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Application of logarithmic mean temperature difference to heat exchangers

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1 Summary of key ideas

In this article, we are going to present the basic characteristics that have to be borne in mind when working with **heat transport in heat exchangers.** In these devices**,** the variation in temperature along the length of the tubes through which the heating and cooling fluids flow is not constant. For that reason, we have to use the logarithmic mean temperature (ΔT_{ml}) in the following equation (Bird *et al*., 2006):

$$q=UA\Delta T_{ml}$$

Where:

q: heat flow rate(W) between two points in the system

U: overall heat transfer coefficient ($W/m^2{}^oC$)

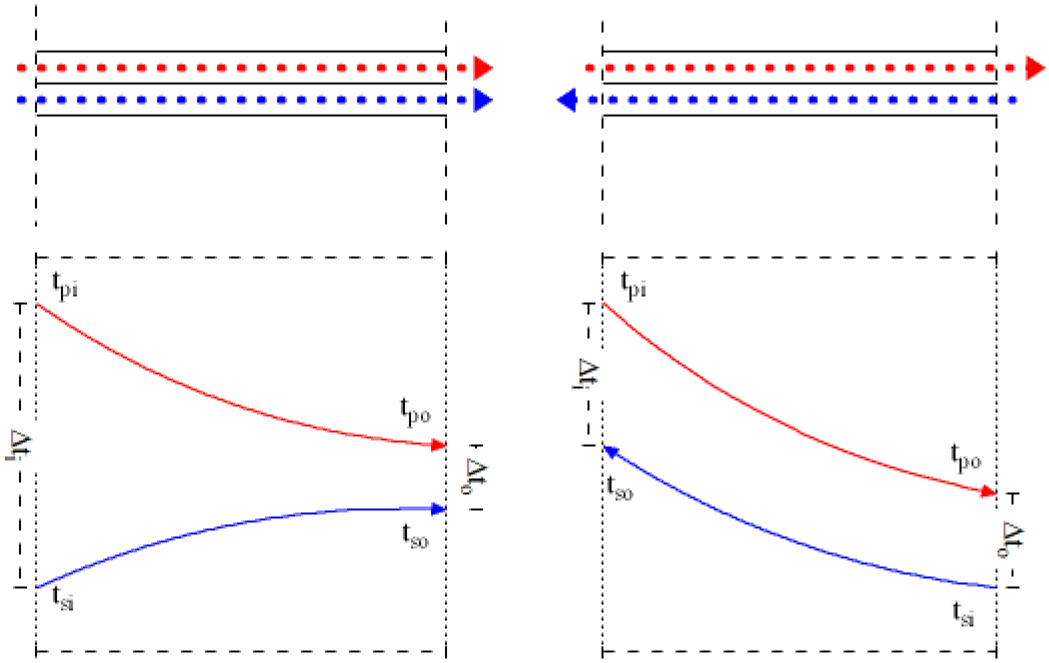
A: area perpendicular to transport (m^2)

ΔT_{ml}: logarithmic mean temperature difference between the inlet and outlet of the exchanger.

In this way we shall be able to adapt more easily to reality and measure more accurately the requirements of the area of heat exchange or the length of the tubes needed for the heat exchanger. In essence, we shall be able to design this type of equipment better.

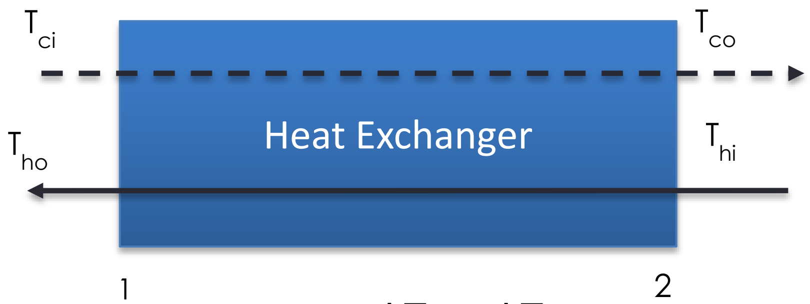
2 Introduction

Heat exchangers are devices that are used to transfer heat between two fluids, or between the surface of a solid and a fluid in movement, and are fundamental elements in many systems: heating, refrigeration, air conditioning, energy production and chemical processing. In their simplest version, we can find concentric tube heat exchangers, in which the flow of the fluids may either be parallel or countercurrent (Figure 1). As can be seen in the figure, the difference in temperature between the fluids will vary depending on the position in question; this variation is more marked when they flow in parallel than countercurrently. So, we shall use a temperature difference that reflects reality. This value is the logarithmic mean temperature difference and will be calculated by following the formula shown in Figure 2. This expression considers the difference in temperature between the hot fluid and the cold at each end of the heat exchanger, the position being represented by the numbers 1 and 2. It does not matter where we place these numbers, but once their position is fixed, we have to apply the formula consistently with the temperatures of the currents at these positions.



a) Parallel System **b) Countercurrent System**

Figure 1. *Variation in temperature difference along the heat exchanger pathway depending on whether the flow is parallel or countercurrent (Adapted from Alba, 2006)*



$$\Delta T_{ml} = \frac{\Delta T_2 - \Delta T_1}{ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

Figure 2. *Schematic representation of the currents of hot and cold fluid in a heat exchanger and the mathematical expression of the calculation of the logarithmic mean temperature difference (ΔT_{ml}) (Doran, 2013). Each sub-index in the temperature variable (T) refers to the cold fluid inlet($_{ci}$), the cold fluid outlet($_{co}$), the hot fluid inlet($_{hi}$) and the hot fluid outlet($_{ho}$).*

2 Objectives

Once you have read this document carefully, you will be able to:

- Identify in which situations you would have to employ the logarithmic mean temperature difference
- Calculate the logarithmic mean temperature difference (ΔT_{ml}) in function of the temperatures at the ends of a heat exchanger, both of hot fluid and of cold.
- Measure a heat exchanger

In order to see how useful this term is, you should watch the following video (Figure 3).

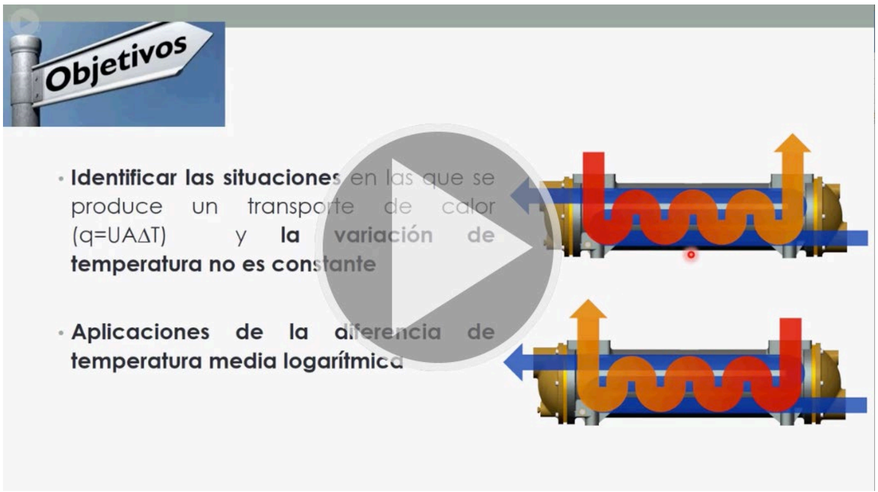


Figure 3. *Video explaining the application of the logarithmic mean temperature difference in heat exchangers (in Spanish) (Castelló, (2020)).* https://media.upv.es/#/portal/video/d9a06900-750b-11ea-b131-2da5ec4e0605

3 Development

As the best way to understand what to do in the event of having to employ the logarithmic mean temperature difference is to see how it is **applied**, in this section we are going to tackle two case studies. Specifically, the following two **situations**:

- A heat exchanger with or without a change of state
- The design of a coil cooler or heater in a fermentation process in which the temperature should be kept constant.

3.1 Heat exchanger with or without a change of state

In order to know how much heat the hot fluid has to transfer when heating up the cold fluid, we can consider an energy balance in the tubes through which the hot fluid must flow. If there are neither variations in the position nor in the diameter of the tube, that energy balance would only take into account the terms of internal energy(Δu) if there is a change between the inlet and outlet temperatures of the fluid in the tube, considering the specific heat of the fluid (C_p), or changes in flow work (ΔPV) if the temperature stays constant on account of a variation in the fluid's state. In this last case, we would consider the latent heat (ΔH) so as to be able to find out the mass flow rate requirements of the fluid (m) (Figure 4).

$$q = \Delta u + \Delta(PV) = m\,C_p\,(T_s - T_e) + m \cdot \Delta H$$

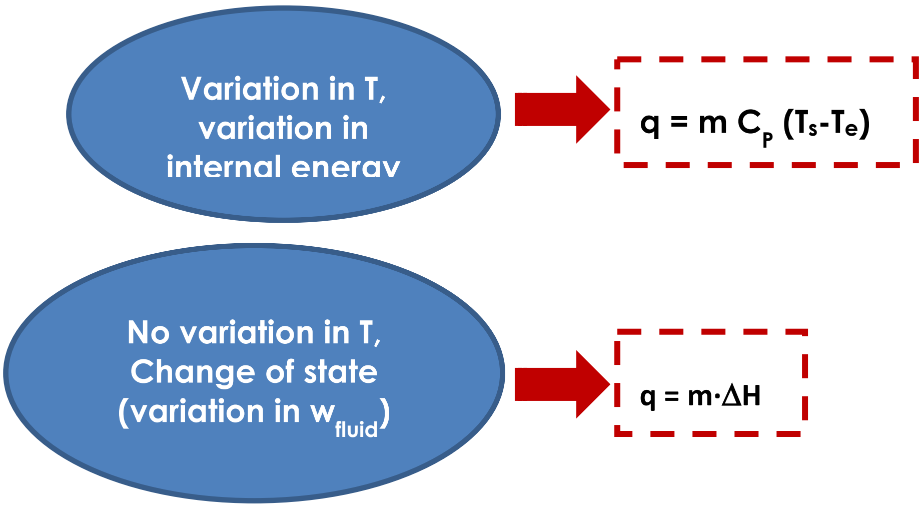


Figure 4. *Diagram showing the calculation of the fluid energy requirements for energy balance depending on whether there is or not a change of state*

Are you ready to solve a case study that considers both situations? Let's take a look!

We are **to design a pasteurizer** to work with 100 kg/s of orange juice that enter at 52 °C and exit at 67 °C (C_{pjuice}=3.88 kJ/kg°C). To that end, we suggest **two possibilities**:

a) Use **overheated steam at 150 °C** (ΔH^v= 2145.7 kJ/kg) as the external fluid which keeps that temperature constant

b) Use countercurrent flow **hot water** as the external fluid which **enters at 95 °C** and **exits at 70 °C** (C_{pw}=4.187 kJ/kg°C).

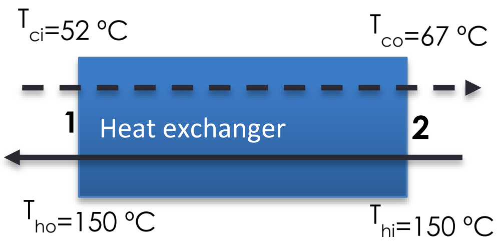
In both cases, the overall heat transfer coefficient (U) is 500 kJ/sm^{2}K.

Calculate:

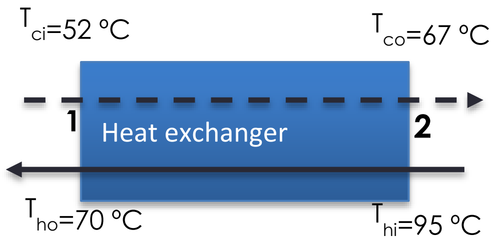
1. The **heat** exchanged
2. The **mass flow rate** of the hot fluid
3. The **area** of heat exchange

1st) To solve it, we first draw a **diagram** in which we show the inlet temperatures of each fluid

Case a



Case b



2nd) We shall calculate the **heat (q)** necessary to **absorb the juice** for pasteurization by means of an energy balance in which we shall use the sensible heat formula:

$$q = m_z C_{pz}(T_{co} - T_{ci}) = 5820\ kJ/s$$

The value of this heat will be the same as that which is transferred by the hot fluid. Therefore, by using the equation corresponding to Figure 4 in each case, we will be able to work out the flow rate of the hot fluid.

3rd) Obtain the **hot fluid mass flow rate**

- In case a (overheated steam): $\boldsymbol{m_v} = \frac{q}{\Delta H^v} = \mathbf{2.71\ kg/s}$
- In case b (hot water): $\boldsymbol{m_w} = \frac{q}{(T_{hi} - T_{ho})} = \mathbf{55.6\ kg/s}$

4th) We shall work out the **area** perpendicular to the transport through the expression of the heat transfer rate: $q = UA\Delta T_{ml}$. To that end, we will have to obtain the ΔT_{ml} in each case, considering randomly selected positions 1 and 2 in the heat exchangers.

- In case a: $\Delta T_{ml} = \frac{(T_{hi} - T_{co}) - (T_{ho} - T_{ci})}{ln\left(\frac{T_{hi} - T_{co}}{T_{ho} - T_{ci}}\right)} = \frac{(150-67)-(150-52)}{ln\left(\frac{150-67}{150-52}\right)} = 90.3\ °C$
- In case b: $\Delta T_{ml} = \frac{(T_{hi} - T_{co}) - (T_{ho} - T_{ci})}{ln\left(\frac{T_{hi} - T_{co}}{T_{ho} - T_{ci}}\right)} = \frac{(95-67)-(70-52)}{ln\left(\frac{95-67}{70-52}\right)} = 22.63\ °C$

Thus, the area in case a would be **0.13 m²** and in case b, **0.51 m²**

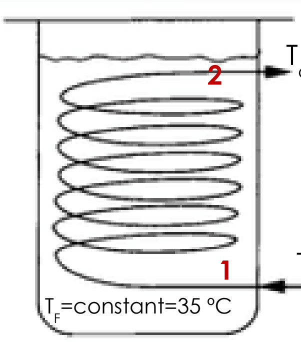
3.2 Coil coolers or heaters in tanks

The tanks that are used in some reactions (bioreactors), such as fermentation, the elaboration of beer and wine or even in the production of medicines, frequently need to maintain the medium at an optimal temperature, a temperature at which the activity of the microorganisms or enzymes can develop in the best possible way. (Alva, 2016). However, the reaction process itself leads to the release or consumption of thermal energy associated with the chemical changes; these cause the temperature of the reaction medium to rise or fall, slowing down or even halting the process. In order to avoid this problem, one widely-used strategy is to install coil coolers or heaters in the tank itself. These devices consist of coiled tubes through which a cooling or heating fluid flows; this fluid is frequently water. The required length and area of the coils may be achieved by combining the calculations obtained by means of the aforementioned heat transfer equation ($q = UA\Delta T_{ml}$) with those derived from the energy balances, both in the middle of the fluid circulating inside the coils and in the fluid that is to be maintained at a steady temperature, which would be equivalent to the hot or cold fluid of the classic exchangers. In the equation $q=UA\Delta T_{ml}$, the variation in temperature with respect to the fluid surrounding it is going to be different along the whole length of the coils.

Let's look at an example.

A **fermenter** that is used for producing an antibiotic must be maintained at **35 °C**. After taking into account the oxygen required by the organism and the agitator's heat dissipation, the maximum **heat transfer rate** is calculated as **550 kW**. The **cooling water** is at **10 °C** and the outlet temperature, calculated via an energy balance, is **25 °C.** The heat transfer coefficient for the fermentation broth is 2150 W/m^2°C and 14000 W/m^2°C for the cooling water. A helical cooling coil is to be installed inside the fermenter. The external diameter of the tube is 8cm, the thickness 5mm and the heat conductivity of the steel is 60 W/m°C. **What length of coil is required?**

1st) Draw a **diagram** in which we show the inlet temperatures of the cooling fluid, as well as the temperature of the fermenter. Also show the remaining data.



2nd) Apply the equation $q=UA\Delta T_{ml}$. To this end, it will be necessary to calculate the total resistance to heat transfer ($1/UA_{reference}$), since the length of the coil (L), which is the unknown variable, will be part of its/the equation's development. We will have to calculate the ΔT_{ml} and clear the L because q is datum.

$$\frac{1}{UA_{ref}} = \sum \frac{1}{hA_i} + \sum \frac{thickness_j}{k_j A_j} = \frac{1}{h_{int}A_{int}} + \frac{1}{h_{ext}A_{ext}} + \frac{thickness_{steel}}{K_{steel}A_{ml}}$$

In terms of the thermal resistance in convection ($\sum \frac{1}{hA_i}$), we will consider the lateral area of a cylinder ($2\pi rL$). As regards the resistance due to the conductivity of the steel coil, we shall have to bear in mind the logarithmic mean area ($A_{ml} = \frac{2\pi L(r_{ext}-r_{int})}{ln\left(\frac{r_{ext}}{r_{int}}\right)}$); this will be used since the area perpendicular to the transport will depend on the tube's radius

Thus, the total resistance will be:

$$\frac{1}{UA_{ref}} = \frac{1}{h_{int}2\pi L r_{int}} + \frac{1}{h_{ext}2\pi L r_{ext}} + \frac{ln\left(\frac{r_{ext}}{r_{int}}\right)}{k_{steel}2\pi L}$$

Substituting the variables for their numerical values, we get:

$$\frac{1}{UA_{ref}} = \frac{0.00253}{L}$$

Next, we shall calculate the mean temperature difference between the coil's cooling fluid and the medium in the fermenter. To that end, let's assume positions 1 and 2 shown in red in the diagram of the fermenter. By applying the formula ΔT_{ml}:

$$\Delta T_{ml} = \frac{\Delta T_2 - \Delta T_1}{ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = \frac{(T_F - T_{co}) - (T_F - T_{ci})}{ln\left(\frac{T_F - T_{co}}{T_F - T_{ci}}\right)} = \frac{(T_{ci} - T_{co})}{ln\left(\frac{T_F - T_{co}}{T_F - T_{ci}}\right)} = 16.37\ °C$$

Lastly, we have to clear the length, substituting the values of q, $1/(UA_{ref})$ and ΔT_{ml} in the equation $q = UA_{ref}\Delta T_{ml}$

$$\boldsymbol{L} = \frac{0.00253 \cdot q}{\Delta Tml} = \mathbf{85}\ \boldsymbol{m}$$

4 Closing statement

In this article, we have seen in which situations it is necessary to make use of the logarithmic mean temperature difference in order to conform to reality and design the systems used to heat or cool fluids with greater precision. To that end, we have combined the equation which allows us to predict the heat transfer rate with the expressions derived from the energy balances so as to determine the energy that the fluids involved in heat exchangers require when absorbing or transferring heat (q).

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