.e., if Nr increases the number of rounds to reach 100% convergence grows. Notice that in PRR it is not possible to know if all the neighbors have been discovered when we set a fixed Nr. If we set higher Nr, this will result in more neighbor discovery time and energy consumption. However, the number of discovered neighbors will probably increase. Moreover, if we set higher Nr, the packets sent will increase.

The total number of rounds that all the nodes are listening in Nr rounds is given in equation 69, being $p = (1 - \frac{1}{N})^N$ the probability of an idle slot (i.e., all the nodes are listening) in a round.

$$E(NIS) = \sum_{k=0}^{Nr} k \cdot \binom{Nr}{k} \cdot p^k \cdot (1-p)^{Nr-k} = Nr \cdot p = Nr \cdot (1 - \frac{1}{N})^N \tag{69}$$

The percentage of idle slots is given in equation 71.

$$PIS = \frac{E(NIS)}{Nr} = \frac{Nr \cdot (1 - \frac{1}{N})^N}{Nr} \tag{70}$$

$$PIS = (1 - \frac{1}{N})^N \tag{71}$$

Furthermore, if we increase Nr, the percentage of idle slots will not vary.

B. Hello

In Hello, the time is also slotted in rounds (of duration ω), and in every round each node transmits a single packet beginning in a randomly chosen time instant t_i of duration τ and listens for the rest of the slot $\omega - \tau$. Moreover, the protocol is not handshake-based, no nodes stop contending, thus the number of rounds (Nr), after which the protocol finishes, must be carefully set.

The variables used for the analysis of Hello are defined in Table IV.

The protocol follows a uniform distribution $U(0, t_w)$ for the times t_i since for all the intervals of equal length (τ) in the distribution in their range ([0, $\omega - \tau$]) are equally probable. This distribution is the same as for CDH, but in this case we take into account that no nodes stop contending, thus there are always N nodes contending.

The average neighbor discovery time (T_t) in seconds is obtained from equation 72, taking into account that there are Nr rounds and the round duration is ω.

$$T_t = Nr \cdot \omega \tag{72}$$

We obtain the expected energy consumption $E(P_1)$ in round 1 from equation 43. As for the average energy consumption by each node in Joules, i.e., the average energy consumption by each node in Nr rounds ($E(P)$) is given in equation 73.

$$E(P) = \frac{1}{N} \cdot \sum_{i=1}^{Nr} E(P_1) = Nr \cdot [E_{tx} \cdot \tau + E_l \cdot (\omega - \tau)] \tag{73}$$

TABLE IV

VARIABLE DEFINITIONS FOR HELLO.

Variable	*Definition*
Nr	The number of rounds (fixed value).
ω	The time a node is transmitting plus the time a node is listening in seconds, i.e., the slot width.
τ	The time a node is transmitting in seconds.
$\omega - \tau$	The time a node is listening in a round.
N	The number of nodes in the network.
t_i	The time in which a node starts transmitting.
T_t	The average neighbor discovery time in seconds.
$E(P_1)$	The expected energy consumption in a round.
$E(P)$	The average energy consumption by each node in the Nr rounds.
n_1	The number of successfully received packets in one round.
P_{rec}	The total number of packets received by each node in Nr rounds.
T_{hr}	The throughput in packet/s.
P_{sent}	The total number of packets sent in Nr rounds.
E_{tx}	The amount of energy consumed by a node transmitting per second.
E_l	The amount of energy consumed by a node listening per second.
PDR	Packet delivery ratio.
$Fx(k)$	CDF (Cumulative Distribution Function) of the discoveries.

As for the total number of packets sent (P_{sent}), taking into account that N nodes transmit per round, is given in equation 74.

$$P_{sent} = Nr \cdot N \tag{74}$$

Next, we will obtain the packet delivery ratio. First, in 1 round (all the nodes are contending), we use equation 39 from CDH: $n_1 = N \cdot (1-a)^{N-1}$, being $a = \frac{\tau}{\omega - \tau}$. In Nr rounds, the total number of packets received is $Nr \cdot (N-1) \cdot n_1$, and the total number of packets received by each node is (P_{rec}):

$$P_{rec} = Nr \cdot (N-1) \cdot N \cdot (1-a)^{N-1} \tag{75}$$

The packet delivery ratio is obtained from equation 76 using P_{rec} from equation 75 and P_{sent} from equation 74.

$$PDR = \frac{Nr \cdot (N-1) \cdot N \cdot (1-a)^{N-1}}{Nr \cdot N} = (N-1) \cdot (1-a)^{N-1} \tag{76}$$

As for the CDF of discoveries for round k, we take into account that $p_1 = (1-a)^{N-1}$ the probability that 1 node transmits successfully in a round, and Nr the number of rounds fixed.

$$F(k; Nr, p_1) = P(X \leq k) = \sum_{x=0}^{k} \binom{Nr}{x} \cdot p_1^x \cdot (1-p_1)^{Nr-x} \tag{77}$$

Using $I_x(c, d)$, the regularized beta function.

$$F(k; Nr, p_1) = I_{1-p_1}(Nr - k, k+1) \tag{78}$$

For Hello, there are no idle slots, since in every round there are N nodes transmitting. Thus, we will not represent this value in section VI.

We conclude that for Hello, the packet delivery ratio does not depend on the number of rounds Nr, although it depends on N. However, the CDF of discoveries depends on the number of rounds Nr fixed, i.e., if Nr increases, the number of rounds to reach 100% convergence will grow. Notice that in Hello it is not possible to know if all the neighbors have been discovered when we set a fixed Nr. Again, if we set higher Nr, this will result in more neighbor discovery time and energy consumption. However, the number of discovered neighbors will probably increase. Furthermore, if Nr increases the number of packets sent grows.

VI. GRAPHICAL RESULTS

Next, we proceed to present the graphical results obtained from the equations shown above. Hello [8] and PRR [5] have been chosen from the literature for comparison purposes since they are widely used to compare against other proposals. We used ZigBee (CC2420), i.e., $E_{tx} = 0.0522J$ (the energy spent by a node transmitting per second) and $E_l = 0.068J$ (the energy spent by a node listening per second). We also set $\tau = 0.07$ s (the time a node is transmitting a BROADCAST), $\tau_f = 0.000392s$ (time a node is transmitting a feedback packet), a BROADCAST packet size of 2500 bytes and a feedback packet size of 14 bytes. For CDH and Hello we set the slot width $\omega = N \cdot \tau$. We assume that all the nodes have the same transmission range, and the nodes are deployed in a one-hop scenario, i.e., all the nodes are within the transmission range of all the others. The model of CDPRR shown above follows a geometrical distribution $Geo(p_i)$. As for the CDH we used a uniform distribution $U(0, \omega - \tau)$. For comparison purposes we set the duration of Hello Nr to $0.5 \cdot N$ rounds, and for PRR Nr to $10 \cdot N$ rounds, since both reference protocols achieve to discover all the neighbors with high probability (different from 1) setting these number of rounds. The selection of this parameter must be properly carried out since as the number of rounds increases, the number of discovered neighbors, the neighbor

TABLE V

PARAMETERS.

Parameter	***Value***
Radio model	CC2420
E_{tx}	0.0522J
E_l	0.068J
Packet size broadcast s_b	2500 bytes
Packet size feedback s_f	14 bytes
Slot width CDH, Hello	$\omega = N \cdot \tau$
Slot width CDPRR, PRR	τ
Slot feedback CDH	$\omega_f = N \cdot \tau_f$
Slot feedback CDH $(25\tau and 50\tau)$	$\omega_f = 100 \cdot \tau_f$
Slot feedback CDPRR	τ_f
τ	0.07s
τ_f	0.000392s
Number of rounds Hello	$0.5 \cdot N$
Number of rounds PRR	$10 \cdot N$
Varying ω for CDH	$(N-1) \cdot \tau$, $N \cdot \tau$, $2N \cdot \tau$, $3N \cdot \tau$, $50 \cdot \tau$, $25 \cdot \tau$

discovery time and the energy consumption also increase. However, as the number of rounds vary, the packet delivery ratio does not vary. However, as the number of rounds vary, the CDF of discoveries will also vary and we can not find the number of discovered neighbors neither for Hello nor for PRR in an analytical model. For CDH with a fixed slot width, i.e., $50 \cdot \tau$ and $25 \cdot \tau$, we set a feedback duration of $100 \cdot \tau_f$, while for the other slot widths, i.e., $\omega = (N-1) \cdot \tau$, $\omega = N \cdot \tau$, $\omega = 2 \cdot N \cdot \tau$ and $\omega = 3 \cdot N \cdot \tau$ the feedback duration depends on N ($N \cdot \tau_f$).

Table V summarizes the main parameters set to obtain the graphical results.

A. Neighbor discovery time

To obtain the Neighbor discovery time in seconds represented in Figure 5 we use the above equations for L_t.

According to Figure 5, we found that all the protocols present an increasing trend with the number of nodes, and CDH outperforms the other solutions regarding this metric. However, the performance of CDPRR is similar and outperforms PRR with 10N rounds, and Hello with 0.5N rounds is the worst. Therefore, we conclude that CDH is faster than the other solutions since it manages to discover all the neighbors in a reduced amount of time.

As shown in Figure 6, varying the slot width (ω), the best performance is obtained when we set $\omega = 50 \cdot \tau$ for number of nodes above 50, CDH with $\omega = 25 \cdot \tau$, $\omega = (N-1) \cdot \tau$ and $\omega = N \cdot \tau$ present intermediate results, followed by CDH $\omega = 2 \cdot N$ and CDH $\omega = 3 \cdot N$ is the

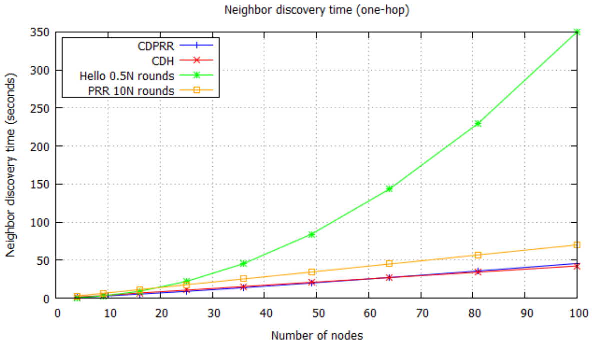

Fig. 5. Neighbor discovery time comparison (one-hop).

worst. CDH with $\omega = 50 \cdot \tau$ for number of nodes above 50 has enough time in each round to discover the neighbors and does not waste time. However, CDH with $\omega = 2 \cdot N$ and CDH with $\omega = 3 \cdot \tau$ waste a lot of time by presenting large rounds.

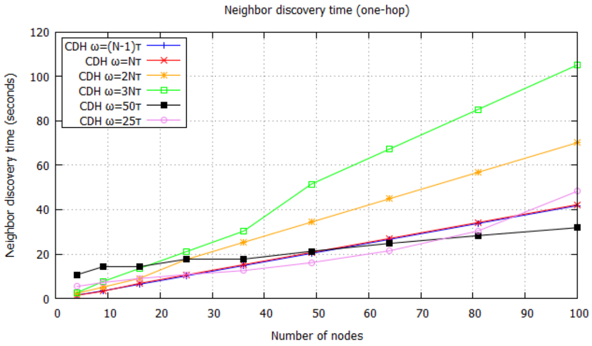

Fig. 6. Neighbor discovery time comparison (one-hop).

B. Energy consumption

Figure 7 presents the energy consumption in Joules obtained by using the above equations of the average energy consumption.

According to Figure 7, we conclude that all the protocols present an increasing trend with the number of nodes, and the results follow the same trend as for latencies in Figure 5, i.e, CDH outperforms the other protocols regarding the average energy consumption. However, CDPRR outperforms PRR with 10N rounds, and Hello with 0.5N rounds is the worst. Again, CDH consumes less energy since the neighbor discovery time is lower. CDPRR consumes less energy than PRR with 10N rounds and Hello with 0.5N rounds since it has a lower neighbor discovery time.

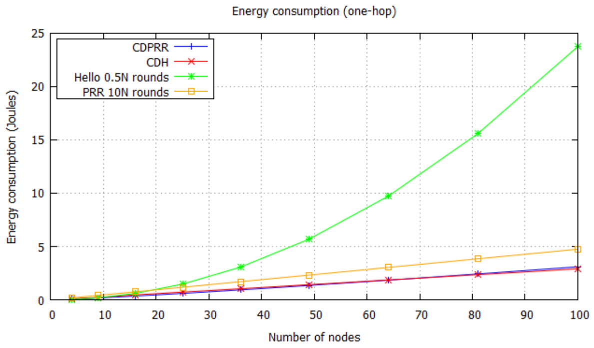

Fig. 7. Energy consumption comparison (one-hop).

Regarding the average energy consumption in Figure 8, varying the slot width (ω), the best results are obtained when we set $\omega = 25 \cdot \tau$ for number of nodes below 75, while CDH with $\omega = 50 \cdot \tau$, $\omega = (N-1) \cdot \tau$ and $\omega = N \cdot \tau$ present intermediate results. Then CDH with $\omega = 2 \cdot N \cdot \tau$ is better than CDH with $\omega = 3 \cdot N \cdot \tau$, which is the worst. The same conclusion as in the neighbor discovery metric is valid for the energy consumption.

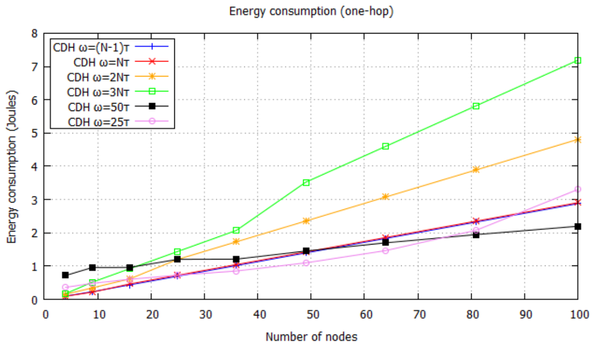

Fig. 8. Energy consumption comparison (one-hop).

Moreover, we conclude that both protocols CDPRR and CDH manage to discover all the neighbors in a one-hop scenario.

C. Number of packets sent

A lower number of packets sent (overhead) means an advantage to the protocol considered.

According to Figure 9, both CDH and CDPRR outperform PRR with 10N rounds and Hello 0.5N rounds is the worst, in terms of number of packets sent. Furthermore, all the protocols follow an increasing trend with the number of nodes. Notice that CDH and CDPRR send less packets since they finish the discovery process in a lower amount of time. Moreover, as time goes by, there are more nodes that have been discovered thus they are listening and do not send packets.

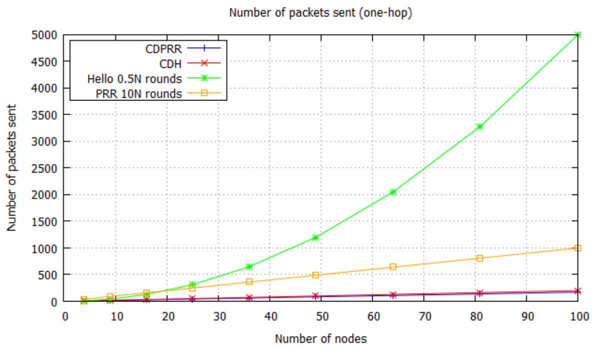

Fig. 9. Number of packets sent comparison (one-hop).

As shown in Figure 10, CDH with $\omega = 2 \cdot N \cdot \tau$ and $\omega = 3 \cdot N \cdot \tau$ present the best results, followed by CDH with $\omega = (N-1) \cdot \tau$ and $\omega = N \cdot \tau$. CDH with $\omega = 50 \cdot \tau$ presents intermediate results, while CDH with $\omega = 25 \cdot \tau$ is the worst. Notice that the results for $\omega = 50 \cdot \tau$ is better than those for $\omega = 25 \cdot \tau$ since more packets sent are received and more neighbors are discovered in a round thus less nodes are contending in the following rounds and the neighbor discovery finishes before therefore the number of packets sent is lower. CDH with $\omega = 2 \cdot N \cdot \tau$ and CDH with $\omega = 3 \cdot N \cdot \tau$ send less number of packets since their round duration is higher resulting in more neighbor discoveries per round and thus the neighbor discovery finishes before and therefore less packets are sent.

Figure 11 shows that the number of packets sent for the feedbacks presents and increasing trend with the number of nodes. The result is almost the same for CDPRR and CDH for any ω, i.e., approximately N^2 as it can be seen in equation 21 and equation 52. Notice that the number of packets sent for the feedbacks are fixed and they are sent in a deterministic way.

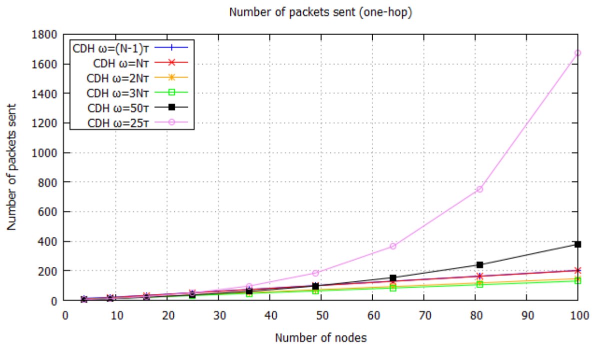

Fig. 10. Number of packets sent comparison (one-hop).

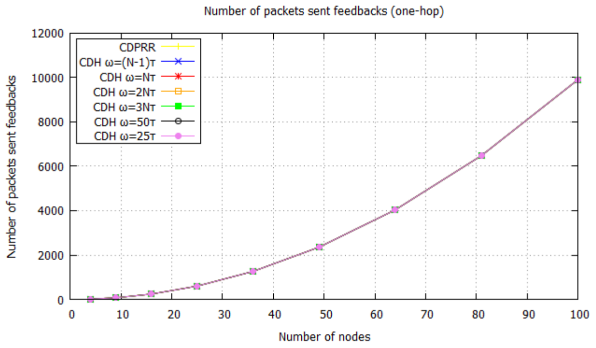

Fig. 11. Number of packets sent feedbacks comparison (one-hop).

D. Packet delivery ratio

In this Section we proceed to present the results obtained regarding the packet delivery ratio, obtained from the equations above. A high packet delivery ratio means an advantage for the protocol considered.

According to Figure 12, we conclude that CDH outperforms the other solutions regarding the packet delivery ratio, Hello 0.5N rounds also presents good results, while CDPRR presents better results than PRR with 10N rounds, which is the worst. A higher amount of packets sent are received in CDH, since as time goes by more nodes have been discovered thus less nodes are sending packets therefore the collisions are reduced and more packets are received.

As shown in Figure 13, CDH with $\omega = 50 \cdot \tau$ is the best for number of nodes below 16, regarding the packet delivery ratio, while CDH with $\omega = 25 \cdot \tau$ is the worst for number of nodes above 25. CDH with $\omega = 3 \cdot N \cdot \tau$ is the best for number of nodes above 16, followed by

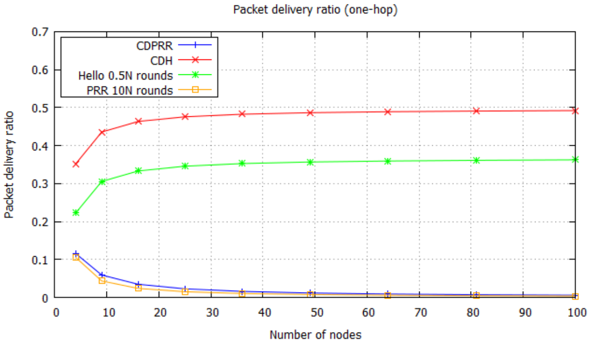

Fig. 12. Packet delivery ratio comparison (one-hop).

$\omega = 2 \cdot N \cdot \tau$. CDH with $\omega = N \cdot \tau$ presents intermediate results, while CDH with $\omega = (N-1) \cdot \tau$ also presents intermediate results, although it is the worst for number of nodes below 25. CDH with $\omega = 50 \cdot \tau$ presents a higher packet delivery ratio for low number of nodes since the round duration is higher thus more packets sent are received. For $\omega = 25 \cdot \tau$ the packet delivery ratio is the worst for a number of nodes above 25 since the round duration is low and produces more collisions thus the number of packets received is lower. CDH with $\omega = 3 \cdot N \cdot \tau$ is the best because the round duration is higher thus more packets are received. Notice that the PDR drops with an increasing number of nodes and this is the expected behavior for $\omega = 25 \cdot \tau$ and $\omega = 50 \cdot \tau$.

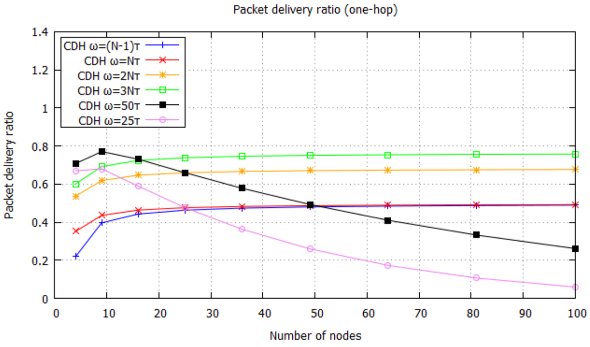

Fig. 13. Packet delivery ratio comparison (one-hop).

Figure 14 includes the results of the packet delivery ratio for the feedbacks. CDH outperforms CDPRR no matter which ω we set for CDH. However, the performance of CDH and CDPRR is almost the same for a number of nodes above 9. This result does not mean that CDPRR loses feedbacks. The only loss of feedback packets correspond to the last termination round since 1 node stopped and does not receive the feedbacks of the other N-1 nodes. Notice that in CDH the packet delivery ratio for the feedbacks is 100% since all the feedback packets are received. The behavior for the packet delivery ratio of both protocols is the expected since the feedbacks are sent in a deterministic way and all the packets sent are received.

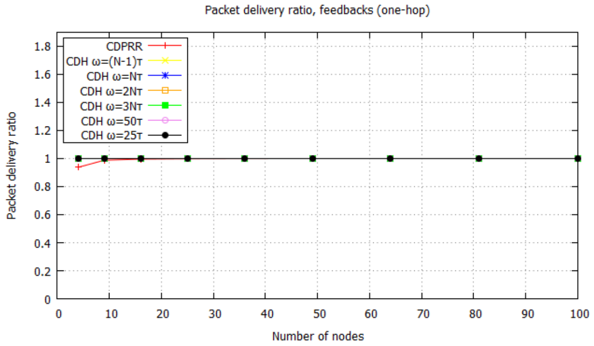

Fig. 14. Packet delivery ratio feedbacks comparison (one-hop).

E. CDF of discoveries

To obtain the CDF of the discoveries, we set a network of N=4 nodes. Similar results are obtained for larger networks.

The sooner the convergence reaches to 100% the better will be the protocol.

According to Figure 15, CDH needs less rounds for the convergence to reach 100% than the other solutions. Furthermore, CDPRR outperforms PRR 10N rounds regarding the CDF of discoveries, and Hello 4N rounds is the worst. If we increase the number of rounds set for Hello, the number of discovered neighbors will grow but the number of rounds to reach 100% convergence of discoveries and the time consumption will also grow. In addition, the duration of the rounds set for CDH and Hello is larger ($\omega = N \cdot \tau$) than for CDPRR and PRR.

As shown in Figure 16, CDH $\omega = 3N \cdot \tau$, $\omega = 50 \cdot \tau$ and $\omega = 25 \cdot \tau$ manage to discover all the neighbors in 2 rounds presenting the best CDF of discoveries, since the slot width is higher than the other solutions and more neighbors are discovered in each round. Then, CDH $\omega = 2N \cdot \tau$ achieves the discovery of all the neighbors in 3 rounds. CDH with $\omega = N \cdot \tau$ manages to discover all the neighbors in 4 rounds, and finally CDH with $\omega = (N-1) \cdot \tau$ is the worst,

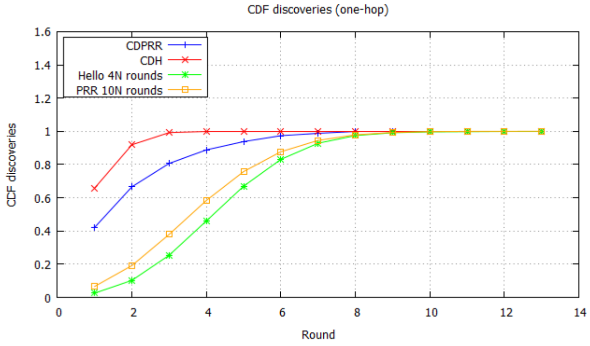

Fig. 15. CDF of discoveries (one-hop).

completing the discovery of all the neighbors in 6 rounds, since the slot width is lower thus less neighbors are discovered in each round.

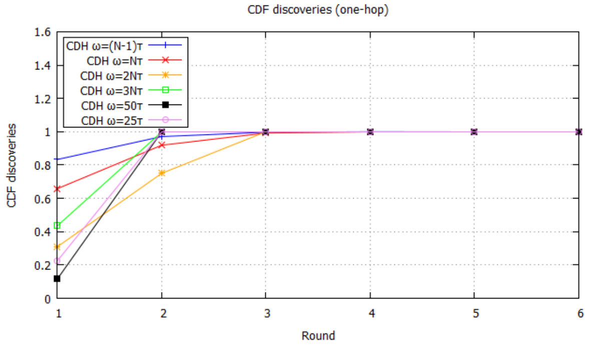

Fig. 16. CDF of discoveries (one-hop).

F. Percentage of idle slots

As for the percentage of idle slots, having more idle slots would be considered as an advantage since less energy is consumed and the number of packets sent is reduced. However, more idle slots produces an increase in the neighbor discovery time.

According to Figure 17, PRR with 10N rounds includes less percentage of idle slots than CDPRR. Furthermore, both protocols follows an increasing asymptotic trend with the number of nodes. CDPRR presents more idle slots since as time goes by there are less nodes contending thus these nodes are listening and the probability of generating idle slots increases.

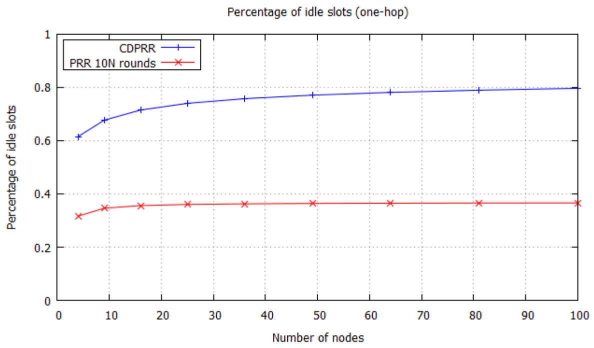

Fig. 17. Percentage of idle slots (one-hop).

VII. DISCUSSION

We found that CDH and CDPRR both present a linear neighbor discovery time $O(N)$, being N the number of nodes in the network.

In addition, both CDH and CDPRR protocols manage to discover all their neighbors with probability 1 in a one-hop scenario, while neither Hello nor PRR manage to discover all the neighbors with probability 1.

Overall, we found that CDPRR and CDH follow more realistic assumptions than existing randomized protocols.

Regarding the percentage of idle slots, having more idle slots would be preferrable since less energy is consumed. Furthermore, when more idle slots occur the number of packets sent is reduced. However, more idle slots increases the neighbor discovery time. Moreover, the nodes which arrive to state S earlier in CDPRR are always listening from then on thus more idle slots are produced.

Packet delivery ratio is a positive phenomenon and a high packet delivery ratio is considered as desirable.

Among its practical limitations, CDPRR presents a low packet delivery ratio, needs to know the number of nodes, and the nodes do not start transmission at different time instants. However, CDH solves these limitations. Both CDPRR and CDH need synchronization in slot boundaries and can not be used in mobile networks, i.e., MANETs, and the time must be slotted. As possible ways to solve these limitations, a synchronization mechanism might be used before beginning the neighbor discovery process, and enhance the protocols to allow nodes join or leave detecting when a new node comes into transmission range of the other nodes or leaves the network, in MANETs.

As practical applications, CDH is fast and spends little energy, it has little overhead (packet sent) and a high packet delivery ratio, which means an advantage, thus it is suitable to be used in practical scenarios in which the batteries can not be recharged frequently, and also when the number of nodes that compose the network is unknown, and both CDH and CDPRR can be used in static wireless ad hoc neighbor discovery environments or spontaneous networks based on trust in which people come together in a meeting to exchange information during a period of time.

Regarding the CDH with fixed slot width, it presents similar results for $\omega = 50 \cdot \tau$ and $\omega = 25\tau$ regarding the neighbor discovery time, energy consumption and the number of rounds to reach a 100% convergence of discoveries, while the performance for $\omega = 25 \cdot \tau$ is worse regarding the number of packets sent and packet delivery ratio.

According to the analytical model and the graphical results we conclude that there is no optimum value for all the metrics varying the slot width ω in CDH. However, CDH with $\omega = 50 \cdot \tau$ and $\omega = 25 \cdot \tau$ present the best results regarding the neighbor discovery time and energy consumption while they present the worst results regarding the number of packets sent, packet delivery ratio, and present intermediate results regarding the CDF of discoveries. As for CDH with $\omega = 2N \cdot \tau$ and $\omega = 3N \cdot \tau$, they present the worst results regarding the neighbor discovery time, energy consumption while they present the best results regarding the number of packets sent and packet delivery ratio, and $\omega = 3N \cdot \tau$ presents the best results regarding the CDF of discoveries. However, CDH with $\omega = (N-1) \cdot \tau$ and $\omega = N \cdot \tau$ present intermediate results in neighbor discovery time, energy consumption, packets sent and packet delivery ratio, while they present the worst results regarding CDF of discoveries.

In case we want to add security to the proposals, a possible solution can be to create a signature of the identifier using the private key and send in the BROADCAST packet the identifier, the public key and the signature. The receiver can check the signature (using the public key) and if there is an error when checking the signature this means that the message has been intercepted or manipulated.

To enhance the proposals to be used in Mobile Ad Hoc Networks (MANETs), we should take into account nodes going in and out of other node's transmission range, by enabling to exchange joining and leaving notifications.

In a realistic scenario in which the number of currently actual working nodes N' is less than the number of nodes known N, in the first sub-slot the probability of collision will be reduced in comparison to when $N' = N$, thus the probability of discovery will increase. The number of rounds after which the protocol finishes will be reduced thus the neighbor discovery time will also be reduced since the protocol ends when the N' nodes have been discovered (and $N' < N$). The total energy consumed will be reduced, and the packets sent will also be reduced. The packet delivery ratio will increase since less nodes are contending thus the probability of collision is reduced and the number of packets received increases. The percentage of idle slots will also increase since there are less nodes in the network. Moreover, in the second sub-slot less feedback packets are sent since there are less nodes in the network, the energy consumed decreases and the packet delivery ratio will not change. However, the protocol will not manage to discover all the neighbors with probability 1.

VIII. CONCLUSION

In this work we have carried out an analytical study about two randomized neighbor discovery protocols based on collision detection in static wireless ad hoc networks for one-hop environments.

For the analytical model, we used a geometric distribution $Geo(p_i)$ for the CDPRR protocol and a uniform distribution $U(0, t_u)$ for the CDH protocol.

To validate and compare the protocols we obtained a mathematical model for CDH, CDPRR and two reference protocols, i.e. Hello and PRR from the literature, and represented the results in several graphics using the equations obtained, regarding six metrics: neighbor discovery time, energy consumption, overhead (number of packets sent), packet delivery ratio, CDF of discoveries, and percentage of idle slots.

According to the analytical results obtained in a one-hop setting, we found that CDH outperforms the other solutions regarding the neighbor discovery time, energy consumption, number of packets sent, packet delivery ratio and CDF of discoveries, while CDPRR achieves good results and it is better than Hello and PRR regarding neighbor discovery time, energy consumption, CDF of discoveries and packets sent. Furthermore, we found that CDPRR presents more percentage of idle slots than PRR, which is a clear advantage regarding the energy consumed and the number of packets sent.

We also focused on the study in CDH when the slot width (ω) is varied, and we demonstrated that for CDH the number of nodes in the network can be unknown, i.e., we can set a slot width ω that does not depend on the number of nodes, and still provide reasonable results.

As future research work, we plan to model other randomized neighbor discovery protocols, spontaneous ad hoc trusted neighbor network creation protocols, and neighbor discovery protocols for MANETs.

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