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Research Paper

A critical analysis of the AHRI polynomials for scroll compressor characterization

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ABSTRACT

This paper presents the analysis of the energy consumption and mass flow rate of scroll-type compressors. The study has included the data of several AHRI reports (especially AHRI 11 and AHRI 21) and data from other sources. A total of 7 different scroll compressors of different sizes have been considered in the study, some of them tested with various refrigerants (R134a, R32, R410A, R404a…). For all the studied compressors and refrigerants, the compressor energy consumption and mass flow rate values have been analyzed. The main objective is to better understand the dependence of these variables on the operating conditions and the refrigerant used. The analyzed data include tests following different superheat control, i.e., constant superheat or constant return temperature, so the effect of the inlet temperature on these variables is also discussed. As the main novelty of this study, the analysis of the response surfaces has allowed the authors to evaluate the most suitable correlation to use, including an analysis of the necessary experimental tests and where to place them to increase the model's accuracy. It was found that using the condensing and evaporating pressure terms is more universal than the classical temperature domain. In scroll compressors, AHRI polynomials overfits the compressor performance introducing significant deviations in the interpolation and extrapolation capabilities if the experimental data are not properly selected. Finally, it was found that lower degree polynomials are more suitable for this kind of compressor and has also the advantage of requiring fewer experimental point measurements to characterize the compressor with the corresponding cost-savings.

1. Introduction

The precise characterization of compressors is critical in order to reproduce the performance of vapor compression systems as this component is the main contributor to the energy consumption and define the heating/cooling capacity of these systems. An extensive review of all the theoretical approaches followed in the compressor modelling is found in [6] and [13].

In these reviews several approaches depending on the objectives of the model is found, from very detailed models based on physical principles and the internal geometry of the different components to black box model in which based on some correlations depending on external working parameters the compressor behavior is described. The first kind of models have the main target of assisting in the compressor design but from the point of view of vapor compressor system modeling are not useful as they require much information that usually is not available. Therefore, when the target is to analyze vapor compression systems the most common followed approach is this kind of black box model as this approach usually requires only the fitting of some compressor coefficients and requires a very low computational time.

In this category of black box models, the correlations used could be based in some physical principle, in that case sometimes they are called semi-empirical correlations, or they are functions that try to fit the real behavior and the parameters used do not have any relation with any physical effect, these are the pure empirical model. Some examples of these semi-empirical models can be found in see for example [14,19,8,36,39,9,16,37], and the recent paper by [13]. The most representative of the pure empirical models are the AHRI polynomials [1].

As it was reported by [7] semiempirical approaches compared to pure empirical have the advantages of requiring less amount of data and are more suitable to perform extrapolations from an experimental test matrix but from the other side usually the functions used are compressor design dependent and when the amount of experimental data available are "enough" they show a worse agreement than the pure empirical

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Nomenclature

h_{is}	Enthalpy of refrigerant at compressor outlet (isentropic compression) (kJ/kg)
h_{suc}	Enthalpy of refrigerant at compressor inlet (kJ/kg)
$\dot{m}_{ref}$	Refrigerant mass flow rate (kg/h)
P_{cond}	Condensation pressure (bar)
P_{evap}	Evaporation pressure (bar)
P_r	Pressure ratio (-)
SH	Superheat (K)
T_{cond}	Condensation temperature (°C)
T_{evap}	Evaporation temperature (°C)
T_{suc}	Return temperature (compressor inlet) (°C)
$\dot{V}$	Swept volume (m^3/s)
$\dot{W}_c$	Compressor energy consumption (kW)
η_c	Compressor efficiency (%)
η_v	Volumetric efficiency (%)
ρ_{suc}	Density at compressor inlet (kg/m^3)

Acronym

DoE	Design of Experiments
D-OPT	D-optimal criterion
LT/MT/HT	Low/Middle/High temperature conditions
MRE	Maximum relative error (%)
OD	Optimal designs
OLS	Ordinary Least Squares
RMSE	Root Mean Square Error (W - kg/h)

ones.

These last points have made that pure empirical model, and particularly AHRI polynomial has been the most common alternative followed when the interest is in modelling vapor compression systems. The AHRI polynomial describes the compressor performance (mass flow rate and energy consumption) based on the 10 coefficients of a polynomial of third degree depending on evaporating and condensing temperatures. However, the use of third-degree polynomials presents some problems that must be considered:

- It could introduce important deviations in the prediction of compressor behavior if experimental compressor data are not enough or they are not properly distributed as it is pointed by [14].
- The number of experimental data required to have a confident curve is significantly high. The third-degree polynomials give great flexibility to reproduce complex response surfaces. However, the significant number of terms may over-fitting the simplest response surfaces, and cubic terms could introduce interpolation/extrapolation problems. Therefore, the adjustment of linear regression models should always aim at finding the most compact and simple model that offers the required accuracy.

And based on them, some questions arise: Is it mandatory the use of all the coefficients of the third-degree polynomials? Which are the minimum number of points required to have a good accuracy in the prediction of the compressor performance and where to place them? This topic was recently researched by [4,3], and [7], and with this considerations in mind some works like [30] for rotary compressors, the functional proposed by [5], and the proposed by [18] have tried to find avoid it.

Nevertheless, and even AHRI polynomials are widely adopted by the industry, up to now there is no clear explanation in the literature about this topic or at least a mathematical analysis which justify the use of third-degree polynomials to characterize compressors. One reason is the lack of a wide database of compressor performance over the whole compressor envelop for any compressor design and refrigerants based on experimental results. This experimental database is especially important. Although manufacturers supply the compressor performance map in a wide range of operating conditions, it is usually not clear if that information comes from internal experimental tests or corresponds to values estimated from some model or interpolation procedure. From that point of view, it has been especially relevant a project founded by AHRI through "*Low-GWP Alternative Refrigerants Evaluation Program*" which supports a massive test campaign of compressors working with different refrigerants.

Using all the experimental tests performed under this program and other experimental compressor data like the one supplied by [8], this paper analyzes the energy consumption and mass flow rate of scroll compressors.

The main novelty of this study is that based on this huge amount of experimental data for compressors of several manufacturers, refrigerants, applications, a detailed analysis of the response surfaces of the energy consumption and mass flow rate in scroll compressors has been done. This analysis has allowed the determination of the optimum number of polynomial terms in the AHRI polynomials which unfortunately is a topic which is not addressed in the present compressor characterization standard [1]). This fact will allow preventing possible overfitting and obtain more extrapolation/interpolation capabilities from these polynomials. In addition, this analysis has allowed the determination of the optimum number of tests in order to maximize the information for compressor characterization. There are other more sophisticated approaches available in the literature as as non-parametric models [12] or neural networks [17]. However, these tools lack the simplicity of the AHRI polynomials, significantly reducing the potential users of them. Furthermore, most of these works available has been tested only with a reduced number of compressors/refrigerants; thus, the global applicability of them must be still tested with wider amount of data. From that perspective, the results presented in this paper offers a clear justification and are supported by a wide spectrum of experimental information and will allow the reduction of time dedicated to characterizing a compressor properly with the corresponding cost-savings. Other relevant results have been the advantages of using pressure terms in the models instead of temperatures or the increased robustness of the models against extrapolations/interpolations due to the significant reduction of terms.

2. Compressor performance data

A few years ago, AHRI disclosed a series of performance data for different compressors (scroll and piston), with conventional and new refrigerants and mixtures. These experimental results are included in several reports within the AHRI "Low-GWP Alternative Refrigerants Evaluation Program". This study has considered all those AHRI reports containing scroll compressor tests: AHRI 11 [32,31], AHRI 21 [32], AHRI 24 [22], AHRI 33 [33], AHRI 34 [23], AHRI 36 [24], AHRI 38 [25], AHRI 39 [26], AHRI 58 [27], AHRI 65 [28], and AHRI 66 [34], and additionally the performance data published in [8]; totaling 8 different scroll compressors, and 13 different refrigerants: R134a, R32, R410A, R404A, R447A, R454B, DR5, DR7, L40, L41a, L41b, ARM31a, D2Y65. In these tests not only the evaporating and condensing temperatures were changed but also some of the tests were done at constant superheat but also in some cases at constant temperature at the compressor inlet, which allow the analysis of the influence of this variable on compressor performance.

The analysis of compressor consumption data show two different trends depending on the application range: for MT and HT conditions, i.

Table 1
Main compressor characteristics and tested refrigerants.

Source	Model	Manufac.	Disp. (freq.) (cm^3) (Hz)	Refrigerants tested	Test points	Conditions by refrigerant test	
AHRI 21	ZS21KAE-PFV	Copeland	50.96 (60)	R404A/ARM31a/D2Y65/L40	191/186/183/173	SH = 11 K	SC = 6 K
						SH = 22 K	
AHRI 11	ZP21K5E-PFV	Copeland	20.32 (60)	R410A/R32/DR5/L41a	196/166/189/186	T_{suc} = 18 °C	———
Cuevas(2009)	–	–	54.25 (50)	R134a	18	SH = 6.8 K	

Table 2
New refrigerant's composition (Mass%).

Source	Name	Composition
AHRI 21	ARM-31a	R32/R134a/R1234yf (28/21/51)
	D2Y-65	R32/R1234yf (35/65)
	L40	R32/R152a/R1234yf/R1234ze(E) (40/10/20/30)
	R32/R134a	R32/R134a (50/50)
AHRI 11	DR-5	R32/R1234yf (72.5/27.5)
	L41a	R32/R1234yf/R1234ze(E) (73/15/12)

e., low, and medium values of pressure ratio, the compressor energy consumption is almost independent of the evaporation temperature. While, for LT condition, i.e., high values of pressure ratio, the compressor energy consumption decrease significantly with the reduction of the evaporation temperature with some kind of hyperbolic behavior. AHRI 11 and AHRI 21 compressor were selected as representative of the M−HT and LT conditions respectively. These compressors were selected as they have the densest test matrix in its corresponding category and were tested with different refrigerants. The results presented in this paper will be shown in terms of these two compressors, but the same analysis was verified with the rest of compressors measured in the reports. Therefore, the presented conclusions are of application to all the compressors presented in that project. This information is presented as supplementary material. The compressor studied in [8] belongs to the M−HT category.

Table 1 shows the main characteristics of these compressors, and [8] which was tested in a different framework.

Finally, Table 2 shows the Mass% compositions of the tested refrigerants' mixtures. The mixtures' thermophysical properties analyzed have been obtained with the NIST's Refprop software package [15], evaporation and condensation temperatures are considered at dew point.

3. Compressor performance analysis

The characterization of compressor performance from the point of view of vapor compression systems depends on a volumetric variable that could be the mass flow or the volumetric efficiency and on an energy consumption variable that could be the energy consumption itself or other like the compressor efficiency.

The compressor and volumetric efficiency are given by the expressions (1) and (2) and traditionally has been very attractive variables for compressor modelling as they are adimesional and do not show a strong dependence on compressor size or used refrigerant. This fact suggests the idea of general models and, they present a quite strong dependence on the pressure ratio.

$$\eta_c = \frac{\dot{m}_{ref} \bullet (h_{is} - h_{suc})}{\dot{W}_c} \tag{1}$$

$$\eta_v = \frac{\dot{m}_{ref}}{\rho_{suc}\dot{V}} \tag{2}$$

These facts have made that some authors like [20,29]; and [18] have used these variables in order to build a model to reproduce compressor behavior.

3.1. Energy consumption analysis

Fig. 1 represents compressor efficiency as a function of pressure ratio for the compressors AHRI 11 and AHRI 21, all the experimental points with the reference refrigerants, R410A and R404A have been included. Three sets of data were measured, corresponding to three different conditions at the suction: constant superheat of 11.11 K, constant superheat of 22.22 K, and constant return temperature 18 °C.

Fig. 1 shows the expected dependence of compressor efficiency with pressure ratio for scroll compressors. Fig. 1 also confirms that the pressure ratio has a strong influence in this variable. However, the figure also demonstrates that, for these compressors, with a wide range of experimental points, other variables like evaporating and condensing temperatures or the compressor inlet temperature must be considered too as it is shown by the wide scattering across the average trend in the figure. The usual statement that the compressor efficiency has its optimum at a certain pressure ratio is a great simplification. It is only valid

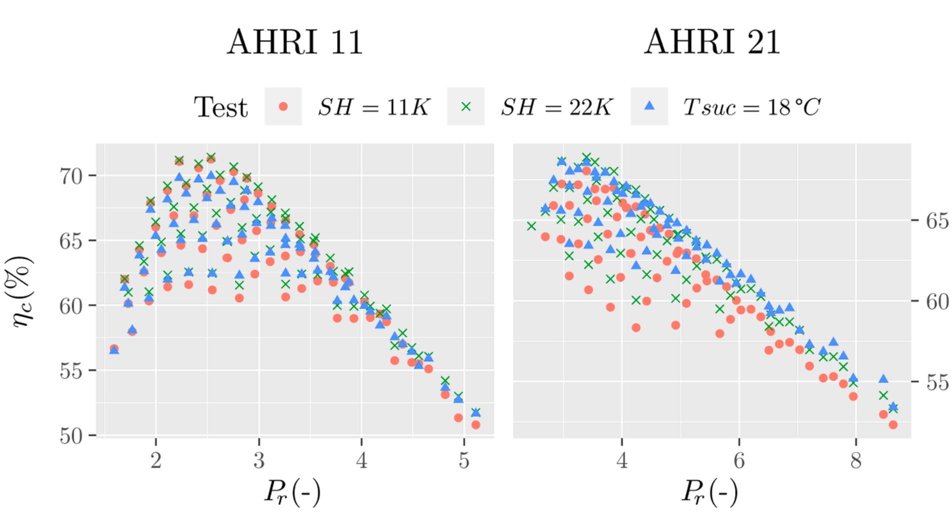

Fig. 1. Compressor efficiency as a function of pressure ratio for compressors included in the test campaign AHRI 11 and AHRI 21.

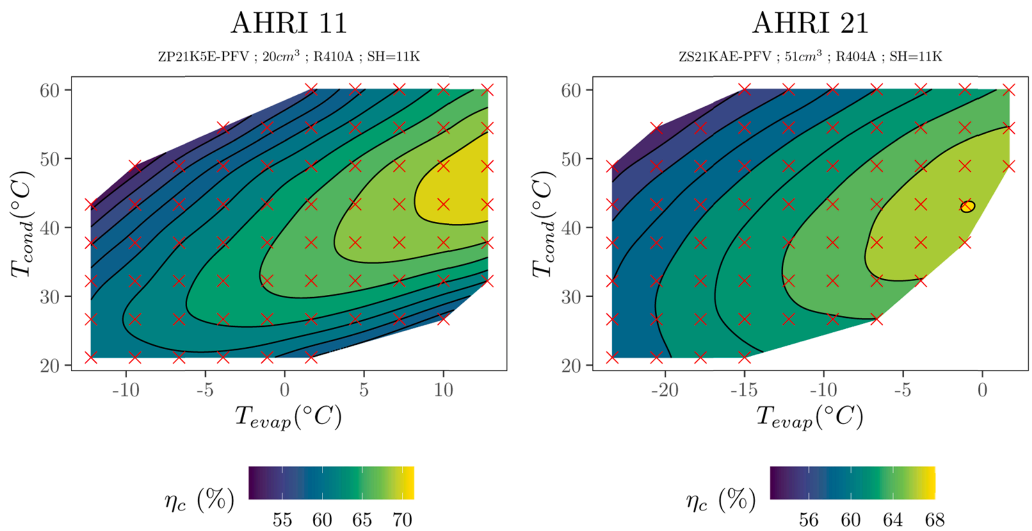

Fig. 2. Compressor efficiency as a function of evaporating and condensing temperature for compressors included in the test campaign AHRI 11 and AHRI 21 with a superheat of 11 K.

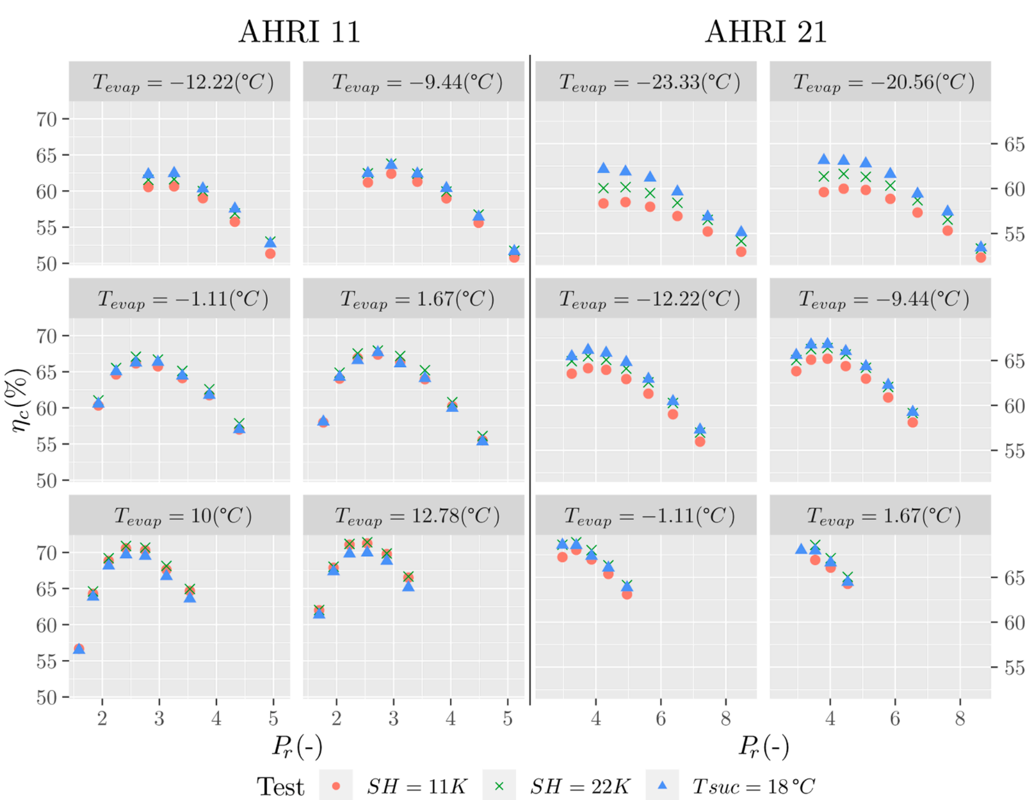

Fig. 3. Compressor efficiency versus pressure ratio of compressors ZP21K5E-PFV (AHRI 11; R410A) and ZS21KAE-PFV (AHRI 21; R404A). Compressor efficiency at given evaporation temperatures.

for compressors with an approximately constant evaporating range, like compressors for air conditioning or chillers. Furthermore, the local optimum is shifted to higher pressure ratios with evaporation pressure.

Fig. 2 plots compressor efficiency as a function of evaporating and condensing temperatures for a constant superheat of 11 K. The figure shows that compressor efficiency depends on both variables and that the absolute maximum of the efficiency as a function of evaporating and condensing temperatures is only placed inside the operation map of the system for the AHRI 21 compressor.

Additionally, from the experimental data of the reports, it can be stated that the compressor inlet temperature has a noticeable influence on compressor efficiency, as it tends to rise as superheat rises. In general, the highest efficiencies are found for superheats of 22 K, except at low evaporation temperatures where a return temperature 18 °C is imposed and the superheat is higher than 22 K. This can be seen in Fig. 3 where the compressor efficiency is plotted as a function of the evaporating temperature for each condensing temperature.

Thus, from all this analysis, it is stated that compressor efficiency does not depend only on pressure ratio but also on evaporating and condensing conditions plus the superheat which made the development

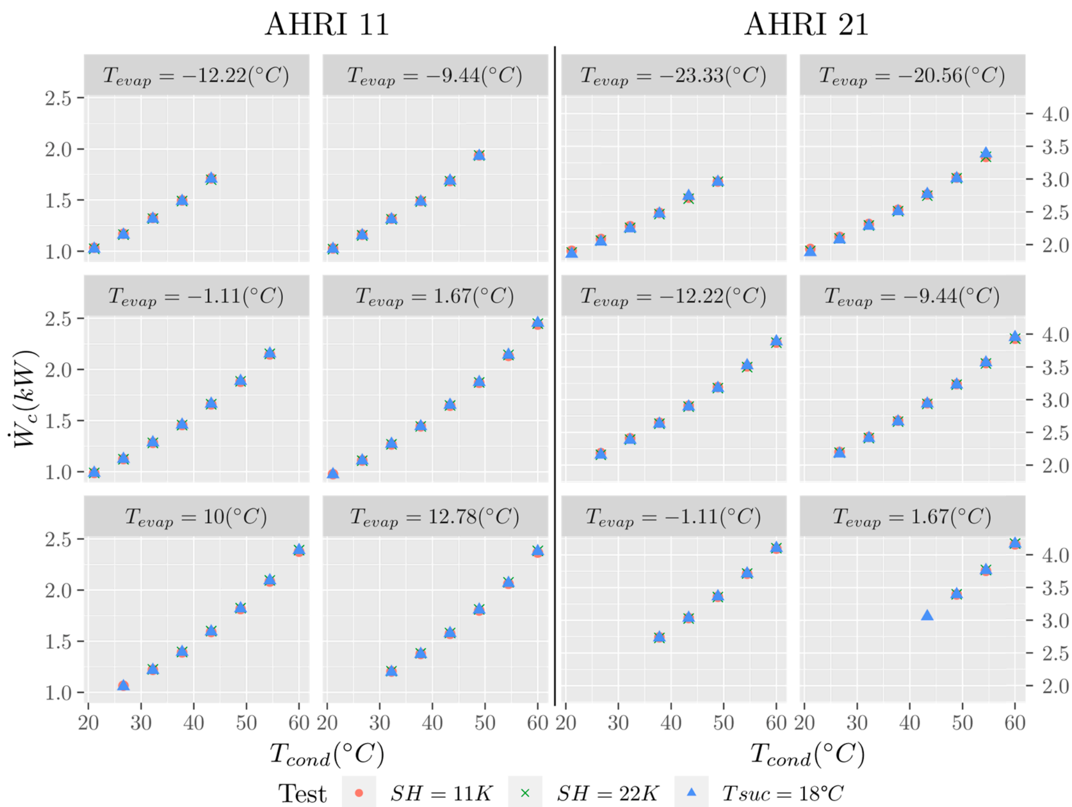

Fig. 4. Compressor consumption versus condensation temperatures of compressors ZP21K5E-PFV (AHRI 11; R410A) and ZS21KAE-PFV (AHRI 21; R404A) at different evaporation temperatures levels.

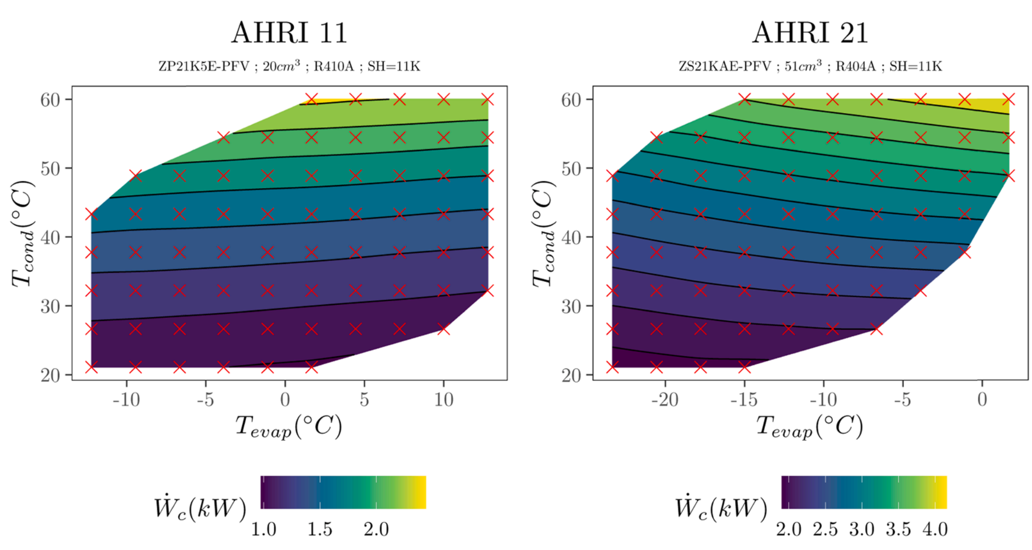

Fig. 5. Compressor energy consumption as a function of evaporating and condensing temperatures.

of a function representing this variable in order to characterize the energy consumption compressor behavior challenging task. On the other side, Fig. 4 represent the compressor energy consumption as a function of condensing and evaporating temperatures, and it is seen that the influence of compressor inlet temperature has been reduced significantly.

Fig. 5 represents the AHRI 11 and AHRI 21 compressors consumption vs condensing and evaporating temperatures. The figure shows that the condensing temperature is the variable with a highest influence on scroll compressors energy consumption, the evaporating temperature also has an influence but of lower level. The figures also reveals that the energy consumption shows a simpler dependence on these variables than compressor efficiency. From the construction of pure empirical models, Fig. 5 clearly shows that energy consumption is a more suitable variable to build this kind of models. It presents a monotonous behavior with smooth trends for the entire working map. Hence, if an adjusted polynomial model is used to predict the compressor energy consumption it will contain less term than a polynomial used to predict the compressor efficiency. This will have as a consequence that this polynomial will be more robust, will have less extrapolation problems and will require less experimental points to predict accurately the compressor behavior.

Fig. 5 shows the compressor consumption maps of compressors ZS21KAE-PFV (AHRI 21) and ZP21K5E-PFV (AHRI 11) for their

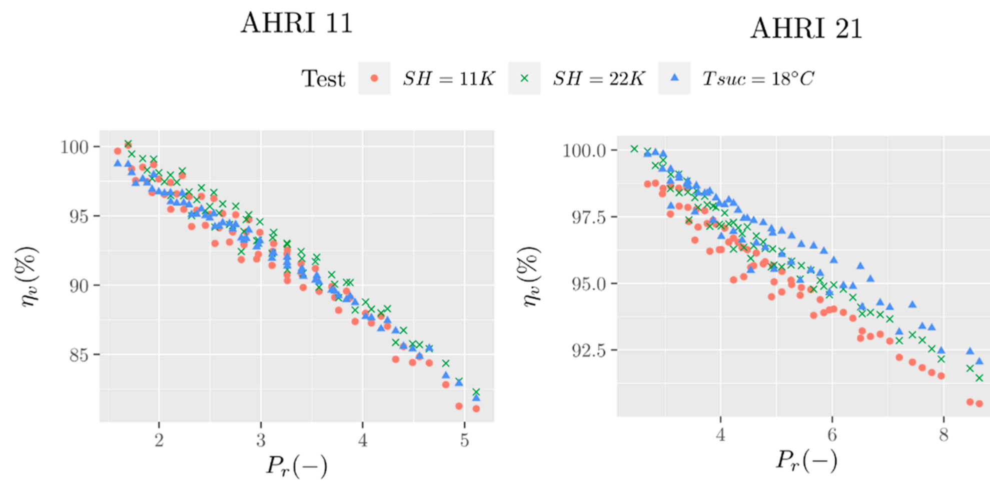

Fig. 6. Volumetric efficiency versus pressure ratio of compressors ZP21K5E-PFV (AHRI 11) and ZS21KAE-PFV (AHRI 21) for their corresponding reference refrigerant, R410A, and R404A.

reference refrigerant, R404A and R410A respectively.

As can be seen, and as mentioned above, the energy consumption of scroll compressors mainly depends on the condensing temperature, and it increases with it. There is a slight dependence on the evaporating temperature, but in this case, the energy consumption dependency with that variable will depend on the application range. For instance, for AHRI 21 compressor, which has been tested at low temperatures, energy consumption decreases with evaporating temperature, while the opposite behavior is observed for AHRI 11 compressor.

All M−HT analyzed scroll compressors of the referenced AHRI reports, and the one tested by [8], show this slight increase of the compressor consumption, almost linear, with the decrease of the evaporation temperature. While the compressors of reports AHRI 21, AHRI 34 and AHRI 36 show the consumption decreasing trend with the decrease of the evaporation temperature. The compressors of reports AHRI 34, AHRI 36 are for LT applications and in fact they are Liquid injection type. Therefore, it can be concluded that the dependence of the scroll compressors consumption with the evaporation temperature is weak, and it depends on the application range, slightly hyperbolic decreasing for LT applications while it is slightly linear increasing for MT-HT applications. Fig. 5 is replicated for the rest of the compressors analyzed. All the working maps generated are included as supplementary material.

3.2. Mass flow analysis

Fig. 6 shows the volumetric efficiency of compressors ZS21KAE-PFV (AHRI 21) and ZP21K5E-PFV (AHRI 11) for their corresponding reference refrigerants R404A, and R410A, respectively, and the three different inlet conditions.

As can be observed, this figure shows a clear primary dependence of η_v with pressure ratio as many references in the Literature describe, with a decreasing trend when the pressure ratio increase. However, it also becomes clear that the relationship is not strictly linear but more complex, and there are also other influences. The significant impact of the inlet conditions on the volumetric efficiency is evident, with higher volumetric efficiencies at higher superheats. This behavior is well known, and there are ways to catch up this effect and correct it in estimating the mass flowrate. The most employed correction is the one proposed by Dabiri [10]. There is also an influence of the evaporating and condensing temperatures, not explained by the pressure ratio. One can see that there are groups of points distinguishable in Fig. 6, corresponding to the same evaporating (T_{evap}) or condensing temperatures (T_{cond}). Therefore, volumetric efficiency is a good parameter to characterize the compressor mass flowrate when a simple correlation is required. However, it is not the right way to characterize it in the general case. In fact, the AHRI standard [1] is based on the direct correlation of

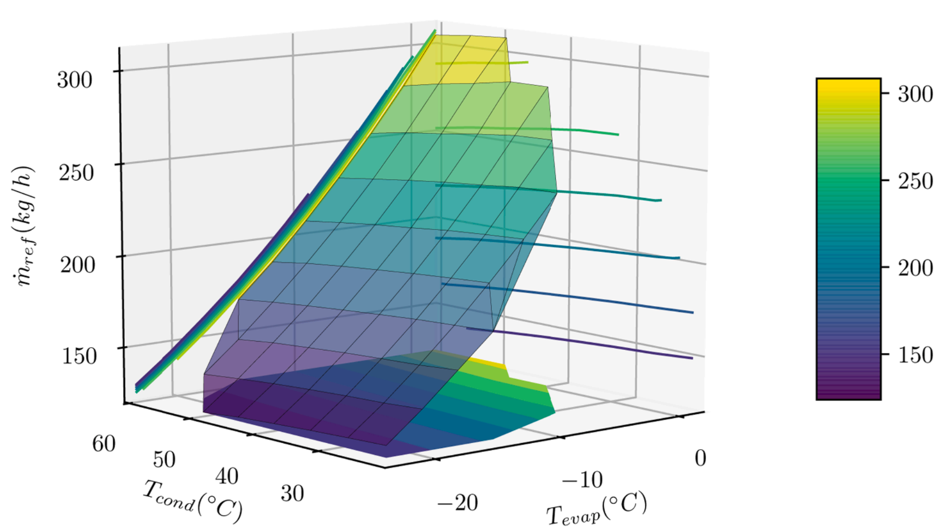

Fig. 7. 3D plot of mass flowrate versus evaporation and condensation temperatures of compressor ZS21KAE-PFV with refrigerant R404A (SH = 11 K).

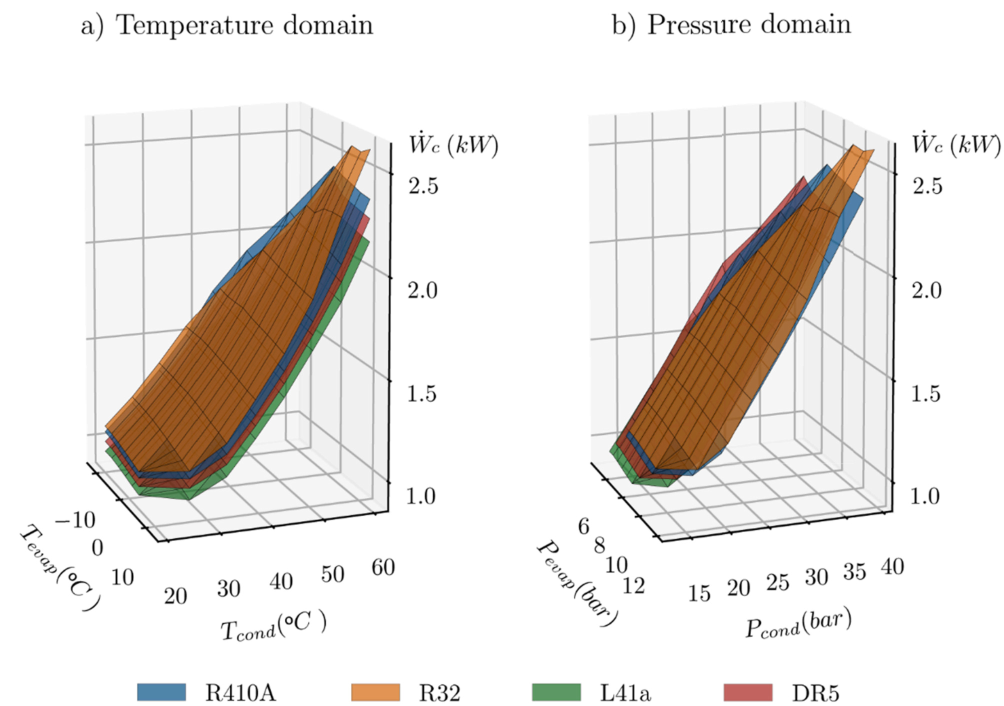

Fig. 8. 3D representation of energy consumption as a function of evaporating and condensing temperatures (left) and evaporating and condensing pressures (right) for ARHI-11 compressor.

the compressor flowrate.

Fig. 7 shows the mass flowrate of compressor ZP21K5E-PFV (AHRI 21) in a 3D plot as a function of evaporating and condensing temperatures for the case with constant superheat SH = 11 K. As it can be observed, at constant superheat, the mass flow rate surface is quite smooth, mainly dependent on the evaporating temperature, with a very slight curvature, and a much weaker dependence on the condensing temperature, again almost linear.

The authors have analyzed the mass flowrate data of all AHRI reports referred above and the test data included in [8]. They have found that the trends observed in Fig. 7 are the same for all compressors and refrigerants.

4. Compressor correlations evaluated

From all these information about the shape of the experimental

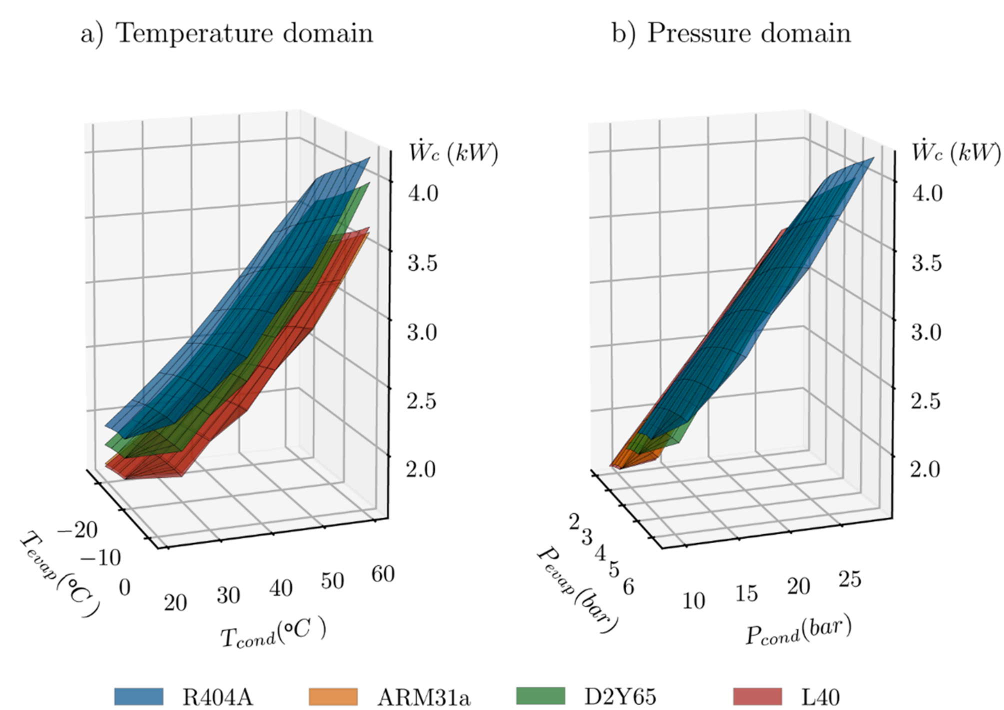

Fig. 9. 3D representation of energy consumption as a function of evaporating and condensing temperatures (left) and evaporating and condensing pressures (right) for AHRI 21 compressor.

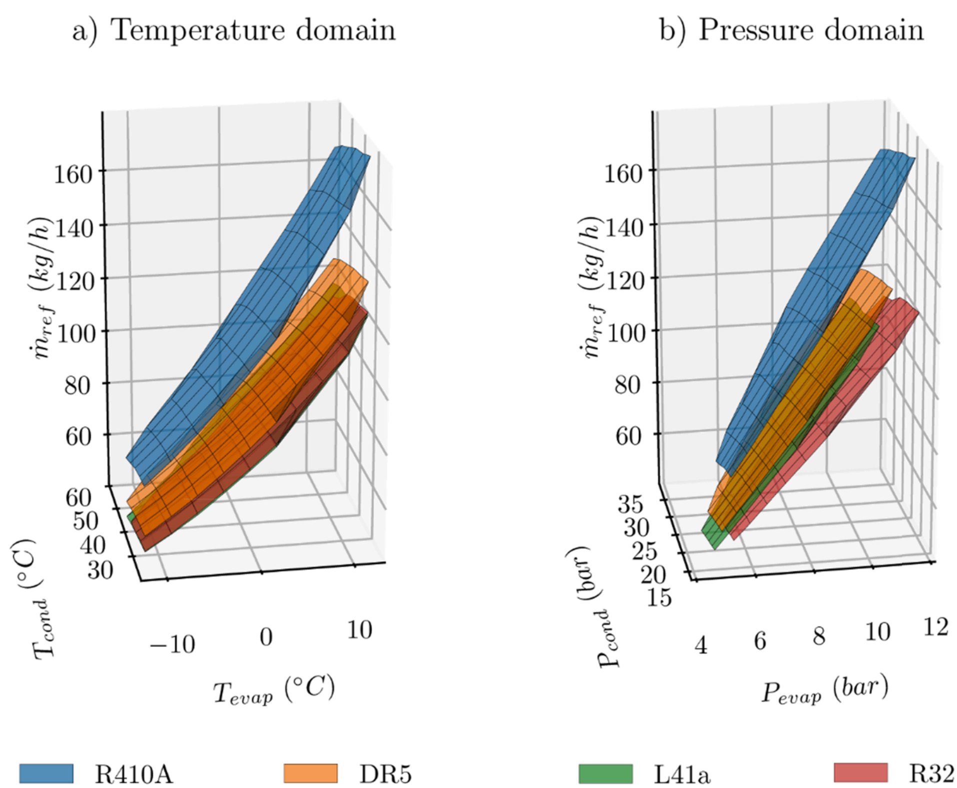

Fig. 10. 3D representation of mass flow rate as a function of evaporating and condensing temperatures (left) and evaporating and condensing pressures (right) for ARHI-11 compressor.

points for the compressor energy consumption and mass flow rate as a function of evaporating and condensing conditions, in this section, the suitability of ARHI polynomials to reproduce the compressor behavior is going to be studied, and the significance of the different terms of that correlation analyzed. From it, the best correlation for scroll compressors based on these polynomials is proposed.

4.1. Correlation for energy consumption

Looking at the surface representing the compressor consumption versus the condensation and evaporation temperatures (Fig. 5), it is obvious that the 10 coefficients AHRI polynomial [1] are able to fit these results if enough data properly distributed are available. However, the question is if all the coefficients are necessary to reproduce the behavior or the same degree of goodness can be obtained using polynomials with a lower number of coefficients. Regarding to that question, a polynomial with only 6 coefficients like the one used by [30] for rotary compressors can be used without reducing significantly the prediction capabilities of the AHRI polynomials. Thus, based on the authors' experience, the polynomial supplied by expression (3) could represent properly the energy consumption of scroll and rotary compressors.

$$\dot{W}_c = C_0 + C_1 T_{evap} + C_2 T_{cond} + C_3 T_{evap} T_{cond} + C_4 T_{evap}^2 + C_5 T_{cond}^2 \quad (3)$$

Though, when the compressor energy consumption is represented for different refrigerants as a function of evaporating and condensing temperatures, although the surfaces show the same trend, they are at different levels. This displacement almost disappears when the representation is made as a function of refrigerant saturation pressures instead of temperatures.

Fig. 8 and Fig. 9 show the compressor energy consumption for AHRI 11 and AHRI 21 compressors as a function of working conditions for 4 different refrigerants. From these figures, it can be seen that the representation as a function of pressures is much more universal than in temperatures. This is completely natural as the compressor does not see temperatures but compresses a reheated gas from the pressure at the inlet (suction) up to the pressure at the outlet (discharge). Of course, the temperature influences the density of the racked refrigerant and, therefore, the mass flow rate and heat transfer, but with minor changes in the compressor consumption if the pressure domain is selected.

The same representation of Figs. 8 and 9 as a function of evaporating and condensing pressures has been done for all the refrigerants and compressor included in the referenced reports, obtaining similar conclusions. Thus, it can be determined that the compressor energy consumption as a function of inlet and outlet saturation pressures is much more independent of the refrigerant and more representative of the compressor. Furthermore, a correlation of polynomials based on the condensing and evaporating pressures can be as effective as the one based on the dew temperatures. Thus, the pressure approach also includes the advantage of adjusting the model for transcritical cycles, where the condensation temperature does not remain constant.

Right-side of Figs. 8 and 9 shows that the compressor consumption surface is quite flat for the pressure domain representation. Therefore, the pressure domain representation can simplify the objective response surface. A simple linear polynomial containing linear terms on both evaporating and condensing pressures and one cross-term with their product leads to a robust correlation with very decent accuracy for all the analyzed compressors and refrigerants.

$$\text{Correlation 1}: \ \dot{W}_c = C_0 + C_1 P_{evap} + C_2 P_{cond} + C_3 P_{evap} P_{cond} \quad (4)$$

Suppose now one wants to increase the accuracy of the correlation. In that case, and based on the dependences shown in Fig. 5, a second-order dependence on the condensation pressure should be incorporated. This incorporation will improve the fitting at high condensation pressures where the linear behavior is slightly broken. A second order on the evaporating temperature will also improve the goodness of the fit for LT compressors. Adding those two terms to Correlation 1, one gets the second correlation proposed by the authors:

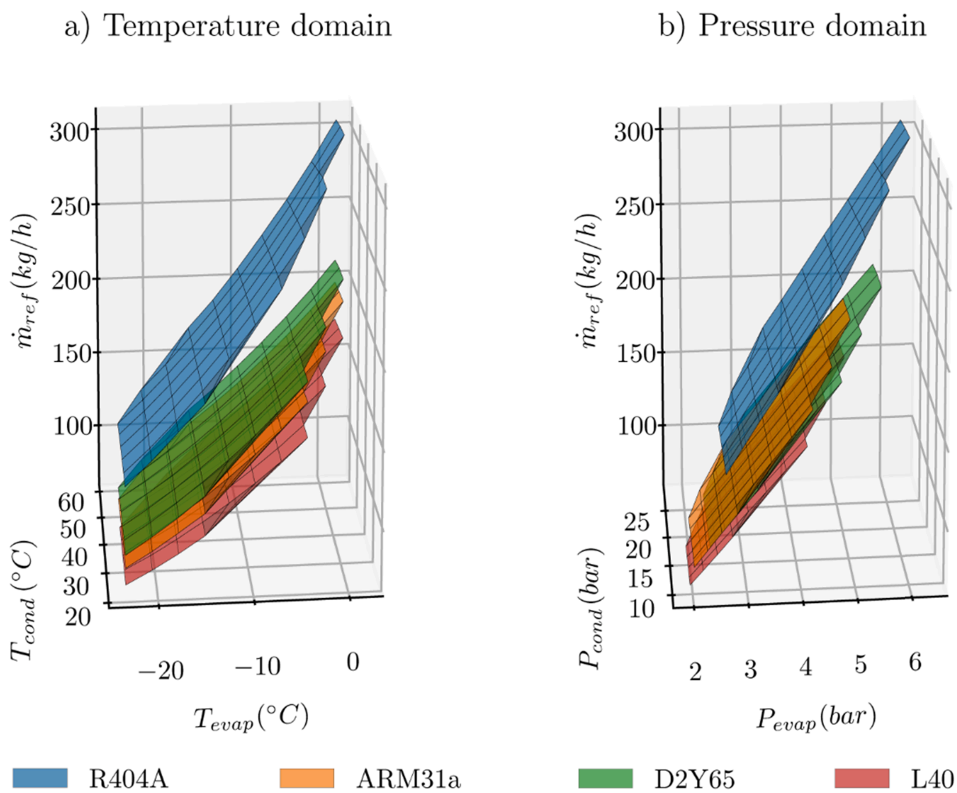

Fig. 11. 3D representation of mass flow rate as a function of evaporating and condensing temperatures (left) and evaporating and condensing pressures (right) for AHRI 21 compressor.

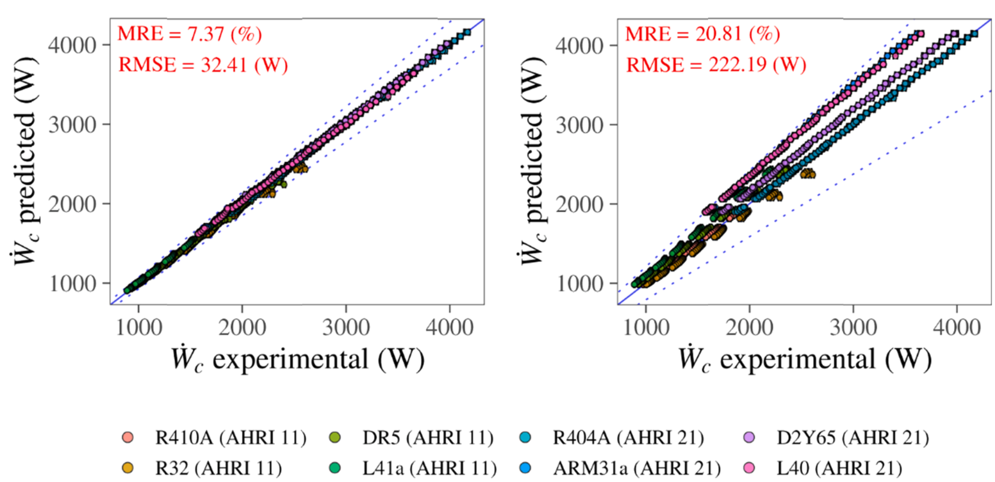

Fig. 12. Energy consumption prediction for other refrigerants (AHRI 11 and AHRI 21).

$$\text{Correlation 2}: \dot{W}_c = C_0 + C_1 P_{evap} + C_2 P_{cond} + C_3 P_{evap} P_{cond} + C_4 P_{evap}^2 + C_5 P_{cond}^2 \quad (5)$$

Finally, the correlation 2 but in the temperatures domain which is the one proposed by [30] for rotary compressors of variable speed is going to be analyzed.

$$\text{Correlation 3}: \dot{W}_c = C_0 + C_1 T_{evap} + C_2 T_{cond} + C_3 T_{evap} T_{cond} + C_4 T_{evap}^2 + C_5 T_{cond}^2 \quad (6)$$

The evaporating and condensing temperatures correspond to dew temperatures.

4.2. Correlation for mass flow

The same behavior observed in the energy consumption can be extrapolated to refrigerant mass flow. In this case, when the representation is made in terms of pressures, the mass flow dependence is more linear, and the surfaces are in some way regularized (Fig. 10 and Fig. 11). However, the surfaces corresponding to different refrigerants do not converge when they are represented as a function of pressures. This is because the refrigerant density at the compressor inlet strongly influences the mass flow.

In this case, the correlations analyzed in terms of pressures have been simplified as:

$$\text{Correlation 1}: \dot{m}_{ref} = C_0 + C_1 P_{evap} + C_2 P_{cond} \quad (7)$$

Table 3
Correlation results. Energy consumption (AHRI 11, 21, Cuevas&Lebrun).

	Correlation 1	MRE (%)	RMSE (W)	Correlation 2	MRE (%)	RMSE (W)	Correlation 3	MRE (%)	RMSE (W)	Fluid	Source
c0	7.7e-01 ± 5.1e-02***	2.34	21.50	5.9e-01 ± 4.2e-02***	1.54	13.44	1.4e + 00 ± 4.9e-02***	2.19	14.04	R404A	AHRI 21
c1	−2.1e-03 ± 1.3e-02			1.1e-01 ± 1.5e-02***			−1.1e-02 ± 2.3e-03***				
c2	8.8e-02 ± 2.5e-03***			8.3e-02 ± 3.0e-03***			2.1e-02 ± 1.9e-03***				
c3	5.0e-03 ± 5.9e-04***			7.1e-03 ± 5.1e-04***			4.8e-04 ± 3.6e-05***				
c4				−1.8e-02 ± 2.1e-03***			−2.1e-04 ± 4.7e-05***				
c5				−8.7e-05 ± 8.8e-05+			4.2e-04 ± 1.9e-05***				
c0	6.7e-01 ± 3.8e-02***	1.89	17.42	5.2e-01 ± 2.7e-02***	1.29	9.40	1.2e + 00 ± 3.5e-02***	1.65	9.73	ARM31a	
c1	1.6e-02 ± 1.2e-02**			1.2e-01 ± 1.2e-02***			−8.7e-03 ± 1.7e-03***				
c2	8.5e-02 ± 2.2e-03***			8.3e-02 ± 2.2e-03***			1.8e-02 ± 1.4e-03***				
c3	5.6e-03 ± 6.4e-04***			8.6e-03 ± 4.9e-04***			4.5e-04 ± 2.6e-05***				
c4				−2.2e-02 ± 2.1e-03***			−1.6e-04 ± 3.3e-05***				
c5				−2.5e-04 ± 7.7e-05***			3.6e-04 ± 1.4e-05***				
c0	7.1e-01 ± 4.1e-02***	2.61	18.41	5.6e-01 ± 3.2e-02***	1.61	11.01	1.3e + 00 ± 4.3e-02***	1.85	11.98	D2Y65	
c1	1.1e-03 ± 1.1e-02			1.0e-01 ± 1.3e-02***			−1.0e-02 ± 2.1e-03***				
c2	8.7e-02 ± 2.2e-03***			8.4e-02 ± 2.4e-03***			1.9e-02 ± 1.7e-03***				
c3	5.2e-03 ± 5.6e-04***			7.7e-03 ± 4.9e-04***			4.8e-04 ± 3.3e-05***				
c4				−1.9e-02 ± 2.1e-03***			−1.7e-04 ± 4.2e-05***				
c5				−1.5e-04 ± 7.7e-05***			4.3e-04 ± 1.7e-05***				
c0	6.1e-01 ± 3.5e-02***	2.27	15.79	5.1e-01 ± 2.5e-02***	1.50	9.04	1.2e + 00 ± 3.7e-02***	1.29	9.12	L40	
c1	1.5e-02 ± 1.1e-02**			1.1e-01 ± 1.2e-02***			−8.7e-03 ± 1.9e-03***				
c2	9.1e-02 ± 2.2e-03***			8.5e-02 ± 2.2e-03***			1.6e-02 ± 1.5e-03***				
c3	5.2e-03 ± 6.2e-04***			8.1e-03 ± 5.9e-04***			4.3e-04 ± 3.0e-05***				
c4				−2.2e-02 ± 2.4e-03***			−1.6e-04 ± 3.6e-05***				
c5				−1.1e-04 ± 8.4e-05**			4.1e-04 ± 1.5e-05***				
c0	6.1e-01 ± 4.0e-02***	1.46	15.50	5.3e-01 ± 2.7e-02***	0.91	8.89	1.3e + 00 ± 8.3e-02***	1.28	13.62	*R*32/R134a	
c1	−5.8e-03 ± 1.0e-02			1.1e-01 ± 1.5e-02***			−1.1e-02 ± 4.8e-03***				
c2	1.0e-01 ± 2.5e-03***			8.8e-02 ± 2.5e-03***			1.4e-02 ± 3.4e-03***				
c3	5.0e-03 ± 5.8e-04***			7.9e-03 ± 9.1e-04***			4.8e-04 ± 8.5e-05***				
c4				−2.1e-02 ± 3.2e-03***			−2.3e-04 ± 9.0e-05***				
c5				9.1e-05 ± 1.1e-04			5.7e-04 ± 3.6e-05***				
c0	2.8e-01 ± 3.2e-02***	2.57	12.86	3.2e-01 ± 1.7e-02***	0.72	4.95	7.7e-01 ± 1.5e-02***	1.21	7.26	R410A	AHRI 11
c1	−3.0e-02 ± 3.8e-03***			−1.1e-02 ± 3.3e-03***			−3.9e-03 ± 7.0e-04***				
c2	5.8e-02 ± 1.3e-03***			4.8e-02 ± 8.0e-04***			1.0e-03 ± 8.1e-04*				
c3	4.3e-04 ± 1.5e-04***			−1.3e-04 ± 7.7e-05***			−2.1e-05 ± 1.8e-05*				
c4				−2.9e-04 ± 2.2e-04**			−7.6e-05 ± 2.2e-05***				
c5				2.9e-04 ± 1.9e-05***			4.5e-04 ± 1.0e-05***				
c0		2.71	16.88		3.19	12.53		3.12	14.43	*R*32	

(*continued on next page*)

Table 3 (*continued*)

	Correlation 1	MRE (%)	RMSE (W)	Correlation 2	MRE (%)	RMSE (W)	Correlation 3	MRE (%)	RMSE (W)	Fluid	Source
	4.0e-01 ± 4.9e-02***			4.0e-01 ± 4.4e-02***			8.0e-01 ± 3.7e-02***				
c1	−4.5e-02 ± 5.4e-03***			−1.6e-02 ± 9.1e-03***			−6.2e-03 ± 1.9e-03***				
	c2			5.2e-02 ± 2.2e-03***			4.3e-02 ± 2.3e-03***				
	6.1e-04 ± 2.1e-03										
	c3			1.3e-03 ± 2.2e-04***			5.3e-04 ± 2.9e-04***				
	6.8e-05 ± 5.4e-05*										
	c4		−6.1e-04 ± 6.3e-04+	−7.1e-05 ± 5.2e-05**							
c5		3.2e-04 ± 6.4e-05***	4.8e-04 ± 2.8e-05***								
c0	2.9e-01 ± 4.6e-02***	4.95	17.94	3.6e-01 ± 3.6e-02***	4.19	10.61	7.8e-01 ± 2.8e-02***	4.62	13.00	DR5	
c1	−3.3e-02 ± 5.7e-03***			−1.3e-02 ± 7.8e-03**			−2.7e-03 ± 1.3e-03***				
c2	5.8e-02 ± 2.1e-03***			4.5e-02 ± 1.9e-03***			−1.7e-03 ± 1.5e-03*				
c3	6.8e-04 ± 2.5e-04***			−2.4e-04 ± 2.0e-04*			−3.6e-05 ± 3.5e-05*				
c4				4.0e-05 ± 5.6e-04			−4.1e-05 ± 4.1e-05*				
c5				4.2e-04 ± 4.7e-05***			4.6e-04 ± 1.9e-05***				
c0	3.0e-01 ± 2.7e-02***	2.29	10.48	3.0e-01 ± 1.6e-02***	1.02	4.64	6.9e-01 ± 1.4e-02***	1.25	6.33	L41a	
c1	−3.1e-02 ± 3.5e-03***			−7.4e-03 ± 3.5e-03***			−4.3e-03 ± 6.7e-04***				
c2	5.5e-02 ± 1.3e-03***			4.7e-02 ± 8.3e-04***			1.6e-03 ± 7.6e-04***				
c3	7.3e-04 ± 1.6e-04***			2.0e-04 ± 9.9e-05***			2.4e-05 ± 1.8e-05**				
c4				−8.1e-04 ± 2.7e-04***			−8.3e-05 ± 2.0e-05***				
c5				2.7e-04 ± 2.3e-05***			3.9e-04 ± 9.7e-06***				
c0	9.2e-02 ± 2.8e-01	3.16	60.75	1.8e-01 ± 3.0e-01	2.58	51.17	1.4e + 00 ± 6.6e-01***	3.05	54.48	R134a	Cuevas
c1	−3.8e-02 ± 3.3e-02*			−2.5e-03 ± 4.6e-02			2.0e-02 ± 2.1e-02+				
c2	1.5e-01 ± 1.1e-02***			1.3e-01 ± 2.8e-02***			−3.1e-02 ± 2.4e-02*				
c3	−6.8e-04 ± 1.1e-03			−1.1e-03 ± 2.4e-03			−1.7e-04 ± 3.4e-04				
c4				−9.5e-04 ± 2.6e-03			−2.7e-04 ± 2.1e-04*				
c5				5.2e-04 ± 8.5e-04			7.7e-04 ± 2.2e-04***				

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$; Confidence interval of 95 % for regression coefficients.

$$\text{Correlation 2}: \ \dot{m}_{ref} = C_0 + C_1 P_{evap} + C_2 P_{cond} + C_5 P_{evap} P_{cond} \tag{8}$$

In the same way, the previous correlation is also performed for the temperature domain. This correlation is precisely the same proposed by [30] for rotary compressors of variable speed, referred to as Correlation 3.

$$\text{Correlation 3}: \ \dot{m}_{ref} = C_0 + C_1 T_{evap} + C_2 T_{cond} + C_3 T_{evap} T_{cond} + C_4 T_{evap}^2 + C_5 T_{cond}^2 \tag{9}$$

In this case, reducing the number of coefficients in the polynomial does not reduce the number of tests required as the energy polynomial requires them but improves the stability and sensibility of the obtained results when extrapolating from the experimental data is done.

5. Comparison of the correlations

5.1. Energy consumption correlation

Table 3 shows the fitting results obtained for the compressors AHRI 21 and AHRI 11, each one tested with 4 refrigerants for the compressor tested by [8]. The fitting has included the 3 correlations proposed plus the AHRI polynomials, but the last ones were not included in the table as they did not improve the quality of the fitting and some of the coefficients were not statistically significant. The Table includes the values of the coefficients obtained for the different correlations, the **maximum relative error** (MRE) in **%** and the **Root Mean Square Error** (RMSE) in **W**. These errors are plotted in Fig. 13 to simplify the comparison. The correlations are fitted to all available test points for each compressor and refrigerant, including all different suction conditions. The coefficients are meant to provide the **compressor consumption** in **kW** with **temperatures** expressed in °**C** and **pressures** in **bar.** A summary table like Table 3 is also included in the supplementary material for the rest of the scroll compressors.

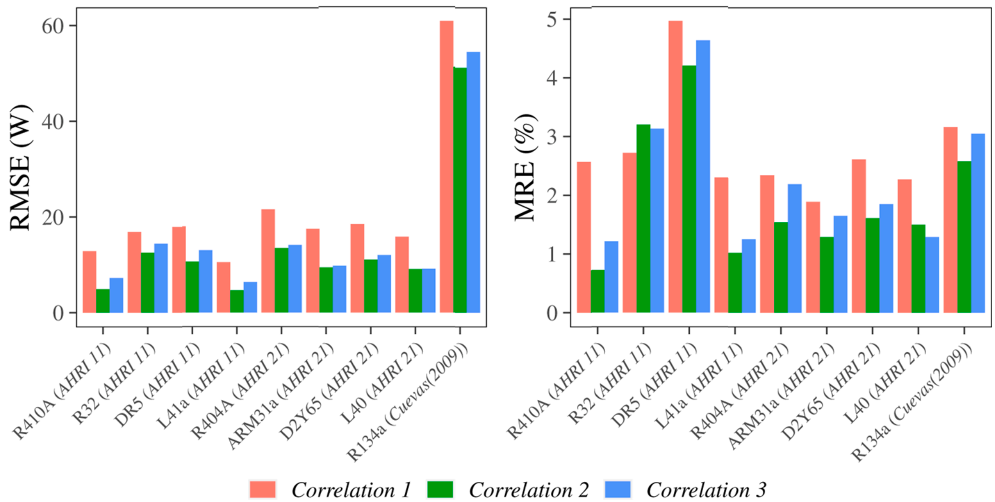

Fig. 13. Error bars plots for the MRE and RMSE in the prediction of the energy consumption.

As shown in Table 3, both MRE and RMSE are very low, for all the analyzed correlations, providing a very good representation of the compressor consumption across the entire envelope. Typically, the analysis of the significance of the coefficients in the model makes use of statistical terms such as the p-value (p), where a term with p-value < 0.05 is considered as significant. Therefore, this table also includes the confidence intervals and p-value for each coefficient and the three correlations were fitted with the same amount of data for each compressor and refrigerant.

We can observe a low prediction error when analyzing firstly correlations 1 and 2 (pressure domain). Correlation 1 allows characterizing the power consumption by including only two linear terms (P_{evap} and P_{cond}) and the interaction term ($P_{evap} \times P_{cond}$). As shown in the analysis performed in section 4.1 on the dependence and shape of the response surfaces for the energy consumption, the use of the pressure domain results in the linearization of the dependencies with P_{evap} and P_{cond}. Therefore, considering the pressure domain allows to characterize the energy consumption without quadratic terms and a low prediction error. Thus, we obtain a simple and compact correlation that will need fewer experimental tests for the adjustment. On the other hand, if we want to increase the accuracy of the model, the addition of the quadratic terms in the correlation 2 allows us to decrease the prediction error slightly. Moreover, we can see that the p-values and the confidence intervals obtained for the coefficients of correlation 2 are still low, so the addition of these terms is statistically significant. Thus, adding coefficients allows for a better fitting of the experimental results but probably imply the necessity of a higher number of experimental points. Finally, correlation 3 (temperature domain) obtains slightly lower prediction errors than correlation 1 but slightly higher than correlation 2. In this case, we can see that all terms included in the correlation are statistically significant due to the greater complexity of the surface when using the temperature domain, where the quadratic terms have greater relevance. Therefore, the highest accuracy is reached with Correlation 2, proving that the correlation with pressures is better than with temperatures, and as discussed above, less dependent on the employed refrigerant.

Moreover, as mentioned in section 4.1, using pressures instead of temperatures allows a correlation more independently of the refrigerant. For example, Fig. 12 shows the results of extrapolating with other refrigerants. This figure shows correlation 2, adjusting its coefficients with the base refrigerants (AHRI 11, R410A and AHRI 21, R404a) and extrapolating the predicted energy consumption values for the other refrigerants available in the dataset.

Finally, the compressor of [8] was selected because their results were obtained in a totally different framework, hence it could be a good indicator about the general application of the obtained results. In this case, their polynomials show slightly higher RMSE when fitted to those results, but its surface is very similar to the other MT-HT compressors of the report. Therefore, under the authors opinion higher experimental uncertainty could explain the slightly higher deviation.

5.2. Mass flow comparison

In this case, the accuracy level reached is like the one compared with the correlations proposed for the energy consumption and ARHI polynomials overdetermine the number of parameters required to fit the experimental data in this compressor design. As a main result, we can observe that the pressure domain eliminates the use of quadratic terms due to the linearization of the response surfaces. In general, the three proposed correlations show similar prediction errors with the additional advantage of including fewer terms in the models fitted in pressures. Thus, we obtain a simple and compact correlations that will need fewer experimental tests for the adjustment. Then, adding the interaction term ($P_{evap} \times P_{cond}$) slightly decreases the prediction errors between correlations 1 and 2. Table 4 shows the results obtained from this analysis, also including the corresponding p-value and confidence interval for each coefficient. In general, all the coefficients included in the correlations are statistically significant (low p-values).

The Table includes the values of the coefficients (estimates) for correlation 7, 8, and 9, as well as the **maximum relative error** (MRE) in **%** and the **Root Mean Square Error** (RMSE) in **kg/h**. These errors are plotted in Fig. 14 in order to simplify the comparison. The coefficients are meant to provide the compressor **mass flowrate** in **kg/h** with **temperatures** expressed in °**C** and **pressures** in **bar**. A summary table like Table 4 is also included in the supplementary material for the rest of scroll compressors.

Regarding the suction conditions used, for each compressor and refrigerant, the correlations are fitted to the data tested at constant SH (SH = 11 K in AHRI reports and SH = 6.8 K in [8]. The use of the Dabiri correlation [10] for correcting suction conditions has been tested on the data analyzed with good results.

6. Experimental points required

One essential question about properly characterizing the compressor performance is how many points and where to place them in the working domain. This question has been widely addressed in the field of Design of Experiments (DoE), including classical methodologies and more sophisticated methods like computer-aided calculations in Optimal Designs (OD). This last typology, OD, has the advantage of selecting points in non-regular domains [2], like the temperatures/

Table 4
Correlation results. Mass flowrate (AHRI 11, 21, Cuevas&Lebrun).

	Correlation 1	MRE (%)	RMSE (kg/h)	Correlation 2	MRE (%)	RMSE (kg/h)	Correlation 3	MRE (%)	RMSE (kg/h)	Fluid	Source
c0	4.8e + 00 ± 9.7e-01***	1.09	0.80	−4.7e + 00 ± 2.3e + 00***	0.85	0.53	3.1e + 02 ± 3.2e + 00***	0.75	0.49	R404A	AHRI 21
c1	5.1e + 01 ± 2.1e-01***			5.3e + 01 ± 5.7e-01***			9.7e + 00 ± 1.6e-01***				
c2	−7.1e-01 ± 4.2e-02***			−2.4e-01 ± 1.1e-01***			−1.7e-02 ± 1.2e-01				
c3				−1.2e-01 ± 2.7e-02***			−5.0e-03 ± 2.4e-03***				
c4							9.5e-02 ± 3.1e-03***				
c5							−4.4e-03 ± 1.2e-03***				
c0	1.4e + 00 ± 3.7e-01***	0.73	0.33	−7.9e-01 ± 1.1e + 00	0.92	0.29	2.0e + 02 ± 1.7e + 00***	0.92	0.28	ARM31a	
c1	4.1e + 01 ± 1.0e-01***			4.1e + 01 ± 3.4e-01***			6.6e + 00 ± 8.2e-02***				
c2	−5.3e-01 ± 1.8e-02***			−4.0e-01 ± 6.2e-02***			−7.2e-02 ± 6.7e-02*				
c3				−3.8e-02 ± 1.8e-02***			−1.2e-03 ± 1.2e-03+				
c4							6.8e-02 ± 1.7e-03***				
c5							−1.9e-03 ± 6.9e-04***				
c0	2.6e + 00 ± 6.4e-01***	1.24	0.55	−6.4e-01 ± 1.9e + 00	1.62	0.50	2.1e + 02 ± 2.6e + 00***	1.33	0.44	D2Y65	
c1	3.9e + 01 ± 1.5e-01***			4.0e + 01 ± 5.3e-01***			6.8e + 00 ± 1.3e-01***				
c2	−5.3e-01 ± 2.8e-02***			−3.6e-01 ± 1.0e-01***			5.8e-02 ± 1.0e-01				
c3				−4.5e-02 ± 2.6e-02***			2.1e-04 ± 1.9e-03				
c4							6.7e-02 ± 2.7e-03***				
c5							−3.5e-03 ± 1.1e-03***				
c0	1.3e + 00 ± 4.8e-01***	1.04	0.42	−4.3e-01 ± 1.5e + 00	1.29	0.41	1.7e + 02 ± 2.1e + 00***	1.04	0.32	L40	
c1	3.7e + 01 ± 1.4e-01***			3.7e + 01 ± 4.8e-01***			5.7e + 00 ± 1.1e-01***				
c2	−5.0e-01 ± 2.6e-02***			−4.0e-01 ± 9.3e-02***			2.1e-01 ± 8.5e-02***				
c3				−3.2e-02 ± 2.7e-02*			2.3e-03 ± 1.7e-03**				
c4							6.1e-02 ± 2.2e-03***				
c5							−4.8e-03 ± 8.7e-04***				
c0	2.3e + 00 ± 7.8e-01***	1.76	0.61	1.8e + 00 ± 2.7e + 00	1.75	0.61	1.7e + 02 ± 5.2e + 00***	1.33	0.53	*R*32/ R134a	
c1	3.5e + 01 ± 2.5e-01***			3.5e + 01 ± 7.2e-01***			5.9e + 00 ± 3.1e-01***				
c2	−8.1e-01 ± 4.6e-02***			−7.8e-01 ± 1.7e-01***			4.1e-01 ± 2.1e-01***				
c3				−7.6e-03 ± 3.9e-02			7.5e-03 ± 5.2e-03**				
c4							6.6e-02 ± 6.1e-03***				
c5							−8.8e-03 ± 2.2e-03***				
c0	−4.5e + 00 ± 7.5e-01***	1.61	0.61	−1.0e + 00 ± 2.5e + 00	1.37	0.57	1.2e + 02 ± 1.5e + 00***	1.42	0.40	R410A	AHRI 11
c1	1.7e + 01 ± 8.2e-02***			1.6e + 01 ± 3.0e-01***			4.1e + 00 ± 6.7e-02***				
c2	−6.7e-01 ± 2.4e-02***			−8.2e-01 ± 1.1e-01***			1.2e-01 ± 7.8e-02**				
c3				1.7e-02 ± 1.2e-02**			4.0e-03 ± 1.7e-03***				
c4							5.6e-02 ± 2.1e-03***				
c5							−6.6e-03 ± 9.8e-04***				
c0		2.24	0.86		1.67	0.70		2.16	0.54	*R*32	

(*continued on next page*)

Table 4 (*continued*)

	Correlation 1	MRE (%)	RMSE (kg/h)	Correlation 2	MRE (%)	RMSE (kg/h)	Correlation 3	MRE (%)	RMSE (kg/h)	Fluid	Source
	−3.2e-01 ± 1.1e + 00			−9.3e + 00 ± 3.5e + 00***			7.7e + 01 ± 2.3e + 00***				
c1	1.1e + 01 ± 1.3e-01***			1.2e + 01 ± 3.8e-01***			3.0e + 00 ± 1.2e-01***				
	c2			−6.3e-01 ± 3.9e-02***			−2.4e-01 ± 1.5e-01**				
	3.4e-01 ± 1.3e-01***										
	c3						−4.2e-02 ± 1.6e-02***				
	9.1e-05 ± 3.3e-03										
	c4			3.5e-02 ± 3.3e-03***							
c5			−9.1e-03 ± 1.7e-03***								
c0	−2.3e + 00 ± 3.9e-01***	1.33	0.32	−2.8e-01 ± 1.3e + 00	1.02	0.29	8.9e + 01 ± 6.9e-01***	0.65	0.19	DR5	
c1	1.3e + 01 ± 4.6e-02***			1.3e + 01 ± 1.6e-01***			3.1e + 00 ± 3.2e-02***				
c2	−6.1e-01 ± 1.3e-02***			−7.0e-01 ± 5.7e-02***			9.7e-02 ± 3.7e-02***				
c3				1.1e-02 ± 6.9e-03**			3.7e-03 ± 8.2e-04***				
c4							3.9e-02 ± 1.0e-03***				
c5							−5.5e-03 ± 4.7e-04***				
c0	−3.9e + 00 ± 5.0e-01***	1.87	0.41	1.4e + 00 ± 1.2e + 00*	1.22	0.27	8.3e + 01 ± 9.4e-01***	0.99	0.25	L41a	
c1	1.3e + 01 ± 6.4e-02***			1.2e + 01 ± 1.5e-01***			2.8e + 00 ± 4.5e-02***				
c2	−5.3e-01 ± 1.8e-02***			−7.7e-01 ± 5.4e-02***			−7.4e-02 ± 5.1e-02**				
c3				3.2e-02 ± 6.8e-03***			4.7e-03 ± 1.2e-03***				
c4							3.5e-02 ± 1.4e-03***				
c5							−2.8e-03 ± 6.4e-04***				
c0	−1.2e + 01 ± 1.0e + 01*	3.93	6.03	4.2e + 00 ± 2.6e + 01	4.70	5.65	1.3e + 02 ± 5.4e + 01***	2.76	4.48	R134a	Cuevas
c1	4.8e + 01 ± 1.0e + 00***			4.6e + 01 ± 3.1e + 00***			6.4e-01 ± 1.7e + 00				
c2	−1.7e + 00 ± 5.0e-01***			−2.3e + 00 ± 1.0e + 00***			1.3e + 00 ± 2.0e + 00				
c3				6.6e-02 ± 1.0e-01			7.3e-03 ± 2.8e-02				
c4							1.5e-01 ± 1.7e-02***				
c5							−1.6e-02 ± 1.8e-02+				

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$; Confidence interval of 95 % for regression coefficients.

pressure domains in scroll compressors.

These theories assume that the model describing the data is known and the application domain is defined. Based on that, the authors have selected the D-Optimal criteria described in [11] as it is especially well indicated for linearized problems with a non-regular operation domain. Furthermore, this methodology is well documented, and many open source tools include preprogrammed algorithms, providing an easy and automatic way to perform experimental test matrices to characterize compressors' performance [21] and [38]. This challenge about selecting a proper sample to obtain a statistically significant sample is not included in the current standard [1]. A more detailed analysis comparing the classical DoE and computer-aided experimental designs will be described in a future publication.

Taking as a first example the correlation 3 (temperature domain) for the characterization of mass flow rate and power consumption, Fig. 15 shows the ubication domain of the experimental points for 7, 9, and 11 experimental measurements. These points have been selected with the D-Optimal criteria as mentioned above. The samples show that: major part of tests must be placed at the operating limits of the compressor. Then, a few tests must be placed in the center. This result is similar to the one obtained by [4].

Once the location of tests points was determined, a fit of the models was done using the samples proposed by the Fedorov technic and the whole matrix of experimental points (SH = 11 K, SH = 22 K, andT_{suc} = 18 °C for $\dot{W}_c$ model and SH = 11 K for $\dot{m}_{ref}$ model) in order to check the prediction error for all the experimental data available. Table 5 shows the results obtained considering the Shao functional (Correlation 3) and 9 experimental measurements. This table also includes in brackets the MRE and RMSE obtained when fitting the original AHRI polynomial and selecting a sample of 11 test points. The authors found that 9 experimental measurements obtained the best results between sample size and prediction accuracy for the models adjusted.

As it can be seen, Maximum Relative Error (MRE), Round Mean Square Error (RMSE) and the differences on the polynomial parameters when 9 experimental points were selected began to be very small, being this number a rational number of required points to characterize scroll

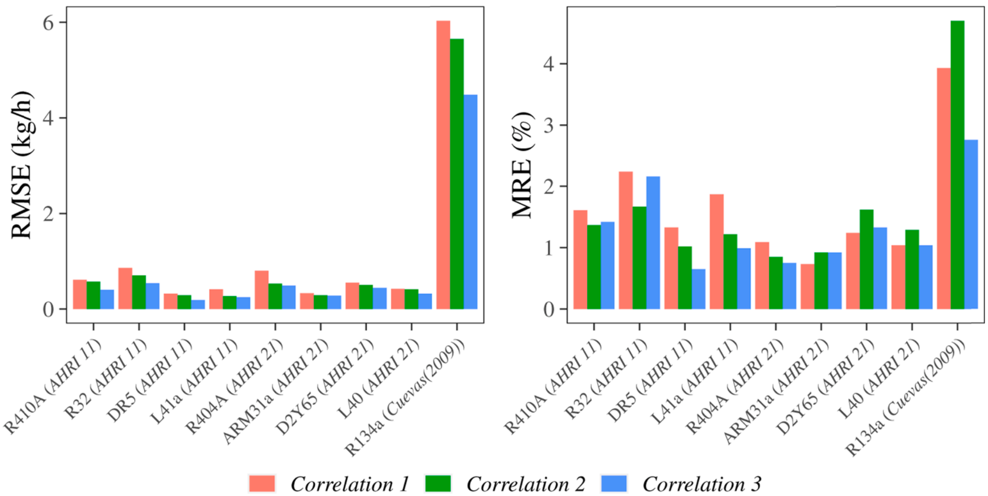

Fig. 14. Error bars plots for the MRE and RMSE in the prediction of the mass flow rate.

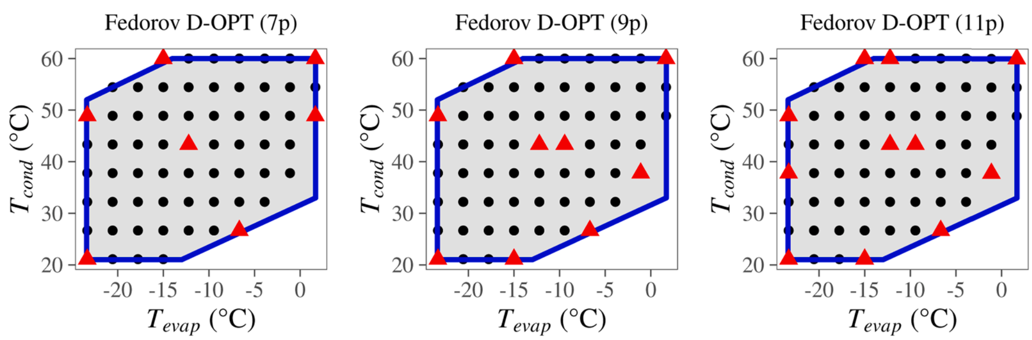

Fig. 15. Optimal Design. Fedorov (7, 9 11 points). AHRI 21 R404A.

Table 5
Regression model adjusted with Fedorov sample (AHRI 21 R404A and Correlation 3).

	All points		Sample Fedorov 9 points	
	$\dot{W}_c$	$\dot{m}_{ref}$	$\dot{W}_c$	$\dot{m}_{ref}$
c0	1.4e + 00 ± 4.9e-02***	3.1e + 02 ± 3.2e + 00***	1.3e + 00 ± 1.3e-01***	3.1e + 02 ± 6.9e + 00***
c1	−1.1e-02 ± 2.3e-03***	9.7e + 00 ± 1.6e-01***	−1.4e-02 ± 6.3e-03**	9.7e + 00 ± 3.4e-01***
c2	2.1e-02 ± 1.9e-03***	−1.7e-02 ± 1.2e-01	2.2e-02 ± 5.5e-03***	−7.7e-02 ± 2.7e-01
c3	4.8e-04 ± 3.6e-05***	−5.0e-03 ± 2.4e-03***	5.4e-04 ± 9.1e-05***	−3.5e-03 ± 4.7e-03+
c4	−2.1e-04 ± 4.7e-05***	9.5e-02 ± 3.1e-03***	−2.5e-04 ± 1.4e-04*	9.4e-02 ± 6.9e-03***
c5	4.2e-04 ± 1.9e-05***	−4.4e-03 ± 1.2e-03***	4.1e-04 ± 6.0e-05***	−3.3e-03 ± 2.8e-03*
Num. Obs.	191	63	9	9
MRE (%)	2.194	0.751	2.504 (2.630[a])	0.872 (1.013[a])
RMSE (W-kg/h)	14.043	0.491	15.691 (16.193[a])	0.584 (0.605[a])
Range (W-kg/h)	[1856,4172]	[124,308]	[1856,4172]	[124,308]

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$; Confidence interval of 95 % for regression coefficients.
[a] MRE and RMSE for the original AHRI polynomial fitted with 11 experimental points.

compressors. Moreover, looking at the MRE and RMSE values in brackets, one can see that the AHRI polynomial fit does not improve the accuracy and slightly increase the prediction error.

On the other hand, it must be noted that test points in the compressor limit usually have a higher experimental error than points in the center. The fitting process had this point weighting the relative influence on the final solution of the different points with its error. Therefore, the Inverse-variance weighting was selected rather than the classical Ordinary Least Squares (OLS) adjustment. The regression adjustment includes a vector of weights with the same length as the experimental sample. This vector is constructed as the inverse experimental variance, i.e., the inverse of the square of the combined standard uncertainty [35].

Similar results can be reproduced considering the pressure domain. In this case, the experimental design is constructed in the same way as described above. We must consider that the optimal design methodologies must know the functional to be applied. This means that in the case of having different functionals for the characterization of the energy consumption and mass flow rate, we should consider the functional with more terms when planning the experimental design. Therefore, if correlation 2 is selected for the energy consumption, we will obtain an experimental design equivalent to the previous one (we have the same polynomial terms between correlation 2 and 3) and selecting as a proper correlation for the mass flow rate the correlation 2 due to its higher precision.

However, if it is desired to decrease the experimental cost with only a slight increase in the prediction error, another interesting option should be to select correlation 1 for the energy consumption. In this case, the proper correlation for the mass flow rate is still correlation 2 due to the experimental sample must be able to adjust the functional for the energy consumption. Therefore, selecting correlation 1 for the energy

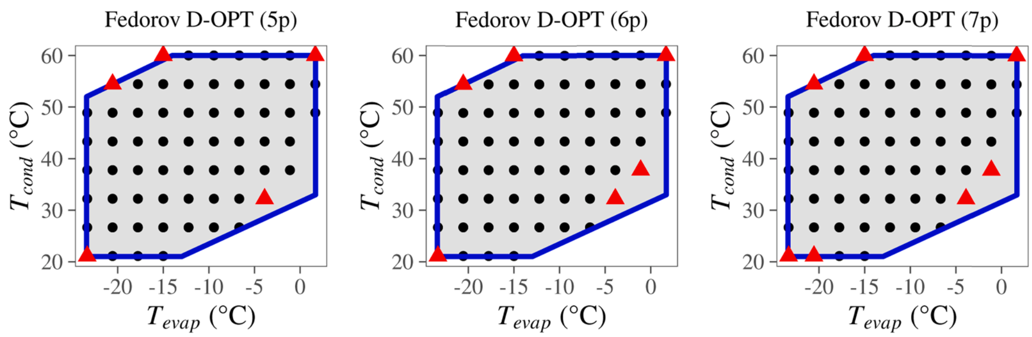

Fig. 16. Optimal Design. Fedorov (5, 6 7 points). AHRI 21 R404A.

Table 6
Regression model adjusted with Fedorov sample (AHRI 21 R404A and Correlation 1, 2).

	All points		Sample Fedorov 6 points	
	$\dot{W}_c$	$\dot{m}_{ref}$	$\dot{W}_c$	$\dot{m}_{ref}$
c0	7.7e-01 ± 5.1e-02***	−4.7e + 00 ± 2.3e + 00***	8.4e-01 ± 2.1e-01**	−3.4e + 00 ± 1.6e + 00*
c1	−2.1e-03 ± 1.3e-02	5.3e + 01 ± 5.7e-01***	−1.7e-02 ± 5.6e-02	5.3e + 01 ± 4.7e-01***
c2	8.8e-02 ± 2.5e-03***	−2.4e-01 ± 1.1e-01***	8.5e-02 ± 1.2e-02**	−3.2e-01 ± 7.6e-02**
c3	5.0e-03 ± 5.9e-04***	−1.2e-01 ± 2.7e-02***	5.4e-03 ± 3.0e-03*	−9.2e-02 ± 2.2e-02**
Num. Obs.	191	63	6	6
MRE (%)	2.345	0.850	2.608 (2.630[a])	0.818 (1.013[a])
RMSE (W-kg/h)	21.495	0.532	25.120 (16.193[a])	0.594 (0.605[a])
Range (W-kg/h)	[1856,4172]	[124,308]	[1856,4172]	[124,308]

+ $p < 0.1$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$; Confidence interval of 95 % for regression coefficients.
[a] MRE and RMSE for the original AHRI polynomial fitted with 11 experimental points.

consumption and correlation 2 for the mass flow rate, Fig. 16 shows the ubication domain of the experimental points for 5, 6, and 7 experimental measurements.

The results obtained are similar to the previous ones. The samples show that: major part of tests must be placed at the operating limits of the compressor. However, if the polynomial model does not include quadratic terms, the experimental sample will not include center points. This does not result in a significant increase in the prediction error. As shown in Table 6, selecting a sample of 6 points results in a prediction error similar to the previous case, with only a slight increase in the RMSE. In this case, 6 experimental measurements obtained the best results between sample size and prediction accuracy for the models adjusted.

Finally, illustrating the importance of obtaining a good experimental design, the robustness of the proposed correlations and the AHRI polynomial against random samples will be analyzed. Sometimes the compressor data supplied have not been obtained using an appropriate experimental design. Depending on the model used, this can lead to significant prediction errors when interpolating or extrapolating data. This can be especially critical in the case of fitting the original AHRI polynomial, where cubic terms add further instability to the model. Considering the original AHRI polynomial and the models proposed in this work, Fig. 17 includes a box plot showing the prediction errors (MRE and RMSE) for the entire data set and selecting a total of 50 random samples. The sample size selected in all models has been 11 points in order to be able to fit the largest model (original AHRI polynomial).

The results have shown that the models proposed in this work obtain greater stability and robustness than the original AHRI polynomial, thanks to the elimination of the cubic terms which are not required for scroll compressors.

7. Conclusions

A thorough analysis of scroll compressor's energy consumption and mass flow rate characteristics has been performed. The study has analyzed all scroll compressor results included in the AHRI reports corresponding to the AHRI Low-GWP Alternative Refrigerants Evaluation Program. As the main novelty of the study, the authors have been able to establish the most advantageous polynomial to use, including an analysis of the necessary experimental tests and where to place them to increase the model's accuracy. The following main conclusions can be drawn from the performed study:

- When the compressor is measured in a wide range of operating conditions, compressor and volumetric efficiencies show a complex shape inside its envelope. It is clearly sensitive to suction conditions (superheat). In contrast, the compressor consumption and mass flow rate are represented by smooth surfaces when plotted versus the evaporation and condensation temperatures (or pressures). For the energy consumption, it shows very little dependence on the superheat. Therefore, compressor consumption and mass flow rate are easier to characterize by fitting a polynomial than compressor efficiencies.
- For scroll compressors, it is not necessary to employ a 10 coefficients polynomial, as proposed in [1], to characterize the compressor. The much compact expression proposed by [30] is accurate enough and requires many less test points to be fitted to.
- The correspondence with the [30] results could indicate that all the conclusions of the paper can be extrapolated to rotary compressors. Unfortunately, the experimental database for these compressors were not so large and it was not possible to confirm this statement.
- The authors have found that the correlation will be smoother and more linear if the compressor consumption and mass flow rate are correlated with the condensation and evaporation pressures. Furthermore, it will also depend less on the used refrigerant for energy consumption and extends its applicability to transcritical cycles.
- The energy consumption of scroll compressors is quite plane and with smooth trends. A simple correlation with linear terms on the condensation and evaporation pressures together with a cross-term with their product requires only 4 coefficients. It provides a very simple and robust representation. Regarding the mass flow rate, correlation with linear terms on the condensation and evaporation pressures requires only 3 coefficients and provides a very simple and robust representation.

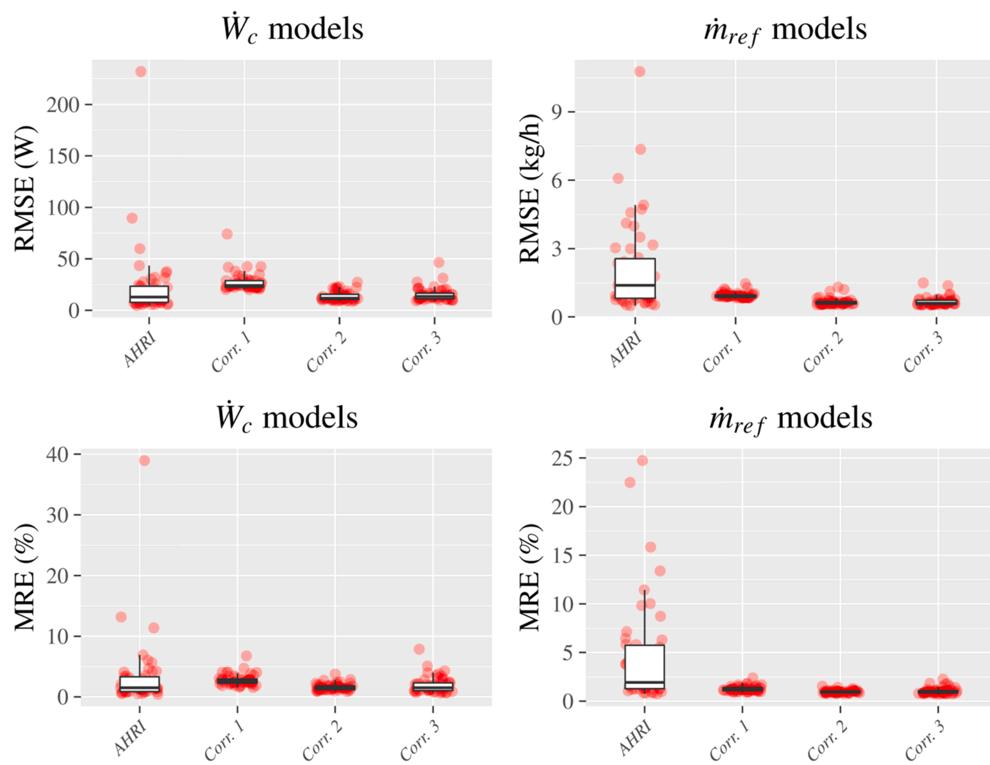

Fig. 17. Random samples error models (AHRI 21 R404A).

- This representation of these variables has an advantage in requiring fewer experimental point measurements to characterize the compressor energy consumption and mass flow rate appropriately than ARHI polynomials. But the most important thing is that the used polynomials for scroll compressors are quite linear and do not require cubic terms. Therefore, obtaining a good compressor performance prediction in this approach is less sensible to where the points are measured and the error when extrapolation from experimental data is performed.
- Although the standard does not specify anything about sampling selection, OD methodologies can be used to select samples and perform the experimental test matrix in the compressor's field. Using the D-Optimal criterion and a proper size for the experimental sample will supply good results. In this sense, to increase the model's accuracy, 9 points is an adequate size for correlation 3 (temperature domain) and 2 (pressure domain). On the other hand, this sample size can be reduced to 6 points using correlation 1 for the prediction of the energy consumption, decreasing the experimentation costs and without a significant loss in accuracy. In this case, the authors recommend using correlation 2 for the mass flow rate because the sample size is enough to obtain a good adjustment.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.applthermaleng.2022.119432.

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