An attempt to analyse Iterative Data Snooping and L_1-norm based on Monte Carlo simulation in the context of leveling networks

The goal of this paper is to evaluate the outlier identification performance of iterative Data Snooping (IDS) and L_1-norm in leveling networks by considering the redundancy of the network, number and size of the outliers. For this purpose, several Monte-Carlo experiments were conducted into three different leveling networks configurations. In addition, a new way to compare the results of IDS based on Least Squares (LS) residuals and robust estimators such as the L_1-norm has also been developed and presented. Two different scenarios were considered in that comparison: (i) both IDS and L_1-norm evaluated with the same threshold values; (ii) both IDS and L_1-norm compared with the same false positive rates. In latter case, a Monte-Carlo approach was applied to control the false positive rates. The question of which of them performs better depends on the viewpoint. From the perspective of analysis only according to the success rate, it is shown that L_1-norm performs better than IDS for the case of networks with low redundancy ($\bar{r} < 0.5$), especially for cases where more than one outlier is present in the dataset. In the relationship between false positive rate and outlier identification success rate, however, IDS performs better than L_1-norm. In that case, IDS with a critical value of 3.29 has the best cost-benefit ratio, independently of the levelling network configuration, number and size of outliers.

Keywords: Data Snooping; L_1-norm; Outliers; Leveling networks; Success rate; False positive rate

Subject classification codes: include these here if the journal requires them

Introduction

Quality control for outlier identification purposes is a routinely task in geodetic data analysis. Data Snooping (DS), initially proposed by Baarda (1968), still remains the most applied strategy (Rofatto et al., 2020a). The DS is a statistical testing procedure based on the residuals of a least-squares (LS) estimation process. Since DS identifies one outlier at time, its application is often performed iteratively (Iterative Data Snooping – IDS) until there is no outlier identified in the LS-residuals (Teunissen, 2006; Klein et al., 2019). However, since LS is sensitive to outliers, there are alternative strategies, such as the use of robust estimators like the L_1-norm. Here, the term “robust” means that these estimators are “insensitive” to (a certain amount of) outliers (Hekimoglu, 2005). Other alternative procedures have also been proposed, such as for outliers in two directions (Hekimoglu and Erenoglu, 2013).

Although major advances have occurred recently, there are still a number of gaps which paves way for further investigations. For example, Hekimoglu (2005), Eshagh et al. (2007), Baselga (2007), Knight and Wang (2009) and Sisman (2010) have not covered experiments with leveling networks. Hekimoglu and Erenoglu (2007) and Erenoglu and Hekimoglu (2010) have not addressed cases with multiple outliers, whereas Baselga et al. (2020) have analyzed only one single leveling network configuration. There are also researches where only the residuals between LS and L1-norm have been confronted (Marshall and Bethel, 1996; Yetkin and Inal, 2011; Gašincová and Gašinec, 2013; Inal et al., 2018).

These studies show that there is still no consensus about which one has a better performance. Here, on the other hand, we try to compare L_1-norm and IDS by considering the redundancy of the geodetic network, number and size of the outliers. These factors play an essential role for the comparison between IDS and L_1-norm. (Hekimoglu, 2005; Erenoglu and Hekimoglu, 2010). We focus on the L_1-norm and IDS for the case where leveling networks is in play. For this purpose, several Monte-Carlo (MC) experiments were conducted into three different leveling networks configuration. In addition, a new way to compare the results of IDS and robust estimators was developed and will also be presented.

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Theoretical background

In this section, we present the theoretical foundation involved with IDS and L_1-norm in order to support the comparison between them. Further details can be found in the related references.

Iterative Data snooping (IDS)

DS is a statistical testing procedure based on LS-residuals. Although it can also be applied based on the L1-residuals instead of the LS-residuals (Gao et al., 1992; Schwars and Kok, 1993; Junhuan, 2005; Amiri-Simkooei; 2018). The LS estimation process (or L_2-norm) seeks to minimize the (weighted) sum of the squared residuals:

$$\hat{v}^T W \hat{v} = min \tag{1}$$

where "$\hat{v}$" is the $n \times 1$ residuals vector and "W" is the $n \times n$ weight matrix of the observations. In the linearized Gauss-Markov model, the vector of $n \times 1$ LS-residuals is given by:

$$\hat{v} = A\delta\hat{x} - l \tag{2}$$

where "A" is the $n \times u$ design (or Jacobian) matrix, "$\delta\hat{x}$" is the $u \times 1$ vector of corrections for the initial parameters or unknowns and "l" is the $n \times 1$ vector of reduced observations (Klein et al., 2019).

LS is not a robust estimator, i.e., the LS solution is sensitive to outliers in the observations. Therefore, the key of the DS is that each observation is individually tested against a possible outlier by means of the following test statistic:

$$w_i = \frac{c_i^T W \hat{v}}{\sqrt{c_i^T W \Sigma_v W c_i}}$$

(3)

where "Σ_v" is the $n \times n$ covariance matrix of the residuals and "c_i" is the $n \times 1$ unit vector relating to the i^{th} observation being tested (Baarda, 1968; Teunissen, 2006).

Since the test statistics "w_i" follow the normal distribution and observational errors spread among all residuals, the observation flagged as being the outlying one is that whose test statistic satisfies the following inequalities:

$$|w_i| > \left|Z_{(\alpha_0/2)}\right|; \; |w_i| > \left|w_j\right| \; \forall \; i \neq j \quad (4)$$

where "$Z_{(\alpha_0/2)}$" is the threshold value in the normal distribution for a two-tailed test and the stipulated significance level "α_0" (Amiri-Simkooei, 2018). The choice of the significance level and consequently the threshold value plays a key role in the testing procedure performance (Nowel, 2016; Rofatto et al., 2020a). For the quality control of geodetic networks, the significance level is usually set at $\alpha_0 = 0.001$ (0.1%), which corresponds to a threshold of $\left|Z_{(\alpha_0/2)}\right| = 3.29$. The test statics in Eq. (3) can be replaced by "τ statistics" of the "Tau test" (Pope, 1976) in cases where the variance factor of observations is unknown. This procedure is outside the scope of this paper.

In general, the identified observation is excluded from the model and the LS estimation is performed again with the $(n-1)$ remaining observations. In some specific cases, the observation can be retained after a corrective action, for example, changing it sign. Since DS identifies only one outlier at a time, the LS estimation and DS testing procedure must be applied iteratively until there is no outlier identified in the observations. This procedure is often called "*iterative data snooping*" (IDS) (Teunissen, 2006; Rofatto et al., 2020a).

L_1-norm

The L_1-norm, unlike LS, seeks to minimize the (weighted) sum of the absolute residuals (Amiri-Simkooei, 2003):

$$p^T|\hat{v}| = \sum_{i=1}^{n} p_i \cdot |\hat{v}_i| = min \tag{5}$$

where "p" is the $n \times 1$ vector of observations weights, "p_i" and "$\hat{v}_i$" are the weight and residual of the i^{th} observation, respectively. The objective function in Eq. (5) is valid for uncorrelated observations such as in leveling networks. The case of correlated observations such as GNSS (Global Navigation Satellite Systems) networks is described, for example, in Yetkin and Inal (2011).

The LS is the best linear unbiased estimator (BLUE) when the weight matrix is taken as the inverse of the covariance matrix of the observations ($W = \Sigma_l^{-1}$) and the observational errors follow the (multivariate) normal distribution (Teunissen, 2003). On the other hand, the L_1-norm is less sensitive to outliers than LS and is also an unbiased estimator (Amiri-Simkooei, 2018).

The absolute value function prevents a generalized solution by differentiating the objective function of L_1-norm in Eq. (5), unlike the LS and its differentiable objective function in Eq. (1) (Marshall and Bethel, 1996). Thus, in general, the L_1-norm solution is obtained by means of linear programming. Here, we use the approach of the simplex method, presented in detail in Marshall and Bethel (1996) and Amiri-Simkooei (2003). Other approaches for L_1-norm solution can be found,e.g., Baselga (2007) and Baselga et al. (2020).

L_1-norm is currently underdeveloped due to the relative complexity of its implementation compared to the LS (Amiri-Simkooei, 2018). However, computational advanced techniques can be used efficiently at present (Lehmann, 2015; Rofatto et al., 2020a). In this context, L_1-norm has recently been applied, for example, to the deformation analysis of geodetic networks (Nowel, 2016; Amiri-Simkooei et al. 2017).

It is important to note that IDS and robust estimators (e.g., L_1-norm) deal with outliers differently: IDS excludes the outlying observation and updates the LS estimation; whereas the L_1-norm minimizes the objective function in Eq. (5) without any adaptation of excluding observations. Therefore, a fair comparison of the results of both methods should be applied in order to evaluate their performance in the presence of outliers. In the next section, we provide a new approach to compare IDS and L_1-norm.

An approach to compare IDS and L1-norm

The IDS performance against outliers can be evaluated, for example, by means of MC experiments with outliers intentionally inserted into the dataset. Thus, the success rate of the IDS is the ratio between the number of MC experiments which only the outlying observations were correctly identified and the total number of MC experiments (Rofatto et al., 2020a). However, since robust estimators (e.g., L_1-norm) do not exclude observations, a success rate for the L_1-norm cannot be obtained so directly.

On the other hand, the performance of robust estimators against outliers can be evaluated by the concept of the "breakdown point". The breakdown point indicates the maximum proportion of outliers which the estimator is able to tolerate (Hekimoglu, 2005). However, since the breakdown point of LS is zero (Hekimoglu and Erenoglu, 2007), it cannot be used to compare the IDS based on LS-residuals with robust estimators such as the L_1-norm.

In case of uncorrelated observations where the weight matrix W is diagonal,, the statistical test w_i in Eq. (3) becomes the "standardized residual" (SR) of each observation (Teunissen, 2006), that is, the ratio between the residual ($\hat{v}_i$) and its correspondent standard deviation ($\sigma_{\hat{v}_i}$), as follows:

$$w_i = \frac{\hat{v}_i}{\sigma_{\hat{v}_i}}$$

(6)

In this case, the i^{th} observation is excluded if $|w_i| = \frac{|\hat{v}_i|}{\sigma_{\hat{v}_i}} > \left|Z_{(\alpha_0/2)}\right|$ and $|w_i| > \left|w_j\right| \; \forall \; i \neq j$. Thus, L_1-norm can be considered successful when the absolute SR of all contaminated observations (and only these) exceed $\left|Z_{(\alpha_0/2)}\right|$. However, the standard deviation of L_1-residuals and LS-residuals are different (Junhuan, 2005). Therefore, the same value for the SR does not guarantee a completely fair comparison.

Regarding these issues, Knight and Wang (2009), for example, applied the "three-sigma rule" for observational residuals to compare the success rates of IDS, L_1-norm and others robust estimators. Other studies based on MC experiments, such as

Hekimoglu (2005), it is not clear whether the "three-sigma rule" for observational residuals was applied for both robust estimators and IDS or the threshold value of IDS was set as $|Z_{(\alpha_0/2)}| = 3.29$ for the SR.

This approach is not completely fair, because the threshold value $Z_{(\alpha_0/2)}$ is related to the significance level, that is, the "false positive rate" of the IDS (Klein et al., 2019). The "false positive rate" of L_1-norm may be different, since LS-residuals and L_1-residuals are different. Here, the "false positive rate" is the rate of experiments in which at least one observation is flagged as outlier, when in fact there is none.

Therefore, MC experiments can be performed by considering a scenario in the absence of outliers. In other words, only random errors are taken into account and the false positive rates of L_1-norm can be obtained for a given threshold value (i.e., number of times that L_1-norm detects outliers, when in fact there is none).. For comparison purposes, the extreme (i.e. maximum absolute) w_i test statistic $|Z_{(\alpha_0/2)}|$ could be stipulated as the threshold value. For example, if the threshold value of IDS is set as $|Z_{(\alpha_0/2)}| = 3.29$; then the false positive rate of L_1-norm is the ratio between the number of MC experiments in which at least one observation residual exceeds $3.29 \times \sigma$ (being σ the standard-deviation of each observation) and the total number of MC experiments. Note that this false positive rate (α'_0) may be different from that pre-stipulated for IDS (α_0).

Once the false positive rate of L_1-norm is derived, the corresponding critical value for the IDS can be obtained by MC experiments ($Z_{(\alpha'_0/2)}$), similarly to Lehmann (2012) and Rofatto et al. (2020a). Therefore, the IDS and L_1-norm are compared in two ways:

- by the same threshold for the residuals (but different false positive rates: α_0 and α'_0); and

- by the same false positive rate: α'_0 (but different thresholds for residuals).

In the first case, the threshold values are the same for both IDS and L_1-norm. In that case, false positive rates are expected to be different between them. This is due to the nature of their objective function (LS w.r.t. sum of squared residuals and L_1-norm w.r.t. sum of absolute residuals). In the second case, the false positive rate is user-controlled in both methods. In that case, the false positive rate of the L_1-norm (α'_0) is obtained for a given threshold. Then, the threshold of the IDS is computed based on that false positive rate (α'_0) from L_1-norm (denoted by $|Z_{(\alpha'_0/2)}|$). Therefore, both L_1-norm and IDS have the same false positive rate at level α'_0. Here, we denoted IDS for the case where the threshold values are taken the same as L_1-norm, whereas IDS' for the case where the false positive rates are equal to those of the L_1-norm. This strategy provides a fair comparison between both methods and can be applied to other robust estimators.. Figure 1 presents a flowchart for the proposed approach.

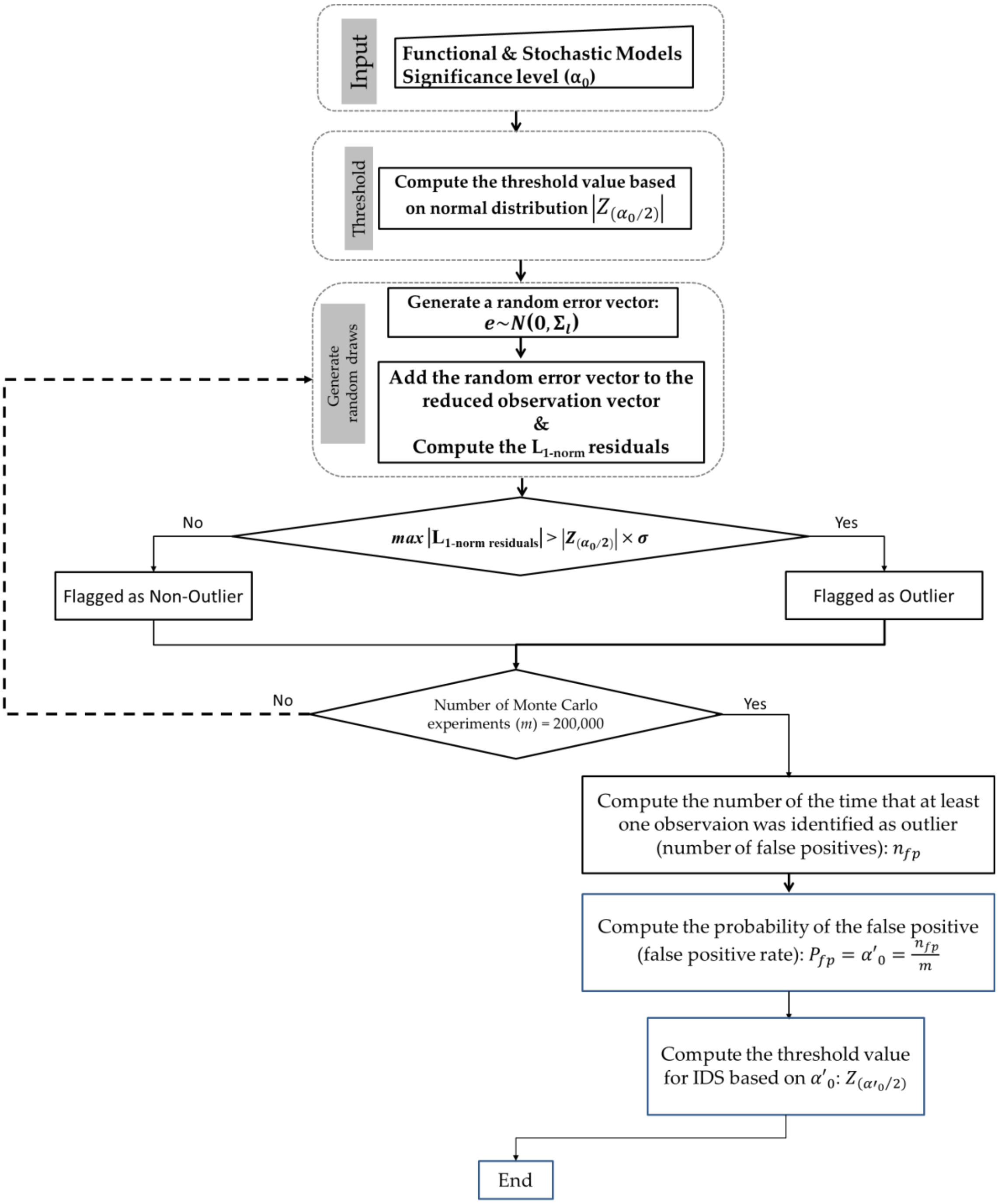

Figure 1. Flowchart of the proposed approach for comparison of IDS and L_1-norm.

Experiments and results

To evaluate the performance of IDS and L_1-norm against outliers, three configurations of leveling networks are taken into account as follows: one of lower redundancy (Network A), one of intermediate redundancy (Network B) and one of higher redundancy (Network C). The networks are presented in Figure 2. Network A consists in one control station, $n = 16$ observations (height differences) and $u = 10$ unknowns (station heights), thus, $n - u = 6$ degrees of freedom. Network B consists in one control station, $n = 20$ observations (height differences) and $u = 10$ unknowns (station heights), thus, $n - u = 10$ degrees of freedom. Network C consists in one control station, $n = 14$ observations (height differences) and $u = 6$ unknowns (station heights), thus, $n - u = 8$ degrees of freedom.

Therefore, the "mean redundancy number" (Teunissen, 2006) of Network A is: $\bar{r} = \frac{16-10}{16} = 0.375$; while the mean redundancy number of Network B is: $\bar{r} = \frac{20-10}{20} = 0.5$ and the mean redundancy number of Network C is: $\bar{r} = \frac{14-6}{14} = 0.571$. More details of Networks A, B and C are provided in Gemael et al. (2015), Suraci et al. (2019) and Ghilani (2010), respectively.

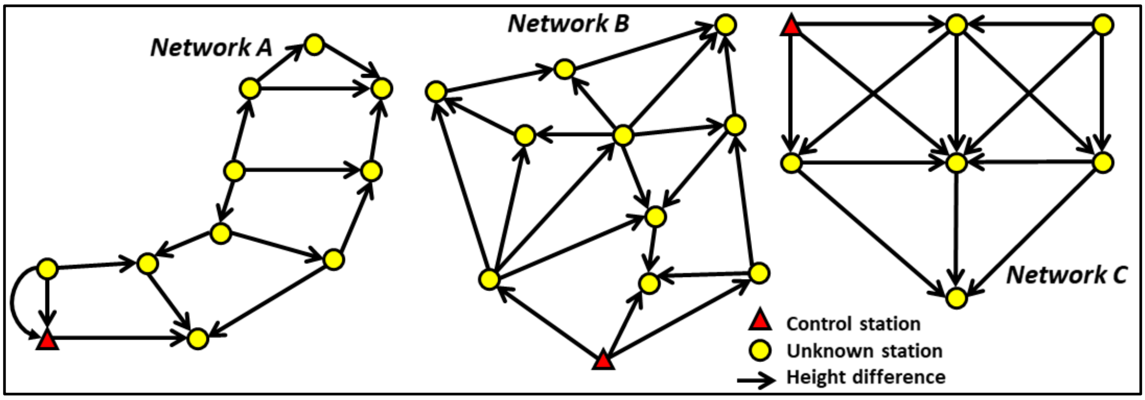

Figure 2. Configuration of Networks A, B and C (Source: adapted from Gemael et al. (2015), Suraci et al. (2019) and Ghilani (2010), respectively)

In MC experiments, initially, random errors of a multivariate normal distribution were generated considering the covariance matrix of observations for each network (the $n \times 1$ random vector "e"), then, the respective L_1-norm residuals was achieved. False positives occur if (at least) one observational residual exceeds $3.29 \times \sigma$, being "σ" the standard deviation of each observation. For each network, this procedure (MC experiment) was run 200,000 times (Rofatto et al., 2020b).

For comparison purposes, the threshold value for IDS was taken as $|Z_{(\alpha_0/2)}| = 3.29$ (with $\alpha_0 = 0.001$). For this threshold ($|Z_{(\alpha_0/2)}| = 3.29$), we obtain the follow false positive rates for L_1-norm:$\alpha'_0 = 0.01$ (10%), $\alpha'_0 = 0.1066$ (10.66%) and $\alpha'_0 = 0.0578$ (5.78%) for networks A, B and C, respectively. Based on these values for α'_0, a new threshold value was obtained for IDS by MC experiments, following the approach of Lehmann (2012) and Rofatto et al. (2020a), which was denoted by IDS' The step-by-step of the applied procedure can be obtained in section 3 of Rofatto et al. (2020a). The threshold values of IDS' were $|Z_{(\alpha'_0/2)}| = 2.5224$, $|Z_{(\alpha'_0/2)}| = 2.7147$ and $|Z_{(\alpha'_0/2)}| = 2.8326$ for networks A, B and C, respectively.

Once that values of α'_0 and $|Z_{(\alpha'_0/2)}|$ were derived, MC experiments with inserted outliers in the simulated random vector "e" were performed in order to evaluate the success rate of IDS, IDS' and L_1-norm against outliers. For both networks, the number of inserted outliers ranges from 1 up to 4 and the size of the outliers ranges from 3σ up to 6σ and from 6σ up to 9σ, where "σ" is the standard deviation of each observation. These outlier sizes are the same as those of Knight and Wang (2009). The outliers were generated according to a uniform distribution and the MC experiments were conduct following the approach of Rofatto et al. (2020b). Figures 3 and 4 present the success rates for Network A; Figures 5 and 6 present the success rates for Network B; and Figures 7 and 8 present the success rates for Network C.

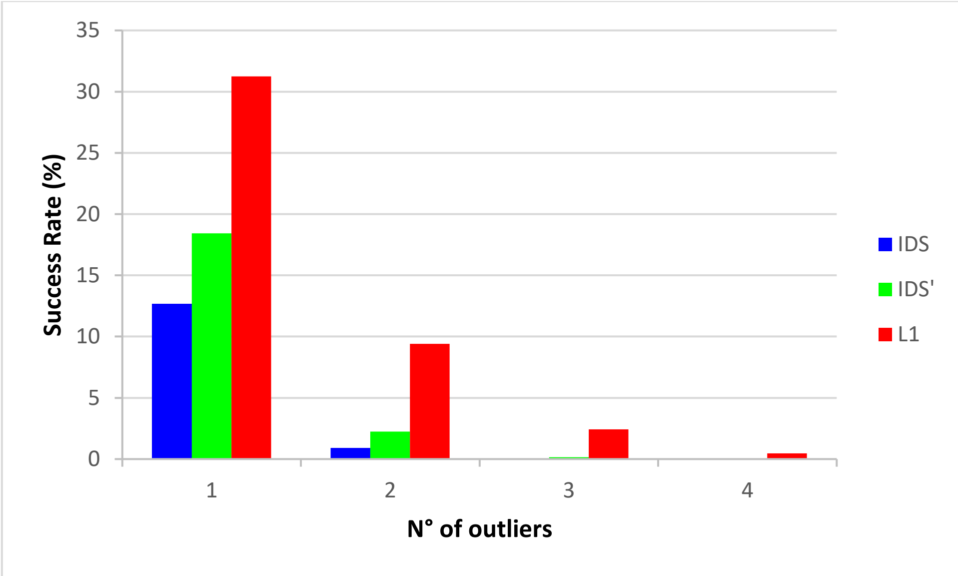

Figure 3. Success rates for Network A (outlier sizes: 3σ – 6σ)

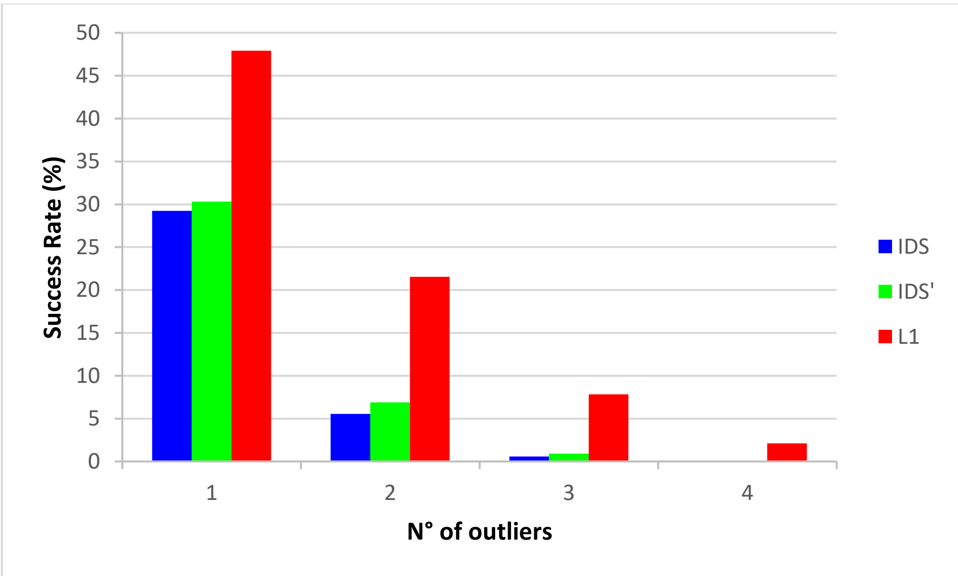

Figure 4. Success rates for Network A (outlier sizes: 6σ – 9σ)

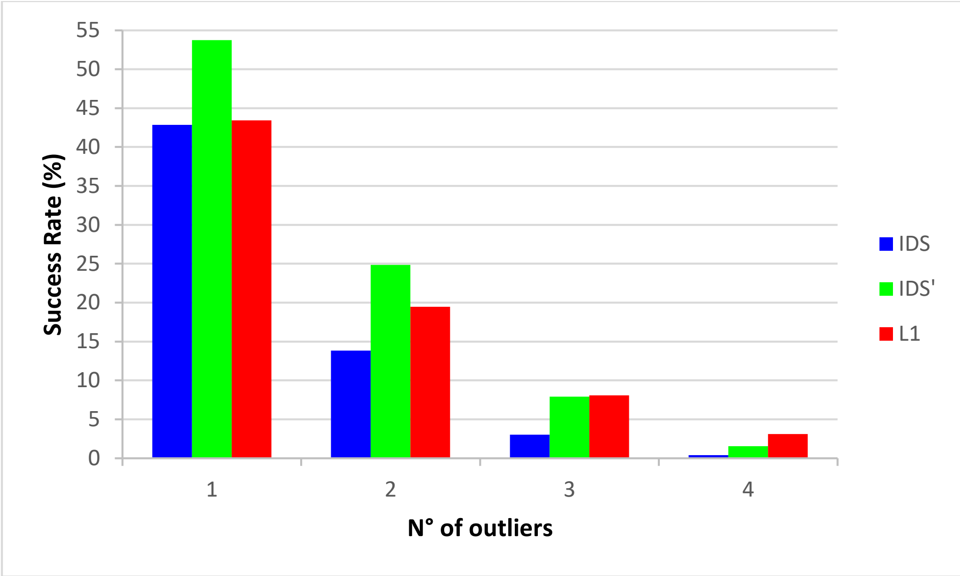

Figure 5. Success rates for Network B (outlier sizes: 3σ – 6σ)

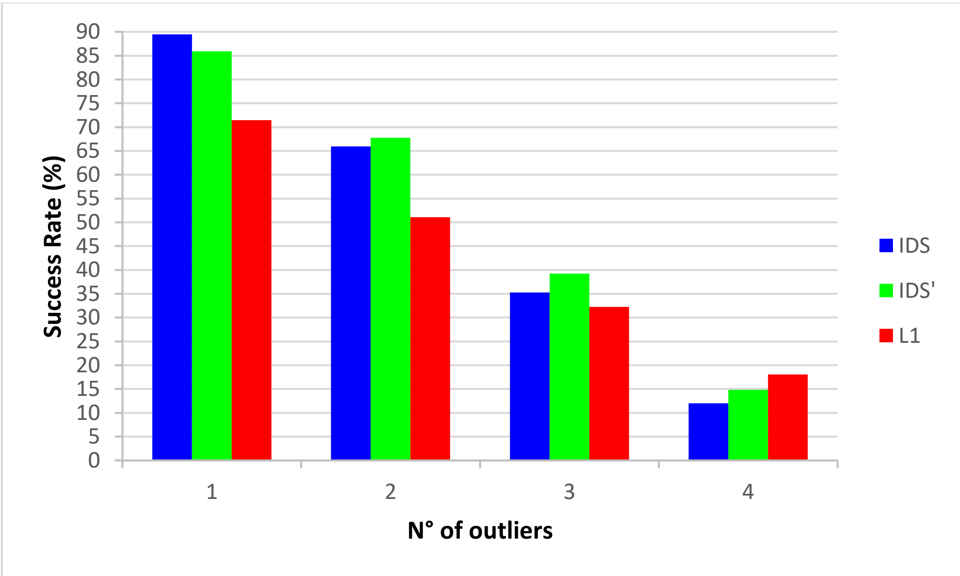

Figure 6. Success rates for Network B (outlier sizes: 6σ – 9σ)

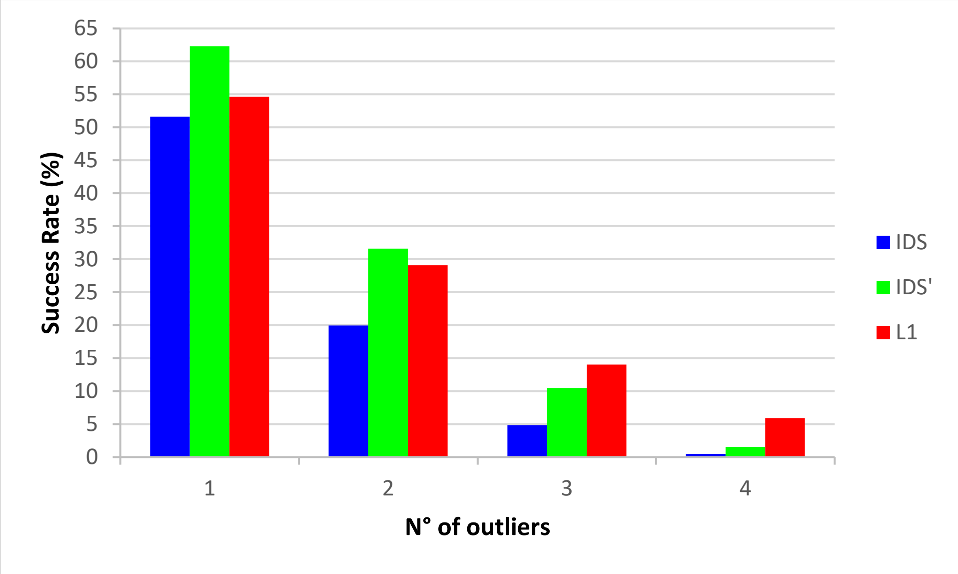

Figure 7. Success rates for Network C (outlier sizes: 3σ – 6σ)

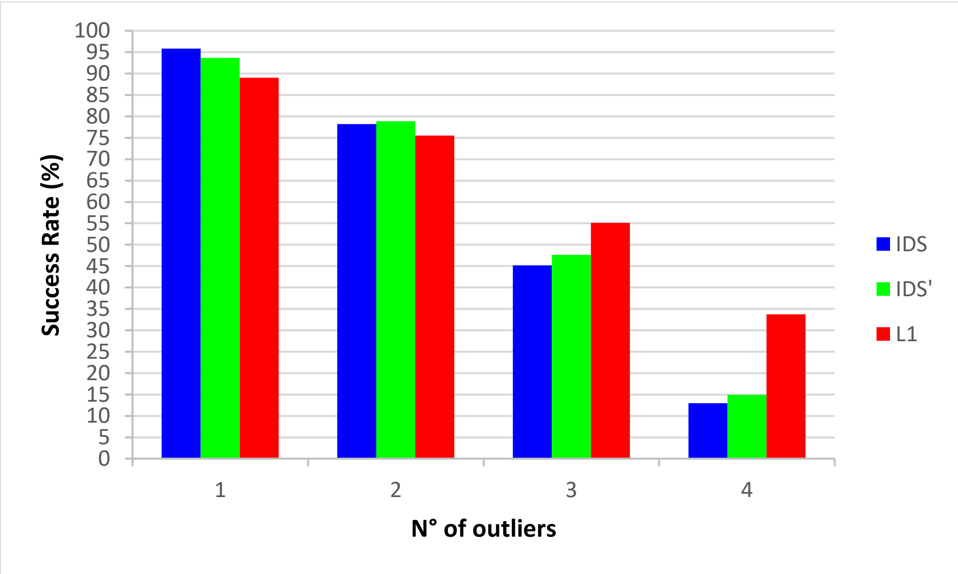

Figure 8. Success rates for Network C (outlier sizes: 6σ – 9σ)

Discussion

The first issue to be addressed is the false positive rate of L_1-norm (10% for Network A, 10.66% for Network B and 5.78% for Network C). To compare with IDS results, MC experiments are needed, since IDS is performed in a multiple alternative hypotheses context (Förstner, 1983; Lehmann, 2012; Yang et al., 2013; Rofatto et al., 2020a). It should be pointed out that the stipulated value of $\alpha_0 = 0.001$ (0.1%) is the false positive rate for a single alternative hypothesis test, i.e., for the case where only one observation is checked.

Regarding this issue, random errors were generated and the LS-residuals were computed, similar to the L_1-norm case. False positives occur in the case of having at least one SR $>\left|Z_{(\alpha_0/2)}\right| = 3.29$. For each network, this procedure (MC experiment) was run 200,000 times. The false positive rates of IDS in this multiple alternative hypothesis context were 0.0098 (0.98%), 0.018 (1.8%) and 0.0132 (1.32%) for the networks A, B and C, respectively. These values were significantly lower than those obtained by the L_1-norm for both networks. The L_1-norm also presented a higher false positive rate than IDS for the experiments with GNSS data provided in Knight and Wang (2009). This can be due to the residual's distribution of LS and L_1-norm.

Regarding the experiments with inserted outliers, the lower the number of outliers and/or the higher the outlier sizes, the higher the success rate for both methods in all networks, as expected. In addition, the higher the mean redundancy number of the network, the higher the success rate, since the correlations for the estimated residuals are lower (Lehmann, 2012; Rofatto et al., 2020a).

For the case of network A (lower redundancy), the success rate of L_1-norm was better than the success rate of IDS and IDS' in all cases. For example, for two outliers with sizes 6σ – 9σ, the success rate of L_1-norm is higher than 20%, whereas both IDS and IDS' are about 5%. In addition, IDS and IDS' were inefficient for three or four outliers despite their sizes (success rates ≈ 0%). It is important to observe that although L_1-norm has performed better than the both IDS and IDS', its success rates were not significant ($<$ 50%).

In the case of network B (intermediate redundancy), the pattern of the results seems more complex. The IDS performs better for one outlier of sizes 6σ–9σ. The IDS' performs better for one or two outliers of sizes 3σ–6σ and for two or three outliers of sizes 6σ–9σ. The L_1-norm performs better for three or four outliers of sizes 3σ–6σ and for four outliers of sizes 6σ–9σ. The IDS presents the worst performances for outliers of sizes 3σ–6σ, whereas the L_1-norm presents the worst performances for outliers of sizes 6σ–9σ (with the exception of the case with four outliers).

For network C (higher redundancy), the IDS again performs better for one outlier of sizes 6σ–9σ. The IDS' performs better for one or two outliers of sizes 3σ–6σ and for two outliers of sizes 6σ–9σ. The L_1-norm performs better for three or four outliers despite their sizes. This better performance of L_1-norm against several outliers has already been noticed, for example, in the 3D geodetic network described in Eshagh et al. (2007). Once again, the IDS presents the worst performances for outliers of sizes 3σ–6σ.

In general, IDS' and L_1-norm provided the best results. The overall mean success rates were ≈ 26%, ≈ 29% and ≈ 30% for IDS, IDS' and L_1-norm, respectively. Although IDS' performs better than L_1-norm in some specific cases, IDS' is unlikely to be applied with such a high false positive rate. In addition, the number and size of outliers is unknown in real applications, which makes difficult to define a strategy on which procedure to apply for each case.

Thus, if we look at the success rate, the general recommendation is to apply the L_1-norm, especially for networks with low redundancy (say $\bar{r} < 0.5$) or networks that have many identified outliers. However, even if L_1-norm was applied for outlier identification, the final solution must be achieved by the LS estimation considering its BLUE properties, as has already been recommended, for example, by Marshall and Bethel (1996).

On the other hand, this conclusion may not be fair, since it does not consider the false positive rate. Thus, considering that the L_1-norm and consequently IDS' had a higher false positive rate than IDS, Tables 1, 2 and 3 present the ratio between the success rate and the false positive rate of both methods for Networks A, B and C, respectively.

Table 1. Ratio values between the success rate and the false positive rate for Network A

Number of outliers	IDS	IDS'	L1
1	**21.4**	2.4	4.0
2	**3.3**	0.5	1.5
3	0.3	0.1	**0.5**
4	0.0	0.0	**0.1**

Table 2. Ratio values between the success rate and the false positive rate for Network B

Number of outliers	IDS	IDS'	L1
1	**36.8**	6.6	5.4
2	**22.2**	4.3	3.3
3	**10.6**	2.2	1.9
4	**3.5**	0.8	1.0

Table 3. Ratio values between the success rate and the false positive rate for Network C

Number of outliers	IDS	IDS'	L1
1	**55.9**	13.5	12.4
2	**37.2**	9.6	9.0
3	**19.0**	5.0	6.0
4	**5.1**	1.4	3.4

The higher the mean redundancy number of the network, the higher the ratio values (Table 1, 2 and 3). It can be noted that in some cases, the false positive rate is higher than the success rate, that is, the ratio value is lower than 1. In general, IDS presents better ratio values between the success rate and the false positive rate, the exception for three and four outliers in Network A. However, in these cases the ratio values are significantly lower for both methods. Thus, if we consider both success rates and false positive rates, then IDS should be applied despite the configuration of the leveling network or the size and number of outliers.

Note that “false positive” not necessarily means “false outlier”. For example, the outlier may not be a blunder or fault (being a “false positive”), but it also may be the result of a random error, being a random (“true”) outlier (Hekimoglu, 2005). The exclusion of false positives is especially critical in leveling networks, which in general the redundancy is lower than in horizontal or GNSS networks.

It should be pointed out that both methods have limitations, which makes it difficult to conclude which is the most efficient. It paves the way for further studies on this topic, considering, for example, an IDS approach based on L_1-residuals such as in Amiri-Simkooei (2018).

Conclusions and recommendations

In this paper, we evaluate the performance of IDS and L_1-norm against outliers in three different leveling networks, by means of several MC experiments. For this purpose, a new way of comparison between the results of IDS and L_1-norm was proposed, considering both the same threshold for the residuals and the same rates of false positives..

If we look only at the success rate, the results suggest that the L_1-norm should be applied for outlier identification instead of the IDS based on LS-residuals, especially for networks with low redundancy (say $\bar{r} < 0.5$) or networks that have many identified outliers. In the relationship between false positive rate and outlier identification success rate, however, IDS performs better than L_1-norm. In the latter case, IDS with critical value of 3.29 has the best cost-benefit ratio, independently of the levelling network configuration, number and size of outliers.

Therefore, the main contribution of this research was the comparison by MC experiments considering both the threshold for the residuals and the false positive rate of both methods. IDS and L_1-norm showed different limitations according to the key factors analyzed.

For future works, one can investigate the effect of "leverage points" in leveling networks, such as Junhuan (2005) provided for triangulation networks. The classes of "controlled and non-controlled observations against outliers" (Hekimoglu et al., 2011) for IDS and L_1-norm can also be addressed. In addition, other observational errors model rather than the normal distribution can be investigated in leveling networks by MC experiments, following the approach described by Lehmann (2015) for linear regressions.

Furthermore, if a robust estimator like L_1-norm is applied for outlier identification, then reliability measures should be derived as pointed out, for example, in Guo et al. (2011). These reliability measures can be obtained by MC experiments following the approach of Rofatto et al. (2020a) for IDS. This is particularly a very interesting issue, inasmuch as L_1-norm tends to project the outliers onto the corresponding residuals more than the LS solution (Junhuan, 2005).

Finally, considering the new way of comparison described here, the L_1-norm and IDS performance against outliers should by fully addressed in other types of geodetic networks, such as horizontal and GNSS networks. The effect of the stochastic model in the performance against outliers can also be investigated.

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