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ORIGINAL PAPER

Geometric overlapping coefficients for calculating the required emitters per plant in drip irrigation

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Abstract

The designer of irrigation systems must consider a complex combination of emitter type, emitter uniformity, hydraulics, topography, desired water distribution, crop salt tolerance, water requirements, water quality, fertilizer injection, soil salinity, cultural practices, and other site-specific conditions. In contrast to the approaches applied for the hydraulic design of irrigation installations, there is not a clear, general and consolidated design criterion for calculating the number required emitters per plant. In most cases, given the wide spectrum of possible scenarios, only guideline recommendations can be found, and the final decision is often based on the subjective experience of the designer or grower. This paper aims at revising, clarifying and refining the existing published guidelines and methodologies for estimating the required emitters per plant in drip irrigation, focussing on the Montalvo approach. The agronomic design should satisfy, among others, two specific conditions: (i) the emitters should wet at least a minimum threshold of the soil area (or volume) corresponding to the plant for ensuring a proper development of the roots; (ii) overlapping between emitter bulbs is required for merging wetted volumes and avoiding salt concentration near the root zone. Relying on this basis, a thorough theoretical geometric analysis of the overlapping between wet bulbs of contiguous emitters is carried out. As a result, Montalvo's overlapping coefficients are deduced here. This author assumes an identical net wetted area for all emitters in the laterals, but it can be stated that the overlapping areas between emitters differ in extreme emitters and interior emitters, as well as in configurations with one lateral per plant row and two laterals per plant row. Therefore, this study proposes new formulations for the computation of the overlapping coefficient, which need to incorporate the number of emitters as an additional variable, as well as to distinguish between the presence of one or two laterals per plant row, and between grouped and non-grouped emitters. In one lateral per plant row, the original overlapping coefficient underestimates the net wetted area by one emitter and thus overestimates the theoretical number of required emitters. In the case of two laterals per plant row, the original overlapping coefficient overestimates the net wetted area in the interior emitters, and thus underestimates the theoretical number of required emitters per plant. The presented formulations are applied in different practical examples covering a wide range of scenarios. The results allow a general overview of the influence of the soil type, the emitter flow rate, and the selected overlapping ratio in the number of required emitters per plant. The revision of guidelines and methods presented here, complemented with other experimental results and models of soil water dynamics under drip irrigation, might contribute to a better decision making of designers and field engineers.

List of symbols

a	Overlapping ratio (percent) for 1 lateral per plant row
a_1	Overlapping ratio (percent) along emitters of the same lateral
a_2	Overlapping ratio (percent) between laterals
Ae	Area wetted per emitter
AneT_{1L}	Total net area wetted by all the emitters of a group
AneT_{2LA}	Total net area wetted by all the emitters of a group with two laterals per plant row and one single overlapping area

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$\text{AneT}_{2\text{LB}}$	Total net area wetted by all the emitters of a group with two laterals per plant row and two different overlapping areas
Ap	Soil area of the plant or shading area of the plant
As	Overlapping area for one lateral per plant row
As_1	Overlapping area along emitters of the same lateral
As_2	Overlapping area between emitters from different laterals
C	Mathematical operator
Dc	Diameter of plant canopy
De	Diameter of wet bulb
Kp	Overlapping coefficient, general meaning
KpM	Overlapping coefficient proposed by Montalvo (2003)
$\text{KpM}_{2\text{LA}}$	Overlapping coefficient KpM generalized for two laterals per plant row and a single overlapping area
$\text{KpM}_{2\text{LB}}$	Overlapping coefficient KpM generalized for two laterals per plant row and two different overlapping areas
$\text{Kp}'_{1\text{L}}$	Overlapping coefficient for one lateral per plant row and grouped of emitters based on n
$\text{Kp}'_{2\text{LA}}$	Overlapping coefficient for grouped emitters in two laterals per row of plants and a single overlapping area based on n
$\text{Kp}'_{2\text{LB}}$	Overlapping coefficient for grouped emitters in two laterals per row of plants and two different overlapping areas based on n
n	Theoretical number of emitters required per plant
n_1	Theoretical number of emitters
n_2	Rounded number of emitters
n_3	Adopted number of emitters
P	Target wetting percent
q	Nominal flow rate of the installed emitter
r	Wet bulb radius
s	Overlapping length
Se	Theoretical required spacing between emitters
θ	Overlapping angle for one lateral per plant row
θ_1	Overlapping angle along emitters of the same lateral
θ_2	Overlapping angle between emitters from different laterals

Introduction

A suitable design and management of irrigation installations can decisively contribute to improve water use efficiency in agriculture. Microirrigation is the frequent application of small quantities of water on or below the soil surface as drops, tiny streams or miniature spray though emitters or applicators placed along a water delivery line. It encompasses a number of methods or concepts; such as bubbler, drip, trickle, mist or spray and subsurface irrigation (ASAE 1988 R2019). In microirrigation, also called localized irrigation, only a part of the soil volume is wetted, and this feature has implications concerning evaporation, transpiration, deep percolation, soil water, nutrient and salinity distributions with respect to crop spatial position and root distributions (Pizarro 1996).

System goals established with defined objectives, system constraints, and desired outcomes might be required for the selection and placement of specific components, sizing and layout of the distribution system, and the development of appropriate operational guidelines and procedures (Clark et al. 2007). For Ayars et al. (2007), the system designer must take into account a complex combination of emitter type, emitter uniformity, hydraulics, topography, desired water distribution uniformity, crop salt tolerance, water requirements, water quality, fertilizer injection, soil salinity, cultural practices, and other site-specific conditions.

The design of the irrigation subunits can be split, among others, in two different stages: the hydraulic design and the agronomic design (Montalvo 2003). The first one involves the determination of parameters such as the pipe diameters, and the required pressure at the beginning of the subunit. The criteria used to determine the mentioned parameters seek basically a uniform delivery of flow rate among emitters, and an average delivered flow rate near the nominal one. The analysis of the hydraulic performance in pipe networks has been amply addressed. Essentially, an energy balance is used to assess the uniformity of flow rate delivery, mainly influenced by head losses and level differences, and the effect of emitter manufacturing homogeneity. Therefore, in the last few decades, many models have been developed for the estimation of friction and local head losses based on very different approaches (Wang and Chen 2020).

However, before tackling the hydraulic design of the subunit, the emitter must already have been selected, i.e. its pressure-flow rate curve must be known. The exponent of the emitter pressure-flow rate curve, or the compensation range in compensating emitters, allows the definition of the maximum allowable pressure variation in the subunit for a given maximum predefined desirable flow rate variation among emitters. Moreover, the flow rate per stretch can be calculated based on the emitter nominal flow rate. Further, the specific position of the emitters in the plot must be also previously defined, i.e. the number of emitters per plant, the separation between emitters in the lateral, and the separation between laterals. The position of the emitters in the plot allows the calculation of the pipe lengths, which is needed for estimating head losses and, eventually, level differences between emitters. And these parameters are defined in the preliminary stage called agronomic design.

The agronomic design requires preliminary knowledge about available emitter designs in pipes. Emitter designs are in general classified into two types, point-source and line-source. In line-source emitters water is discharged, two-dimensionally, from closely spaced perforations, emitters or a porous wall along the lateral line. On the other hand, in point-source emitters, water is discharged, three-dimensionally, from emission points that are individually and relatively widely spaced, usually over 1 m. Multiple-outlet emitters discharge water at two or more emission points (ASAE 1988 R2019). According to Ayars et al. (2007), point-source emitters are spaced generally 0.76 to 1 m apart or according to wider plant spacing arrangements. Point-source emitters are typically used for widely spaced plants such as trees, vines, ornamentals, and shrubs. However, some point-source emitters are also used for closely spaced row crops. Bubbler and microsprinkler emitters are usually classified as point-source systems. Line-source emitters have perforations, holes, porous walls, formed indentations, or molded emitter inserts in the tubing that discharge water at close spacings (0.1 to 0.6 m). Line-source emitter systems are frequently used on small fruits, vegetables, or other closely spaced row crops.

In contrast to the methods applied in the hydraulic design, there is not a clear, general and consolidated design criterion for calculating the number of required emitters per plant. The main factors that determine the selection, spacing and flow rate of the emitters might be known, including plant spacing, plant rooting characteristics, soil texture, and lateral hydraulics (Waller and Yitayew 2016). But in most cases, only guideline recommendations can be found, and just for limited scenarios. Usually, vineyards surface drip irrigation systems have one dripline per vine row, but orchards may have two driplines per tree row (Bielorai 1985). According to Schwankl and Hanson (2007), paired laterals are usually placed slightly away from each side of the tree row (less than 1 m) to provide a larger wetted soil volume. This arrangement also keeps the tree crown dry minimizing the potential for disease. However, it might be difficult to achieve sufficient wetted soil volume with drip irrigation systems (Schwankl et al. 1999). Regarding emitter spacing, for vineyards, one or two plug-in drip emitters are used per vine, usually located within 0.5 m of the vine. For orchards, emitter spacing can be determined, among others, by peak water demand period and maximum acceptable daily operating time. According to Schwankl and Hanson (2007), surface drip irrigation systems should be designed to meet peak water demands with 12 to 15 h of operation. This design criterion allows the system to operate longer when needed to catch up on irrigation after a system is shut-down for repairs or cultural operations. Further, other factors influencing emitter spacing are soil conditions and wetted soil volume conditions. It is desirable to wet at least 40% of the orchard floor with the drip irrigation system. Therefore, emitters should be spaced so that the wetted volume of one emitter overlaps with the adjacent emitter along the drip lateral.

Few studies investigated the effect of dripline and emitter spacing in surface drip irrigation systems on crop yield. Plaut et al. (1988) found no trend in cotton yield for different ratios of dripline spacing to row spacing, with emitter spacing of 0.4 m, and emitter discharge rate of 1.4 l/h. Ayars et al. (1985) considered line spacings of 1.5 m and 2.5 m for cotton in a clay loam, with emitter spacing of 1 m and emitter discharge rate of 2 l/h, and found no yield differences. Emitter spacing of 0.5 m and emitter discharge rates of 2 l/h to 4 l/h led to similar results (e.g. Howell et al. 1987; Russo 1987; Bar-Yosef et al. 1989; Meiri et al. 1992). Regarding collapsible thin-walled emitting hose, no studies have addressed the effect of emitter spacing on row crop yields. Growers have relied on practical experience to determine the most appropriate spacing for their conditions. Accordingly, growers often use emitter spacing for annual crops ranging from 0.2 m to 0.45 m. In California (USA), an emitter spacing of 0.2 m is generally used for strawberry, but spacing of 0.3 m to 0.45 m is used for drip irrigation of vegetables (Schwankl and Hanson 2007). In this regard, it seems difficult to generate enough experimental evidence for drawing general conclusions or defining a general criterion, given the wide spectrum of possible scenarios. So, this decision is often based on the subjective experience of the designer or grower.

Other studies have assessed soil water dynamics under drip irrigation, and several models, mainly numerical, analytical and/or empirical, have been developed to determine the wetting pattern under drip irrigation systems, some of them including the effect of root-absorption (e.g. Al-Ogaidi et al. 2016; Karimi et al. 2020; Shiri et al. 2020; Ozgur et al. 2021; del Vigo et al. 2020, 2022, 2023). According to Schwartzman and Zur (1986), numerical solutions of two or three-dimensional soil water flow models call for detailed information of the physical properties of the soil and for an access to computers. As a result, the use of these methods by field engineers are conditioned by their substantial simplification. Thus, it seems difficult to provide simple models with accurate enough performance and wide generalizability, given the complexity of the problem. On the other hand, despite the soil wetting patterns might be currently simulated using numerical methods with the help of software packages eventually with user-friendly-interfaces (e.g. Šimůnek et al. 2006, 2016; Friedman et al. 2016), designers and field engineers tend to use, if possible, more straight and simple methods for decision making. Thus, a clear and simple general design criterion should be also desirable, and able to be complemented with the practical experience. In this regard, Schwartzman and Zur (1986) made an attempt to propose a simplified procedure for computing the optimal emitter spacing based on a preliminary estimation of the geometry of the wetted soil volume and the cost of the irrigation lateral. The optimal choice of emitter spacing should correspond to the minimal value of lateral cost, while the former should be calculated based on the amount of water to be applied in an irrigation event. The threshold volume per plant that should be wetted seems not to be an input variable of this approach.

Among others, the agronomic design should satisfy two specific conditions: (i) the emitters should wet at least a certain minimum threshold of the soil area/volume corresponding to the plant for ensuring a proper development of the roots, (ii) a certain overlapping between emitter bulbs is required for merging wetted volumes and avoiding salt concentration near the root zone. Based on such basis, a general procedure for estimating the required emitters per plant might be defined, if the plant soil area/volume that should be wetted and the soil area/volume wetted by an emitter could be estimated. This paper aims at revising, clarifying and refining the existing published guidelines and methodologies for estimating the required emitters per plant in drip irrigation based on such approach, mainly build upon the method described by Pizarro (1996), and further developed by Montalvo (2003), through the theoretical assessment of the overlapping between wet bulbs of emitters. This latter is deduced here and refined relying on a thorough geometric analysis of the overlapping between emitters in a horizontal plane. Finally, the proposed equations are applied to practical examples.

General criterion

Different studies (e.g. Atkinson 1983; Black and West 1974; Levin et al. 1979; Willoughby and Cockroft 1974) suggested that it might not be necessary to wet the complete soil area of the plant for ensuring a suitable root development. Accordingly, the general criterion for determining the required number of emitters per plant is based on wetting at least a minimum threshold of the soil area of the plant or of its root volume. There is no definitive recommendation regarding the amount of surface area of an orchard or vineyard floor that should be wetted, although one-third to one-half of the area was suggested (USDA-NRCS 1984). The wetted area seems to be less important in vineyards, where many successful drip irrigation systems wet no more than one-third of the vineyard surface (Schwankl and Hanson 2007).

Keller and Karmelli (1974) suggested to assess the wetted soil volume as a function of the total crop area (i.e. the planting frame). The ASAE standard EP405.1 (1988 R2019) defines the percent area wetted as a percent of the total crop area. Further, the wetted area is defined as the average irrigated soil area in a horizontal plane located at or below the emitter. Thus, this standard recommends that the area wetted, as a percent of the total crop area, may range from a low of 20% for widely spaced crops for irrigation in high rainfall regions, in agreement with Keller (1978), to a high of over 75% for row crops in low rainfall regions, in contrast to the range 33–50% proposed by Keller (1978). However, this criterion might provide different solutions for the same crop and canopy if it is planted under different spacings. To avoid such contradiction, Rodrigo et al. (1997) proposed that the wetted threshold area should be established as a function of the shading area of the crop, i.e. to the horizontal projection area of the canopy, and suggested a minimum threshold of 50%. In any case, an approximate suitable target area of the plant should be fixed, and this decision will depend on the designer.

In drip irrigated soils, the salts tend to be pushed to the edges of the wetted area, and rings of salt can be seen around emitters. If emitter spacing is slightly higher than the wetted diameter, then salts will concentrate between emitters. Thus, emitter spacing should be close enough to prevent salt accumulation between emitters (Waller and Yitayew 2016). According to these authors, to create a line source, the distances between emitters along inline tubing should be less than the wetted diameter (3/4 of the wetted diameter), and the wetted diameters published by Benami and Ofen (1983), respectively, for light, medium, and heavy soil types, and for 2, 4, and 8 l/h emitter flow rates are cited.

Montalvo (2003), based on the previous references, also proposed to limit the separation between two neighbouring emitters below a certain threshold to ensure a minimum overlapping between their wet bulbs to prevent the proliferation of salts in the root zone, and eventually problems of root development. Thus, according to this author, assuming a theoretical horizontal circular wetted area, the overlapping ratio can be defined as a function of the wet bulb radius as:

$$s = \frac{a \cdot r}{100} \tag{1}$$

where s is the overlapping length, a is the overlapping percent and r is the wet bulb radius. This author, based on accepted existing references, recommended a minimum overlapping percent of 15% and a maximum value of 50%. Higher values would result in excessive salt washout and would be uneconomical. From this definition it is possible to establish the theoretical separation required between two emitters, Fig. 1, to ensure a pre-set overlapping value:

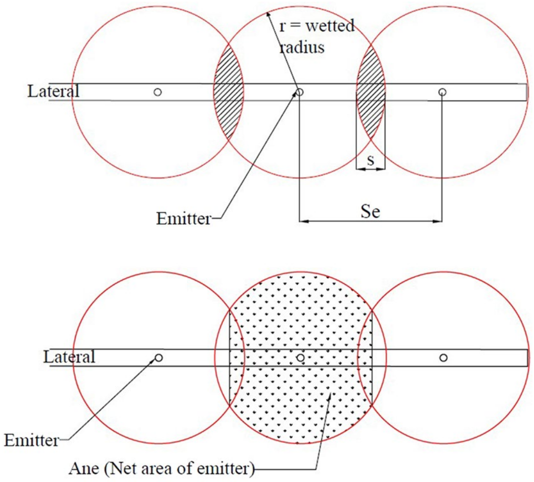

Fig. 1 Overlapping area between neighbouring emitters in one lateral per plant row

$$Se = 2 \cdot r - s = 2 \cdot r - \frac{a \cdot r}{100} = r \cdot \left(2 - \frac{a}{100}\right) \tag{2}$$

where Se is the theoretical required spacing between emitters. Se can be rounded to a commercial spacing (Sad). In this case *a* should be recalculated.

Thus, a general procedure for estimating the required emitters per plant might be defined, if the plant area that should be wetted and the area wetted by an emitter could be estimated. A similar approach could be based on a volume basis. Accordingly, if the area basis is chosen, the procedure consists of dividing the target threshold soil area of the plant by the area wetted by an emitter:

$$n > \frac{\mathrm{Ap} \cdot P}{100 \cdot \mathrm{Ae}} \tag{3}$$

where *n* is the theoretical number of emitters required per plant, *Ap is* the soil area of the plant, *P* is the target wetting percent, and Ae is the area wetted per emitter. Ae could be considered the plane at a certain depth where the wet bulb reaches its maximum width. According to the mentioned recommendations, Ap would correspond to the shaded area of the plant and *P* would be 50 (dimensionless) if the recommendation of Rodrigo et al. (1997) is adopted. A similar threshold can be defined from the basis of the planting frame using a different percent (e.g. ASAE 1988 R2019). In any case the designer must fix an approximate suitable threshold. The shaded area would be calculated as a circular projection of the canopy into the soil, i.e. the area of a circumference with canopy average radius. Subsequently, *n* is rounded up and it is verified that the calculated spacing between emitters, to ensure a specific overlapping, allows their placement in the lateral length corresponding per plant, given by the planting spacing. If this is not the case, two laterals per row of plants must be used. Even if there is enough spacing for n in one lateral, the designer might decide to fix likewise 2 laterals, due to other factors (e.g. hypothetically better development of the root system). This procedure based on Eq. 3 might be thought for trees, where Ap would be large enough to justify more than one emitter.

According to Schwartzman and Zur (1986), in point source emitters, the geometry of the wetted soil volume at the end of an irrigation depends on the soil type, emitter discharge, and the total amount of water in the soil. Further, the geometry of the wetted soil volume is best described by the depth of the wetting, and by the diameter of the wetted soil volume, measured at its widest point, De. To determine Ae or *r*, there might be, among others, three general options: (i) on-site experimental determination (recommended), (ii) to obtain an approximate value from the literature (e.g. Benami and Ofen 1983; Waller and Yitayew 2016), or (iii) to calculate it from empirical formulas. These formulas (e.g. Karmeli et al. 1985; Schwartzman and Zur 1986) are defined for different soil types (e.g. light, medium, heavy, or eventually include the hydraulic conductivity of the soil as input), and usually rely on the nominal flow rate of the installed emitter and/or from the total amount of water in the soil. Alternatively, other more complex models might be applied, eventually with the help of software with user-friendly-interfaces (e.g. Šimůnek et al. 2006, 2016; Friedman et al. 2016) to determine the wetting pattern caused by the emitter in the soil (e.g. del Vigo et al. 2023), and thus estimate r or Ae. In any case, given that the volume of the wet bulb will evolve in time, an average probable *r* should be assigned. Given that the approach relies on estimating a wetted area per emitter and not a wetted volume, the depth of the generated bulb by the chosen emitter should be enough to properly cover the root depth of the plant. According to Schwartzman and Zur (1986), the depth and diameter of the wetted soil volume can be controlled by the proper selection of the emitter discharge.

Pizarro (1996) assumes that Ae adopts a circular area and does not subtract the overlapping area when there are two or more emitters per plant. Montalvo (2003) introduces the concept of net wetted area of the emitter (Ane) for a theoretical calculation of the number of required emitters. Ane takes into account the reduction in the wetted area due to the overlapping between emitters when there are two or more emitters per plant. Further, the concept of overlapping coefficient (Kp) is introduced to estimate Ane as:

$$\mathrm{Ane} = \mathrm{Ae} \cdot \mathrm{Kp} \tag{4}$$

Thus, the theoretical number of required emitters per plant is:

$$n > \frac{\mathrm{Ap} \cdot P}{100 \cdot \mathrm{Ane}} = \frac{\mathrm{Ap} \cdot P}{100 \cdot \mathrm{Ae} \cdot \mathrm{Kp}} \tag{5}$$

Montalvo (2003) defines Kp as the ratio between the net area wetted by an emitter and the total wetted area, and just provides a table with the Kp coefficients as a function of the overlapping ratio. However, the author does not explain explicitly how Ane is calculated. It is noteworthy that, according to this author, Kp does not depend on the number of emitters considered, nor on the diameter of the wet bulb, nor on the number of laterals per plant row.

The following sections present a mathematical development to estimate the required number of emitters per plant through a detailed geometrical analysis of the horizontal overlapping area between emitters under different configurations. Therefore, the estimation of the net soil area wetted by the emitters in each scenario is required.

Analytical calculation of Montalvo's overlapping coefficient Kp

To obtain the Kp values provided by Montalvo (2003) the value of Ane must be defined, for which the value of the overlapping area between two emitters must be calculated. As starting simplifying hypothesis, it must be assumed that the area wetted by an emitter is circular, and that the overlapping area per emitter between two emitters corresponds to the area of a circular segment, calculated as the area of a circular sector minus that of the adjacent triangular portion, Fig. 2:

$$\mathrm{As} = \frac{r^2}{2}(\theta - \sin(\theta)) \tag{6}$$

where As is the overlapping area, r is the wet bulb radius and θ is the overlapping angle.

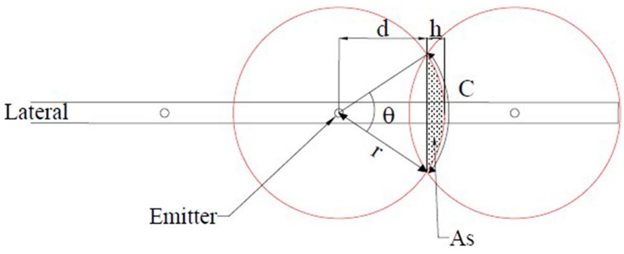

Fig. 2 Area of a circular segment

Next, θ is expressed as a function of the overlapping percent. Taking into account that h is half of the total overlapping length s, and introducing Eq. 1:

$$\theta = 2\arccos\left(\frac{d}{r}\right) = 2\arccos\left(\frac{r-h}{r}\right) = 2\arccos\left(\frac{r - \frac{s}{2}}{r}\right) = 2\arccos\left(\frac{r - \frac{a \cdot r}{2 \cdot 100}}{r}\right)$$

And simplifying:

$$\theta = 2\arccos\left(1 - \frac{a}{200}\right) \tag{7}$$

To provide his Kp coefficients, Montalvo (2003) always assigns to each emitter the same net area, independent of the number of emitters and the number of laterals. Specifically,

$$\mathrm{Ane} = \mathrm{Ae} - 2 \cdot \mathrm{As} \tag{8}$$

This is equivalent to assign to all the emitters the overlapping that exists when each emitter has two neighbouring emitters (interior or non-extreme) on a single lateral, Fig. 1. Therefore, the expression of Kp will be:

$$\mathrm{Kp} = \frac{\mathrm{Ae} - 2\mathrm{As}}{\mathrm{Ae}} = \frac{\pi r^2 - 2 \cdot \left(\frac{r^2}{2}(\theta - \sin(\theta))\right)}{\pi r^2} = 1 - \left(\frac{\theta - \sin(\theta)}{\pi}\right) \tag{9}$$

Or representing θ as a function of a, substituting Eq. 7:

$$\mathrm{KpM} = 1 - \left(\frac{2\arccos\left(1 - \frac{a}{200}\right) - \sin\left(2\arccos\left(1 - \frac{a}{200}\right)\right)}{\pi}\right) \tag{10}$$

which can be written as

$$\mathrm{KpM} = 1 - C \tag{11}$$

where C is the operator

$$C = \left(\frac{2\arccos\left(1 - \frac{a}{200}\right) - \sin\left(2\arccos\left(1 - \frac{a}{200}\right)\right)}{\pi}\right) \tag{12}$$

This equation leads to the overlapping coefficients published by Montalvo (2003), called KpM from now on in this paper. In view of Eqs. 9 and 10, it can be stated that, assuming that the area wetted by an emitter is circular, KpM is independent of the dimensions of the wet bulb, i.e. of r. The calculation of C might be simplified through an estimated value using a potential regression equation between a and C values ($R^2 = 1$), Fig. 3, namely:

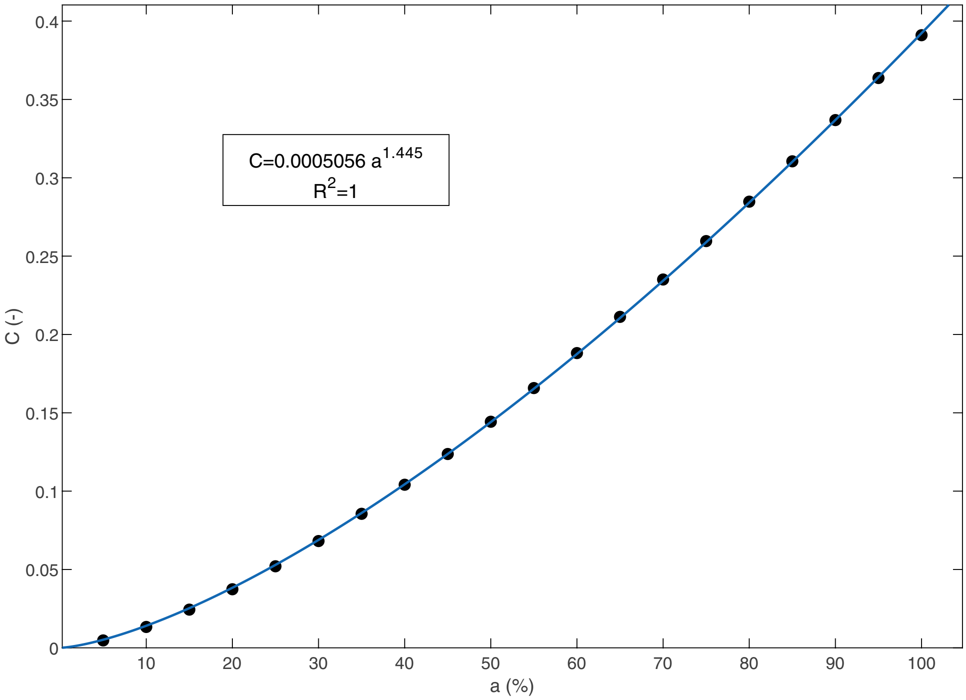

Fig. 3 Alternative function for operator C

$$C = 0.0005056 \cdot a^{1.445} \tag{13}$$

When there is one lateral per plant row, the definition proposed by Montalvo (2003) does not take into account that the extreme emitters have a higher Ane (As would be subtracted only once). The proposed KpM would underestimate the total Ane and overestimate the theoretical number of required emitters. On the other hand, when there are two laterals per plant row, there is an additional overlapping between lateral lines, and the proposed KpM would overestimate the Ane of non-extreme (or interior) emitters, and this would underestimate the theoretical number of required emitters.

Generalization of Montalvo's overlapping coefficient for two laterals per plant row

If a second lateral per row of plants is introduced, the calculation of Kp should be different, because additional As take place, due to the overlapping between neighbouring emitters from the second lateral. This section presents a similar approach to Montalvo's KpM, i.e. independent of the number of emitters, but incorporating the effect of and additional overlapping area. According to Montalvo's approach, the new Kp will assign to all emitters the same net area to simplify the calculation procedure, and it will be omitted that the extreme emitters present a higher Ane than the interior emitters.

The analytical calculation of the new Kp is similar to the adopted in the previous section, with the difference that an additional As must be subtracted from Ae. There might be two scenarios, Fig. 4, depending if the overlapping area between neighbouring emitters along a lateral line is the same as the overlapping area between neighbouring emitters from different lateral lines.

One overlapping ratio between emitters

In this case, it is assumed that the spacing between neighbouring emitters is the same along one lateral line and between laterals, i.e. the spacing of the lateral lines is the same as the spacing between emitters along the lateral. It must be remembered that, theoretically, the overlapping between emitters is fixed for preventing salt concentration in the root zone. The overlapping between driplines should, additionally, not interfere with plot operations. In this case, as stated above, an additional As should be subtracted from Ae to consider the additional overlapping between laterals. Therefore, according to the approach of Montalvo (2003):

$$\mathrm{Ane} = \mathrm{Ae} - 3 \cdot \mathrm{As} \tag{14}$$

This is equivalent to assign to all the emitters the overlapping that exists when each emitter has three neighbouring emitters (two from the same lateral and one from the other lateral). This approach neglects eventual diagonal overlapping from other emitters from the second lateral. Thus:

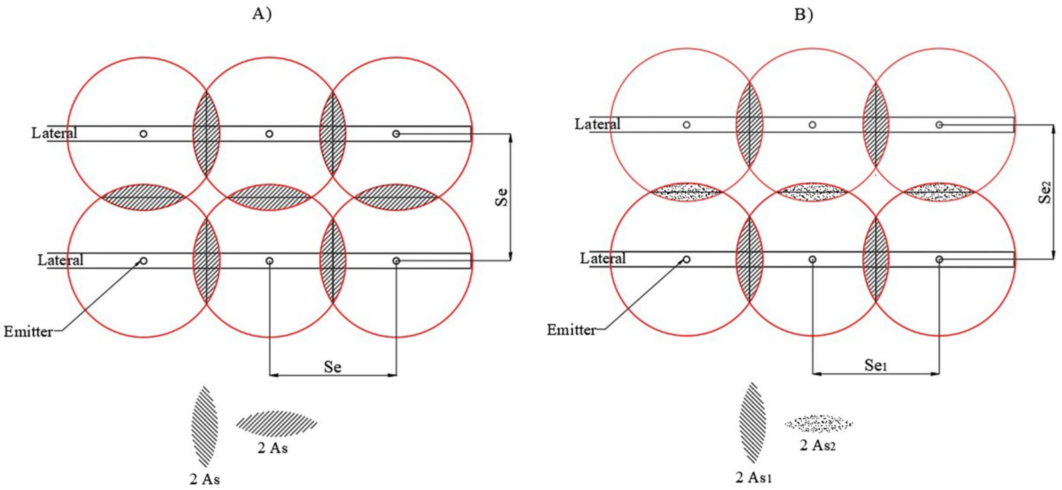

Fig. 4 Overlapping area between neighbouring emitters in two laterals per plant row. **A** One single overlapping area, **B** two different overlapping areas

$$\mathrm{KpM}_{2\mathrm{LA}} = \frac{\mathrm{Ae} - 3\mathrm{As}}{\mathrm{Ae}} = \frac{\pi r^2 - 3 \cdot \left(\frac{r^2}{2}(\theta - \sin(\theta))\right)}{\pi r^2} = 1 - \frac{3}{2}\left(\frac{\theta - \sin(\theta)}{\pi}\right) \tag{15}$$

where $\mathrm{KpM}_{2\mathrm{LA}}$ is the overlapping coefficient independent of n for two laterals per plant row when there is a single constant overlapping area for all emitters. Or representing θ as a function of a, substituting Eq. 7:

$$\mathrm{KpM}_{2\mathrm{LA}} = 1 - \frac{3}{2}\left(\frac{2\arccos\left(1 - \frac{a}{200}\right) - \sin\left(2\arccos\left(1 - \frac{a}{200}\right)\right)}{\pi}\right) \tag{16}$$

Or

$$\mathrm{KpM}_{2\mathrm{LA}} = 1 - \frac{3}{2}C \tag{17}$$

Two different overlapping ratios between emitters

In this case, it is assumed that the spacing between neighbouring emitters along one lateral line is different than between neighbouring emitters from different laterals, i.e. the spacing of the lateral lines is not the same as the spacing between emitters along the lateral. This scenario requires the definition of 2 different overlapping areas, namely one between emitters along the same lateral, As_1, and one between neighbouring emitters from different laterals, As_2. Further, this involves the definition of the two corresponding overlapping angles, namely θ_1 and θ_2, and the two corresponding overlapping ratios, namely a_1 and a_2, respectively. The overlapping between driplines should, additionally, not interfere with plot operations. Thus, the total net area is:

$$\mathrm{Ane} = \mathrm{Ae} - 2 \cdot \mathrm{As}_1 - \mathrm{As}_2 \tag{18}$$

Again, this is equivalent to assign to all the emitters the overlapping that exists when each emitter has three neighbouring emitters (two from the same lateral and one from the other lateral). This approach neglects eventual diagonal overlapping from other emitters of the second lateral. Thus:

$$\mathrm{KpM}_{2\mathrm{LB}} = \frac{\mathrm{Ae} - 2 \cdot \mathrm{As}_1 - \mathrm{As}_2}{\mathrm{Ae}} = \frac{\pi r^2 - 2 \cdot \left(\frac{r^2}{2}\left(\theta_1 - \sin\left(\theta_1\right)\right)\right) - \left(\frac{r^2}{2}\left(\theta_2 - \sin\left(\theta_2\right)\right)\right)}{\pi r^2} = 1 - \left(\frac{\theta_1 - \sin\left(\theta_1\right)}{\pi}\right) - \frac{1}{2}\left(\frac{\theta_2 - \sin\left(\theta_2\right)}{\pi}\right) \tag{19}$$

where $\mathrm{KpM}_{2\mathrm{LB}}$ is the overlapping coefficient independent of n for two laterals per plant row when there are two different overlapping areas between emitters. Or representing θ as a function of a:

$$\mathrm{KpM}_{2\mathrm{LB}} = 1 - \left(\frac{2\arccos\left(1 - \frac{a_1}{200}\right) - \sin\left(2\arccos\left(1 - \frac{a_1}{200}\right)\right)}{\pi}\right) - \frac{1}{2}\left(\frac{2\arccos\left(1 - \frac{a_2}{200}\right) - \sin\left(2\arccos\left(1 - \frac{a_2}{200}\right)\right)}{\pi}\right) \tag{20}$$

Or

$$\mathrm{KpM}_{2\mathrm{LB}} = 1 - C_1 - \frac{1}{2}C_2 \tag{21}$$

where C_1 and C_2 are the operators corresponding to the overlappings a_1 and a_2

Actual overlapping coefficient for one lateral per plant row and grouped emitters

In this section an alternative Kp is obtained for grouped emitters in a single lateral per row of plants, taking into account that Ane is different between extreme and interior emitters. Therefore, in the length of the lateral corresponding to each plant, it is necessary to count the number of As as a function of the number of emitters per group. Thus, As is subtracted one time from the total wetted area of the extreme emitters, while 2As are subtracted from the total wetted area of the rest of emitters. So,

$$\mathrm{AneT}_{1\mathrm{L}} = n \cdot \mathrm{Ae} - 2 \cdot \mathrm{As} \cdot (n - 1) \tag{22}$$

where $\mathrm{AneT}_{1\mathrm{L}}$ corresponds to the total net area wetted by all the emitters of a group, and n is the number of emitters per group. Thus, the alternative definition of Kp would be:

$$\mathrm{Kp}'_{1\mathrm{L}} = \frac{\mathrm{AneT}_{1\mathrm{L}}}{n \cdot \mathrm{Ae}} = \frac{n \cdot \mathrm{Ae} - 2 \cdot \mathrm{As} \cdot (n - 1)}{n \cdot \mathrm{Ae}} = 1 - \frac{2 \cdot \mathrm{As} \cdot (n - 1)}{\mathrm{Ae}} \tag{23}$$

Replacing As using Eq. 6:

Table 1 Comparison between KpM and $\mathrm{Kp}'_{1\mathrm{L}}$ for different overlapping ratios and number of emitters per group

a (%)	KpM (–) Montalvo (2003)	$\mathrm{Kp}'_{1\mathrm{L}}$ (–)						
		$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=10$
5	0.995	0.998	0.997	0.997	0.996	0.996	0.996	0.996
10	0.987	0.993	0.991	0.990	0.989	0.989	0.989	0.988
15	0.976	0.988	0.984	0.982	0.981	0.980	0.979	0.978
20	0.963	0.981	0.975	0.972	0.970	0.969	0.968	0.966
25	0.948	0.974	0.965	0.961	0.958	0.957	0.955	0.953
30	0.932	0.966	0.955	0.949	0.946	0.943	0.942	0.939
35	0.915	0.957	0.943	0.936	0.932	0.929	0.927	0.923
40	0.896	0.948	0.931	0.922	0.917	0.913	0.911	0.906
45	0.876	0.938	0.918	0.907	0.901	0.897	0.894	0.889
50	0.856	0.928	0.904	0.892	0.885	0.880	0.876	0.870
55	0.834	0.917	0.890	0.876	0.867	0.862	0.858	0.851
60	0.812	0.906	0.875	0.859	0.850	0.843	0.839	0.831
65	0.789	0.894	0.859	0.842	0.831	0.824	0.819	0.810

Table 2 Percent variation of $\mathrm{Kp}'_{1\mathrm{L}}$ vs. KpM for different overlapping ratios and number of emitters per group

a (%)	% variation						
	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$	$n=10$
5	0.3	0.2	0.2	0.1	0.1	0.1	0.1
10	0.6	0.4	0.3	0.2	0.2	0.2	0.1
15	1.2	0.8	0.6	0.5	0.4	0.3	0.2
20	1.8	1.2	0.9	0.7	0.6	0.5	0.3
25	2.7	1.8	1.4	1.0	0.9	0.7	0.5
30	3.5	2.4	1.8	1.5	1.2	1.1	0.7
35	4.4	3.0	2.2	1.8	1.5	1.3	0.9
40	5.5	3.8	2.8	2.3	1.9	1.6	1.1
45	6.6	4.6	3.4	2.8	2.3	2.0	1.5
50	7.8	5.3	4.0	3.3	2.7	2.3	1.6
55	9.1	6.3	4.8	3.8	3.2	2.8	2.0
60	10.4	7.2	5.5	4.5	3.7	3.2	2.3
65	11.7	8.1	6.3	5.1	4.2	3.7	2.6

$$\mathrm{Kp}_{1\mathrm{L}}^{'} = 1 - \frac{2 \cdot \frac{r^2}{2}(\theta - \sin(\theta)) \cdot (n-1)}{n \cdot \pi r^2} = 1 - \frac{(n-1) \cdot (\theta - \sin(\theta))}{n \cdot \pi} \tag{24}$$

Or substituting θ as a function of a, with Eq. 7:

$$\mathrm{Kp}_{1\mathrm{L}}^{'} = 1 - \frac{(n-1) \cdot \left(2\arccos\left(1 - \frac{a}{200}\right) - \sin\left(2\arccos\left(1 - \frac{a}{200}\right)\right)\right)}{n \cdot \pi} \tag{25}$$

where $\mathrm{Kp}'_{1\mathrm{L}}$ is the overlapping coefficient for one lateral per plant row and grouped of emitters, a is the overlapping ratio (percent), and n is the number of emitters per group. It should be remembered that n should refer to the number of emitters per plant, not to the total number of emitters on the lateral. Or simplifying

$$\mathrm{Kp}_{1\mathrm{L}}^{'} = 1 - \frac{(n-1)}{n}C \tag{26}$$

A comparison between KpM and $\mathrm{Kp}'_{1\mathrm{L}}$ values for different overlapping ratios and n values is shown in Table 1, while Table 2 presents the corresponding percent variation of $\mathrm{Kp}'_{1\mathrm{L}}$ in comparison to KpM. A maximum number of 10 emitters per plant/group were defined, due to the limitation of lateral length corresponding to each plant. This number depends on the spacing defined between emitters (Se, which can be rounded to a commercially spacing Sad). 10 emitters per plant in one lateral per row is rather a high number, which might correspond, for instance, to a spacing of 5 m between plants and a spacing of 0.5 m between emitters. This scenario just aims at defining a theoretical threshold for performing the comparison.

Given the definition of $\mathrm{Kp}'_{1\mathrm{L}}$, the differences with KpM decrease when increasing n, because the effect of the lower Ane at the 2 extreme emitters is reduced. As explained, n is limited by the planting spacings. Therefore, the effect of the lower Ane of the 2 extreme emitters could be relevant, in a higher degree when n is lower. A comparison of the values in the tables shows that the differences between KpM and $\mathrm{Kp}'_{1\mathrm{L}}$ become greater as the overlapping ratio increases. Since $\mathrm{Kp}'_{1\mathrm{L}}$ requires n as input variable for its calculation, and this is actually the target that wants to be determined once the overlapping coefficient is known, an iterative procedure must be adopted. As initial solution for n there might be two options: (i) calculating n based on KpM, or (ii) considering initially that there is no overlapping between emitters for calculating n. Another option is to use the overlapping coefficient exclusively to verify if the threshold wetted area is achieved, when n is already known.

If there is no grouping of emitters (i.e. the spacing between emitters is always constant along the lateral line, even between extreme emitters from neighbouring plants), KpM could be considered, except in the extreme plants of the row, where there would be an emitter with a higher Ane than the one considered. In this case, it would not make sense to make a specific calculation for the plants at the extremes, and the same solution would be adopted as for the interior plants.

Actual overlapping coefficient for two laterals per plant row and grouped emitters

As already mentioned, if a second lateral per row of plants is introduced, the calculation of Kp should be different, because additional *As* take place, due to the overlapping between neighbouring emitters from the second lateral. In comparison to the Ane proposed by Montalvo (2003), this is only different for interior emitters, where it gets reduced. Compared to the Ane of the previous section, it gets reduced for all emitters. The analytical calculation of the new Kp is similar to the adopted in the previous section, with the difference that the accumulated As count per group of emitters increases one As per emitter, Fig. 4. Again, there might be two scenarios depending if the overlapping between neighbouring emitters along a lateral line is the same as the overlapping between neighbouring emitters from different lateral lines. The overlapping between driplines should, additionally, not interfere with cultural operations.

One overlapping ratio between emitters

Following the same procedure as in the previous section, the total net area per group of emitters will be

$$\mathrm{AneT}_{2\mathrm{LA}} = 2 \cdot n \cdot \mathrm{Ae} - 2 \cdot \mathrm{As} \cdot [(n-1) \cdot 2 + n] \tag{27}$$

where $\mathrm{AneT}_{2\mathrm{LA}}$ corresponds to the total net wetted area per group and n is the number of emitters per lateral and plant group. So, the total number of emitters per group would be *2n*. Thus, Kp would be:

$$\begin{aligned}\mathrm{Kp}_{2\mathrm{LA}}^{'} &= \frac{\mathrm{AneT}_{2\mathrm{LA}}}{2 \cdot n \cdot \mathrm{Ae}} = \frac{2 \cdot n \cdot \mathrm{Ae} - 2 \cdot \mathrm{As} \cdot [(n-1) \cdot 2 + n]}{2 \cdot n \cdot \mathrm{Ae}} \\ &= 1 - \frac{\mathrm{As} \cdot [(n-1) \cdot 2 + n]}{n \cdot \mathrm{Ae}}\end{aligned} \tag{28}$$

where $\mathrm{Kp}'_{2\mathrm{LA}}$ is the overlapping coefficient for grouped emitters in two laterals per row of plants as a function of n when there is only a single overlapping ratio. Replacing As by Eq. 6:

$$\begin{aligned}\mathrm{Kp}_{2\mathrm{LA}}^{'} &= 1 - \frac{\frac{r^2}{2}(\theta - \sin(\theta)) \cdot [(n-1) \cdot 2 + n]}{n \cdot \pi r^2} \\ &= 1 - \frac{(\theta - \sin(\theta)) \cdot (3n-2)}{2 \cdot n \cdot \pi}\end{aligned} \tag{29}$$

Or replacing θ as a function of a, Eq. 7:

Table 3 Comparison between KpM, KpM$_{2L}$ and Kp$'_{2L}$ for different overlapping ratios and number of emitters per group

a (%)	KpM (–) Montalvo (2003)	KpM$_{2LA}$ (–)	Kp$'_{2LA}$ (–)				
			2n = 4	2n = 6	2n = 8	2n = 10	2n = 20
5	0.995	0.993	0.995	0.995	0.994	0.994	0.993
10	0.987	0.980	0.987	0.985	0.983	0.983	0.981
15	0.976	0.963	0.976	0.972	0.970	0.968	0.966
20	0.963	0.944	0.963	0.956	0.953	0.951	0.948
25	0.948	0.922	0.948	0.939	0.935	0.932	0.927
30	0.932	0.898	0.932	0.921	0.915	0.911	0.905
35	0.915	0.872	0.915	0.900	0.893	0.889	0.880
40	0.896	0.844	0.896	0.879	0.870	0.865	0.854
45	0.876	0.814	0.876	0.856	0.845	0.839	0.827
50	0.856	0.784	0.856	0.832	0.820	0.812	0.798
55	0.834	0.751	0.834	0.807	0.793	0.785	0.768
60	0.812	0.718	0.812	0.781	0.765	0.755	0.737
65	0.789	0.683	0.789	0.754	0.736	0.725	0.704

Table 4 Percent variation of Kp$'_{2L}$ vs. KpM for different overlapping ratios and number of emitters per group

a (%)	% variation				
	2n = 4	2n = 6	2n = 8	2n = 10	2n = 20
5	0	0.0	– 0.1	– 0.1	– 0.2
10	0	– 0.2	– 0.4	– 0.4	– 0.6
15	0	– 0.4	– 0.6	– 0.8	– 1.0
20	0	– 0.7	– 1.0	– 1.3	– 1.6
25	0	– 1.0	– 1.4	– 1.7	– 2.3
30	0	– 1.2	– 1.9	– 2.3	– 3.0
35	0	– 1.7	– 2.5	– 2.9	– 4.0
40	0	– 1.9	– 3.0	– 3.6	– 4.9
45	0	– 2.3	– 3.7	– 4.4	– 5.9
50	0	– 2.9	– 4.4	– 5.4	– 7.3
55	0	– 3.3	– 5.2	– 6.2	– 8.6
60	0	– 4.0	– 6.1	– 7.5	– 10.2
65	0	– 4.6	– 7.2	– 8.8	– 12.1

$$\mathrm{Kp}'_{2\mathrm{LA}} = 1 - \frac{\left(2\arccos\left(1 - \frac{a}{200}\right) - \sin\left(2\arccos\left(1 - \frac{a}{200}\right)\right)\right) \cdot (3n - 2)}{2 \cdot n \cdot \pi} \tag{30}$$

Or expressed as a function of the total number of emitters per group n':

$$\mathrm{Kp}'_{2\mathrm{LA}} = 1 - \frac{\left(2\arccos\left(1 - \frac{a}{200}\right) - \sin\left(2\arccos\left(1 - \frac{a}{200}\right)\right)\right) \cdot \left(3n' - 4\right)}{2 \cdot n' \cdot \pi} \tag{31}$$

Or

$$\mathrm{Kp}'_{2\mathrm{LA}} = 1 - \frac{\left(3n' - 4\right)}{2 \cdot n'} C \tag{32}$$

Again, it should be remembered that $2n$ or n' should refer to the number of emitters per plant, not to the total number of emitters of the lateral line. Table 3 shows a comparison between KpM, KpM$_{2L}$ and Kp$'_{2LA}$ for different overlapping ratios and values of n' ($2n$). Table 4 presents the corresponding percent variation of Kp$'_{2L}$ in comparison to KpM. A maximum number of 10 emitters per plant and lateral have been considered, due to the limitation of lateral length corresponding to each plant according to the planting spacing. The maximum number of 10 emitters per lateral line was justified in the previous section.

Kp$'_{2LA}$ is always lower than Kp and the differences between both models increase as the number of emitters per group increases. This is due to the estimation of the total As per group in both models. As mentioned above, for the calculation of Ane in Kp$'_{2LA}$ an additional As is subtracted in the central emitters, while it remains the same in the extreme emitters in comparison to KpM. KpM and Kp$'_{2LA}$ present the same values for two emitters per lateral (four emitters per group/plant), because all four emitters are extreme emitters in this case, and have identical overlapping of 2As, as in the calculation of KpM. KpM$_{2L}$ presents always the lowest values, due to its definition, because *3As* are always subtracted, even in the extreme emitters. The differences decrease with Kp$'_{2LA}$ for an increasing number of emitters per group, because the effect of the extreme emitters gets reduced. The differences between KpM, KpM$_{2L}$ and Kp$'_{2LA}$ increase as the overlapping ratio increases.

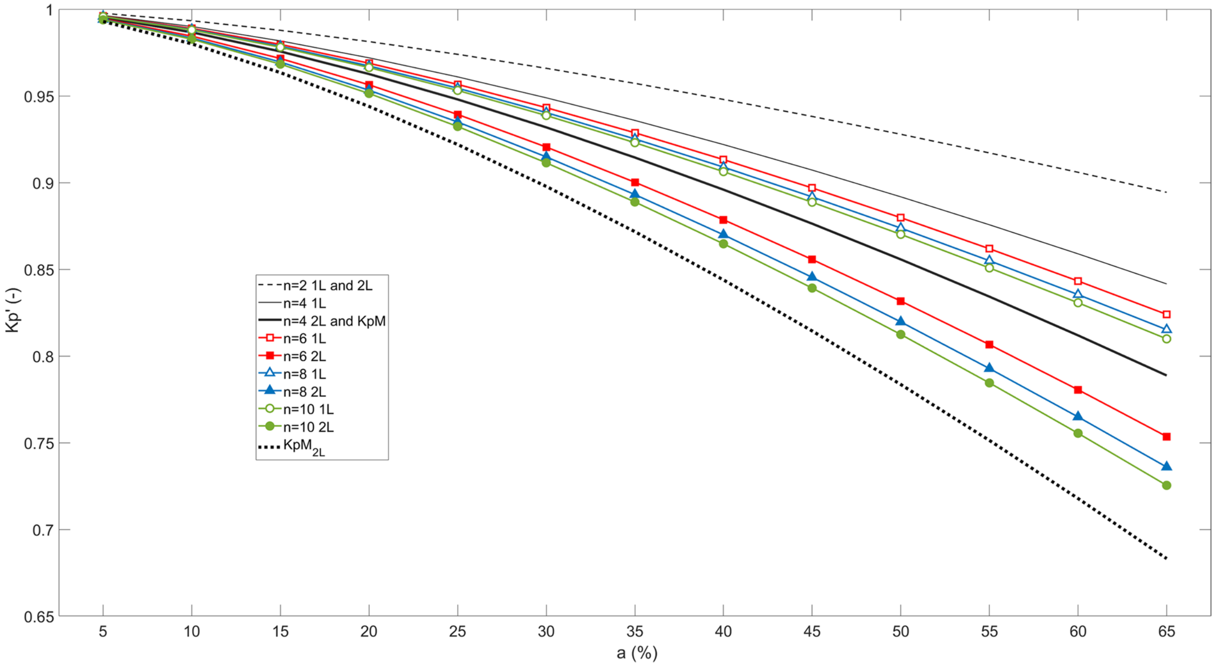

Fig. 5 Comparison between overlapping coefficients for 2, 4, 6, 8 and 10 emitters in 1 and 2 laterals

These results indicate that KpM would underestimate the theoretical number of required emitters. KpM_{2L} would overestimate the theoretical number of required emitters, in a higher degree the lower the number of emitters per group. Similarly to Kp'_{1L}, since Kp'_{2LA} requires *n* as input variable for its calculation, and this is actually the target that wants to be determined once the overlapping coefficient is known, an iterative procedure must be adopted. As initial solution of *n* there might be two options: (i) calculating n based on KpM, or (ii) considering initially that there is no overlapping between emitters for calculating *n*. Another option is to use the Kp exclusively to verify if the condition of minimum wetted area is achieved, when n is already known.

Figure 5 shows a comparison between Kp'_{1L} and Kp'_{2L} for groups of 2, 4, 6, 8 and 10 emitters, arranged on one or two laterals, respectively, and overlapping ratios between 5 and 65%. KpM and KpM_{2L} are also shown. Further, in the case of two laterals only the scenario with a single overlapping is represented (KpM_{2LA} and Kp'_{2LA}). For two emitters Kp'_{1L} and Kp'_{2L} are coincident, because the overlapping area is the same in both cases. When the number of emitters increases, the overlapping coefficient is lower when the emitters are arranged on two laterals and when the overlapping ratio increases, as mentioned above. The Kp'_{2L} and KpM values are the same for four emitters (2 emitters per lateral). As mentioned, KpM_{2L}, due to its definition, presents always the minimum values.

These differences indicate that, if initially Kp'_{1L} is used, because an arrangement of emitters grouped in one lateral per plant row if adopted, and the resulting number of emitters cannot be installed in practice in a single lateral, due to limitation of lateral length between plants, n should be recalculated using Kp'_{2L}, or this should be used to verify that the minimum wetted area threshold is reached.

Two different overlapping ratios between emitters

Again, this scenario requires the definition of two different overlapping areas, namely one between emitters along the same lateral, As_1, and one between neighbouring emitters from different laterals, As_2. Further, this involves the definition of the two corresponding overlapping angles, namely θ_1 and θ_2, and the two corresponding overlapping ratios, namely a_1 and a_2, respectively. Again, the overlapping between driplines should, additionally, not interfere with plot operations. Following the same procedure as in the previous section, the total net area per group of emitters will be

$$AneT_{2LB} = 2 \cdot n \cdot Ae - 2 \cdot As_1 \cdot (n-1) \cdot 2 - 2 \cdot As_2 \cdot n \tag{33}$$

where $AneT_{2LB}$ corresponds to the total net wetted area per group with two different overlapping areas, As_1 is the overlapping area along the same lateral, As_2 is the overlapping area between laterals, and n is the number of emitters per lateral and plant group. So, the total number of emitters per group would be *2n*. Or referred to *n'*:

$$\mathrm{AneT}_{2\mathrm{LB}} = n' \cdot \mathrm{Ae} - 2 \cdot \mathrm{As}_1 \cdot \left(n' - 2\right) - \mathrm{As}_2 \cdot n' \tag{34}$$

Thus:

$$\begin{aligned}\mathrm{Kp}^{'}_{2\mathrm{LB}} &= \frac{\mathrm{AneT}_{2\mathrm{LB}}}{2 \cdot n \cdot \mathrm{Ae}} = \frac{2 \cdot n \cdot \mathrm{Ae} - 2 \cdot \mathrm{As}_1 \cdot (n-1) \cdot 2 - 2 \cdot \mathrm{As}_2 \cdot n}{2 \cdot n \cdot \mathrm{Ae}} \\ &= 1 - \frac{\left[2 \cdot \mathrm{As}_1 \cdot (n-1) + n \cdot \mathrm{As}_2\right]}{n \cdot \mathrm{Ae}}\end{aligned} \tag{35}$$

where $\mathrm{Kp}'_{2\mathrm{LB}}$ is the overlapping coefficient for grouped emitters in two laterals per row of plants with two values of overlapping areas. Replacing As by Eq. 6:

$$\begin{aligned}\mathrm{Kp}^{'}_{2\mathrm{LB}} &= 1 - \frac{\left[2 \cdot \frac{r^2}{2}\left(\theta_1 - \sin\left(\theta_1\right)\right) \cdot (n-1) + n \cdot \frac{r^2}{2}\left(\theta_2 - \sin\left(\theta_2\right)\right)\right]}{n \cdot \pi r^2} \\ &= 1 - \frac{\left[\left(\theta_1 - \sin\left(\theta_1\right)\right) \cdot (n-1) + \frac{n}{2}\left(\theta_2 - \sin\left(\theta_2\right)\right)\right]}{n \cdot \pi}\end{aligned} \tag{36}$$

where θ_1 and θ_2 are the overlapping angles along one lateral and between laterals, respectively. Or replacing θ as a function of a, Eq. 7:

$$\mathrm{Kp}^{'}_{2\mathrm{LB}} = 1 - \frac{\left[\left(2\arccos\left(1 - \frac{a_1}{200}\right) - \sin\left(2\arccos\left(1 - \frac{a_1}{200}\right)\right)\right) \cdot (n-1) + \frac{n}{2}\left(2\arccos\left(1 - \frac{a_2}{200}\right) - \sin\left(2\arccos\left(1 - \frac{a_2}{200}\right)\right)\right)\right]}{n \cdot \pi} \tag{37}$$

where a_1 and a_2 are the overlapping ratios along one lateral and between laterals, respectively. Or

$$\mathrm{Kp}^{'}_{2\mathrm{LB}} = 1 - \frac{\left[C_1 \cdot (n-1) + \frac{n}{2} \cdot C_2\right]}{n} \tag{38}$$

Or expressed as a function of the total number of emitters per group n′:

$$\mathrm{Kp}^{'}_{2\mathrm{LB}} = 1 - \frac{2 \cdot \left[\left(2\arccos\left(1 - \frac{a_1}{200}\right) - \sin\left(2\arccos\left(1 - \frac{a_1}{200}\right)\right)\right) \cdot \left(\frac{n'}{2} - 1\right) + \frac{n'}{4}\left(2\arccos\left(1 - \frac{a_2}{200}\right) - \sin\left(2\arccos\left(1 - \frac{a_2}{200}\right)\right)\right)\right]}{n' * \pi} \tag{39}$$

a_1 and a_2 should be in the range 15–50, as explained before. Or

$$\mathrm{Kp}'_{2\mathrm{LB}} = 1 - \frac{2\left[C_1\left(\frac{n'}{2} - 1\right) + \frac{n'}{4}C_2\right]}{n'} \tag{40}$$

Overlapping coefficient for non-grouped emitters

One lateral per plant row

If the emitters are not grouped, the spacing between emitters is always constant along the lateral line, even between extreme emitters from neighbouring plants. In this scenario, the extreme bulbs of two neighbouring plants present overlapping, too. Regarding the evaluation of the Ane per emitter in this scenario, it can be observed that it is the same for all emitters, with the exception of the two extreme emitters of the lateral. In this case, it seems reasonable to apply to any emitter only the Ane corresponding to an interior emitter, although in the group of the extreme plants Ane is slightly higher, because in the extreme emitters only 1As would be subtracted, not 2As. Thus, if there is only one lateral per row of plants and non-grouped emitters, the original KpM (Montalvo 2003) would be used.

Two laterals per plant row

In the case of non-grouped emitters arranged on two lateral lines per row of plants, again, an additional As should be subtracted, due to the overlapping between laterals. Similarly to the previous scenario, in this case it seems also reasonable to consider for all emitters only the Ane corresponding to an interior emitter. However, in this case the original KpM (Montalvo 2003) would not be strictly valid, because an additional As due to the overlapping between laterals should be subtracted. Therefore, in this case, Ane would be calculated again using Eqs. 14 and 18.

'Thus, the resulting Kp would be the same as those corresponding to the generalization of Montalvo's overlapping coefficient for two laterals per plant row, $\mathrm{KpM}_{2\mathrm{LA}}$, shown in Eq. 16, and $\mathrm{KpM}_{2\mathrm{LB}}$, shown in Eq. 20. It might also make sense to apply the equations for grouped emitters $\mathrm{Kp}'_{1\mathrm{L}}$ and $\mathrm{Kp}'_{2\mathrm{L}}$, because the main objective is to calculate the number of emitters per plant, not per lateral, regardless of the arrangement of the emitters along the lateral. Anyway, if non-grouped emitters are adopted, these new equations could be applied to verify that the minimum wetted threshold area is reached.

Application examples

Figure 6 presents a scheme of the global calculation process. First, the boundary conditions must be fixed, namely (i) nominal flow rate of the emitter (q), (ii) soil type, (iii)

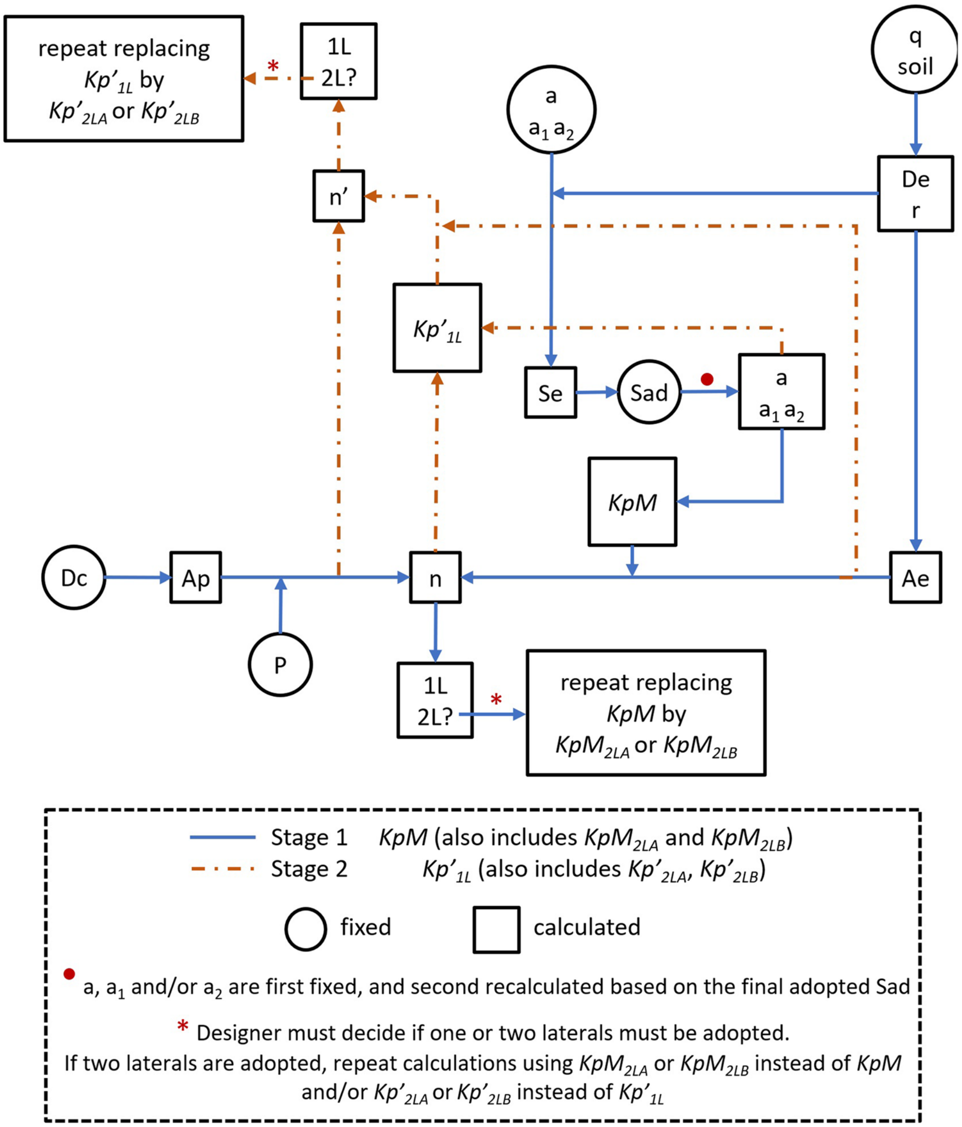

Fig. 6 Scheme for the application of the presented methodology

canopy diameter of the plant (Dc), (iv) desired overlapping percents (a_1 and a_2), (v) target wetting threshold of shaded area (P). (i) and (ii) might be replaced by an estimated or measured maximum diameter of the wet bulb (De). In our examples De (in m) was estimated from q (in l/h) the following expressions (Karmeli et al. 1985):

$$D_e = 0.3 + 0.12 \cdot q \text{ (light)} \tag{41}$$

$$D_e = 0.7 + 0.11 \cdot q \text{ (medium)} \tag{42}$$

$$D_e = 1.2 + 0.10 \cdot q \text{ (heavy)} \tag{43}$$

First, KpM (1 lateral per plant row) must be found out. Therefore Se (Eq. 2) is calculated from De (or r) and a or a_1 (both parameters refer to the overlapping between emitters in same lateral). Se can be rounded to a commercial spacing (Sad), and the actual a or a_1 is recalculated based on Sad and r. Afterwards KpM is calculated from the adopted a or a_1. On the other hand, Ae is calculated using De, while Ap is calculated using Dc. Finally, n can be calculated with Ap, P, and Ane using Eq. 5. If n and Se or Sad are not compatible with the plant spacing (i.e. there is not enough space between plants for n emitters spaced *Sad* meters), two laterals must be adopted. In this case, KpM$_{2LB}$ (Eq. 16) or KpM$_{2LB}$ (Eq. 20) are adopted instead of KpM, and n is recalculated similarly using Eq. 5. Further, if n and Se are compatible with the plant spacing, but the designer imposes 2 laterals from the beginning, e.g. for ensuring a hypothetical more suitable growth of the roots, Eqs. 16 or 20 are adopted from the beginning instead of KpM. If the new approach wants to be applied, the previous resulting n is used together with a_1 for estimating Kp$'_{1L}$ (Eq. 25). The new n or n' is

Table 5 Practical cases studied

Case	Ap (m^2)	Soil type	q (l/h)	De (m)	Ae (m^2)	a_1 (–)	a_2 (–)	Se (m)
1	12.57	Medium	8	1.58	1.96	0.15	0.15	1.46
2						0.50	0.50	1.19
3						0.50	0.15	1.19
4			4	1.14	1.02	0.15	0.15	1.06
5						0.50	0.50	0.86
6						0.50	0.15	0.86
7			2	0.92	0.66	0.15	0.15	0.85
8						0.50	0.50	0.69
9						0.50	0.15	0.69
10		Light (sandy)	8	1.26	1.25	0.15	0.15	1.17
11						0.50	0.50	0.95
12						0.50	0.15	0.95
13			4	0.78	0.48	0.15	0.15	0.72
14						0.50	0.50	0.59
15						0.50	0.15	0.59
16			2	0.54	0.23	0.15	0.15	0.50
17						0.50	0.50	0.41
18						0.50	0.15	0.41
19		Heavy (clay)	8	2.00	3.14	0.15	0.15	1.85
20						0.50	0.50	1.50
21						0.50	0.15	1.50
22			4	1.60	2.01	0.15	0.15	1.48
23						0.50	0.50	1.20
24						0.50	0.15	1.20
25			2	1.4	1.54	0.15	0.15	1.30
26						0.50	0.50	1.05
27						0.50	0.15	1.05

calculated using Eq. 5 based again on Ap, P, and Ane until the resulting n equals the n used as input in Eq. 25. Again, the designer must decide if 1 or 2 laterals must be adopted. If 2 laterals are adopted $\mathrm{Kp}'_{2\mathrm{LA}}$(Eq. 31) or $\mathrm{Kp}'_{2\mathrm{LB}}$(Eq. 39) are used instead of $\mathrm{Kp}'_{1\mathrm{L}}$. Further, if the emitters are not grouped, KpM would be used for the case of 1 lateral per plant row, while Eqs. 16 or 20 would be used for the case of 2 laterals per plant row.

Table 5 summarizes the boundary conditions of the 27 studied practical cases. In particular, a fruit tree with canopy diameter of 4 m was adopted, providing a shaded area of 12.57 m^2, if a circular orthogonal area is assumed. Moreover, a planting spacing of 5 × 4 m is adopted. Three possible emitter nominal flow rates, namely 8 l/h, 4 l/h and 2 l/h, were considered. Moreover, three soil types were considered, namely light (sandy), medium and heavy (clay) soil types. The corresponding De were calculated using Eqs. 41–43. These values might be fixed based on other criteria, such as experimental determination, tables, other references etc. as mentioned in the section "General criterion". Ae is calculated using De assuming a circular area. Finally, 3 combinations of a_1 and a_2 were defined per flow rate and soil type, i.e. two scenarios with identical overlapping between emitters and laterals (15% and 50%, respectively), while a third scenario where emitters of the same lateral presented an overlapping of 50%, while the overlapping between laterals was fixed in 15%. If both percents must be different, it seems reasonable that the overlapping between laterals should be lower than between emitters of the same lateral, or even 0. As mentioned in the introduction, paired laterals are usually placed slightly away from each side of the tree row to provide a larger wetted soil volume. Further, this arrangement also keeps the tree crown dry minimizing the potential for disease. 10 iterations were considered for calculating Kp if it relied on n. All calculations were implemented using the software Matlab version 2021b (The MathWorks, Inc., Natick, MA, USA).

Based on the procedures previously described and Fig. 6, Tables 6 (cases 1–5, 10–14, 19–23) and Table 7 (6–9, 15–18, 24–27) present the corresponding solutions for n. Seven solutions for n were calculated per practical case, namely without KpM, and with KpM, $\mathrm{KpM}_{2\mathrm{LA}}$, $\mathrm{KpM}_{2\mathrm{LB}}$, $\mathrm{Kp}'_{1\mathrm{L}}$, $\mathrm{Kp}'_{2\mathrm{LA}}$, and $\mathrm{Kp}'_{2\mathrm{LB}}$. The cases were arranged for allowing an easier comparison between soil types for identical flow

Table 6 Calculated emitters in practical cases 1–5, 10–14, 19–23

Criterion	Case	Medium soil type				Case	Light soil type				Case	Heavy soil type			
		n_1	n_2	n_3	Kp (n_1)		n_1	n_2	n_3	Kp (n_1)		n_1	n_2	n_3	Kp (n_1)
Without Kp	1	3.20	3	3	0	10	5.04	5	5	0	19	2.00	2	2	0
KpM		3.28	3	3	0.976		5.16	5	5	0.976		2.05	2	2	0.976
KpM_{2LA} KpM_{2LB}		3.33	3	4	0.963		5.23	5	6	0.963		2.08	2	2	0.963
Kp'_{1L}		3.26	3	3	0.983		5.14	5	5	0.980		2.02	2	2	0.988
Kp'_{2LA} Kp'_{2LB}		3.28	3	4	0.978		5.18	5	6	0.973		2.03	2	2	0.988
Without Kp	2	3.20	3	3	0	11	5.04	5	5	0	20	2.00	2	2	0
KpM		3.74	4	4	0.856		5.89	6	6	0.856		2.34	2	2	0.856
KpM_{2LA} KpM_{2LB}		4.09	4	4	0.784		6.43	6	6	0.784		2.55	3	4	0.784
Kp'_{1L}		3.58	4	4	0.896		5.72	6	6	0.881		2.17	2	2	0.922
Kp'_{2LA} Kp'_{2LB}		3.72	4	4	0.861		6.06	6	6	0.831		2.18	2	2	0.916
Without Kp	3	3.20	3	3	0	12	5.04	5	5	0	21	2.00	2	2	0
KpM		3.74	4	4	0.856		5.89	6	6	0.856		2.34	2	2	0.856
KpM_{2LA}		4.09	4	4	0.784		6.43	6	6	0.784		2.55	3	4	0.784
KpM_{2LB}		3.80	4	4	0.844		5.97	6	6	0.844		2.37	2	2	0.844
Kp'_{1L}		3.58	4	4	0.896		5.72	6	6	0.881		2.17	2	2	0.922
Kp'_{2LA}		3.72	4	4	0.861		6.06	6	6	0.831		2.18	2	2	0.916
Kp'_{2LB}		3.46	3	4	0.927		5.63	6	6	0.895		2.03	2	2	0.986
Without Kp	4	6.16	6	6	0	13	13.15	13	13	0	22	3.13	3	3	0
KpM		6.31	6	6	0.976		13.48	13	13	0.976		3.20	3	3	0.976
KpM_{2LA} KpM_{2LB}		6.39	6	6	0.963		13.65	14	14	0.963		3.24	3	4	0.963
Kp'_{1L}		6.28	6	6	0.980		13.45	13	13	0.977		3.18	3	3	0.983
Kp'_{2LA} Kp'_{2LB}		6.34	6	6	0.971		13.60	14	14	0.967		3.19	3	4	0.979
Without Kp	5	6.16	6	6	0	14	13.15	13	13	0	23	3.13	3	3	0
KpM		7.19	7	7	0.856		15.37	15	15	0.856		3.65	4	4	0.856
KpM_{2LA} KpM_{2LB}		7.86	8	8	0.784		16.78	17	18	0.784		3.99	4	4	0.784
Kp'_{1L}		7.03	7	7	0.876		15.20	15	15	0.865		3.48	3	3	0.897
Kp'_{2LA}		7.49	7	8	0.822		16.41	16	16	0.801		3.62	4	4	0.863
Kp'_{2LB}					0.822										

n_1 the theoretical solution (Eq. 5), n_2 rounded solution, n_3 adopted solution

rates and overlapping percents. Finally, three solutions of n are provided, namely n_1, n_2 and n_3. n_1 corresponds to the theoretical solution provided by Eq. 5. n_2 corresponds to the rounded solution, while n_3 corresponds to the adopted solution. n_3 mainly intends to highlight that an odd number of emitters (n_2) cannot be adopted for two laterals, and was increased in one emitter. Similarly, despite this was not done here for fixing n_3, if one lateral per plant row is initially considered, it should be checked if there is enough spacing between plants for n_2 emitters attending to Se. If not, two laterals per plant row should be adopted. Some theoretical solutions might not be adopted even with 2 laterals per plant row, due to lack of spacing. When a_1 and a_2 were identical (15% or 50%), KpM_{2LB} and KpM_{2LB}, as well as Kp'_{2LA} and Kp'_{2LB} provided the same solutions, respectively, and their cells were thus combined.

Table 7 Calculated emitters in of practical cases 6–9, 15–18, 24–27

Criterion	Case	Medium soil type				Case	Light soil type				Case	Heavy soil type			
		n_1	n_2	n_3	Kp (n_1)		n_1	n_2	n_3	Kp (n_1)		n_1	n_2	n_3	Kp (n_1)
Without Kp	6	6.16	6	6	0	15	13.15	13	13	0	24	3.13	3	3	0
KpM		7.19	7	7	0.856		15.37	15	15	0.856		3.65	4	4	0.856
KpM_{2LA}		7.86	8	8	0.784		16.78	17	18	0.784		3.99	4	4	0.784
KpM_{2LB}		7.30	7	8	0.844		15.59	16	16	0.844		3.70	4	4	0.844
Kp'_{1L}		7.03	7	7	0.876		15.20	15	15	0.865		3.48	3	3	0.897
Kp'_{2LA}		7.49	7	8	0.822		16.41	16	16	0.801		3.62	4	4	0.863
Kp'_{2LB}		6.96	7	8	0.885		15.25	15	16	0.862		3.36	3	4	0.929
Without Kp	7	9.45	9	9	0	16	27.43	27	27	0	25	4.08	4	4	0
KpM		9.69	10	10	0.976		28.12	28	28	0.976		4.18	4	4	0.976
KpM_{2LA}		9.81	10	10	0.963		28.48	28	28	0.963		4.24	4	4	0.963
KpM_{2LB}															
Kp'_{1L}		9.66	10	10	0.978		28.10	28	28	0.976		4.16	4	4	0.981
Kp'_{2LA}		9.76	10	10	0.968		28.43	28	28	0.965		4.19	4	4	0.975
Kp'_{2LB}															
Without Kp	8	9.45	9	9	0	17	27.43	27	27	0	26	4.08	4	4	0
KpM		11.05	11	11	0.856		32.06	32	32	0.856		4.77	5	5	0.856
KpM_{2LA}		12.06	12	12	0.784		35.01	35	36	0.784		5.21	5	6	0.784
KpM_{2LB}															
Kp'_{1L}		10.88	11	11	0.869		31.89	32	32	0.860		4.60	5	5	0.887
Kp'_{2LA}		11.69	12	12	0.808		34.64	35	36	0.792		4.84	5	6	0.843
Kp'_{2LB}															
Without Kp	9	9.45	9	9	0	18	27.43	27	27	0	27	4.08	4	4	0
KpM		11.05	11	11	0.856		32.06	32	32	0.856		4.77	5	5	0.856
KpM_{2LA}		12.06	12	12	0.784		35.01	35	36	0.784		5.21	5	6	0.784
KpM_{2LB}		11.21	11	12	0.844		32.52	33	34	0.844		4.84	5	6	0.844
Kp'_{1L}		10.88	11	11	0.869		31.89	32	32	0.860		4.60	5	5	0.887
Kp'_{2LA}		11.69	12	12	0.808		34.64	35	36	0.792		4.84	5	6	0.843
Kp'_{2LB}		10.86	11	12	0.870		32.18	32	32	0.852		4.50	4	4	0.908

n_1 the theoretical solution (Eq. 5), n_2 rounded solution, n_3 adopted solution

As could be expected, the required number of emitters increases from heavy (clay) to light (sandy) soil types, and for decreasing flow rates, because Ae gets smaller, independently of Kp. Accordingly, the range of required emitters fluctuates between 2(clay)-6(sandy) for a flow rate of 8 l/h, between 3(clay)-18(sandy) for a flow rate of 4 l/h, and between 4(clay)-36(sandy) for a flow rate of 2 l/h. These results are strongly dependent on *De* provided by Eqs. 41–43 and may vary for other imposed D_e. It seems not reasonable to represent a general soil type by a single De, thus an on-site accurate De estimation/measurement might be highly recommended. The high number of emitters for sandy soil type and 2 l/h (28 emitters for 15% of overlapping, and 36 emitters for 50% of overlapping) is due to the low estimation of the bulb wetted diameter (De = 0.54 m). This case was fixed for allowing a theoretical comparison with the rest of scenarios, but this solution would be discarded in practice. In practice higher nominal flow rates and lower overlaps would be adopted (the nominal flow rate 8 l/h provides a solution of 5 emitters for 15% of overlapping and of 6 emitters for 50% of overlapping, while the flow rate 4 l/h provides a solution of 13 emitters for 15% of overlapping and of 18 emitters for 50% of overlapping). In this regard, a commercial solution offered by manufacturers in Spain is the dripline with integrated emitters of 2 l/h with regular spacing of 0.3 m or 0.33 m. This corresponds to 24 emitters per plant with two laterals per plant row if the plant spacing is 4 m. Assuming the De given by Eqs. 41–43, this emitter spacing would be theoretically translated into 77% to 152% of overlapping in light and heavy textures, respectively. And according to the proposed Kp methodologies, a theoretical wetting threshold between 26.7 and 20.6% of the shaded area would be achieved. Thus, other solutions would be required.

The differences between the proposed Kp approaches increase for an increasing number of emitters per plant (e.g. sandy texture and 2 l/h). In particular, the approach omitting Kp provides around 1–2 fewer emitters per plant, except with 8 l/h and 15% of overlapping, where few emitters are required in all cases. For extreme cases, low flow rates and high overlaps, between 1 and 3 emitters (medium soil type), and between 2 and 5 emitters (sandy soil type) less are provided if Kp is omitted. Among approaches considering Kp, for a single lateral per plant row, KpM and Kp'_{1L} tend to provide similar results, but Kp_{2L} approaches (i.e. KpM_{2LA}, KpM_{2LB}, Kp'_{2LA}, and Kp'_{2LB}) involve 1 additional emitter, or even 2 if the rounded solution is an odd number, for flow rates of 4 l/h and 2 l/h. As mentioned, for 2 l/h, 50% of overlapping and sandy texture the differences can be higher. Similarly, reducing the overlapping between laterals from 50 to 15% is translated into a decrease of emitters, which can be masked by the need for rounding to an even number. These differences will increase if the overlap between laterals is neglected.

Validity of the general criterion for landscape irrigation

In landscape irrigation it should be questioned whether the general criterion applied in agriculture is still valid. Landscape irrigation might be conditioned, among others, by two differentiating factors: first, the estimation of evapotranspiration requirements in landscape irrigation is not based on a crop coefficient. So, plants might be irrigated with different goals, e.g. to just survive or to thrive, to maintain plant biomass or to have vegetative growth (Waller and Yitayew 2016). Second, the same irrigation subunit might bring together a group of different species in the same area, because the landscape design may respond to criteria other than the irrigation doses that will be applied to the species, e.g. exclusively to aesthetic criteria.

Thus, in landscape irrigation, different species might be irrigated together receiving the same doses as a part of the same grouping. In this scenario, it is highly probable to find species with very different canopies in the same grouping. Moreover, these species will be probably located in the same subunit, too. This involves that they have the same irrigation time. However, the number of required emitters per plant is commonly calculated based on the same criterion of wetting at least a minimum threshold of the soil area of the plant. Thus, the irrigation doses will depend on the canopy, rather than on their specific consumptions. The calculation of emitters per plant relying on their canopy projection will hardly allow adjusting the different doses at the same irrigation time. So, if the general criterion is applied in landscape irrigation, it is highly probable that there will be inconsistencies between the number of emitters assigned per plant and the different doses to be applied within the same subunit.

Therefore, an alternative criterion that might contribute to achieve a more accurate distribution of different irrigation doses within a subunit would be to calculate the number of emitters per plant based on its individual specific evapotranspiration requirements and a single pre-set irrigation time. This criterion would allow to establish a proportionality ratio of emitters among the species of the same subunit. Then, an irrigation time would then be set to meet the needs of one species, and the number of emitters of the rest would be adjusted based on the calculated ratio.

Conclusion

Given the absence of clear, general and consolidated criteria for agronomic design, this paper aimed at revising, clarifying and refining the existing published guidelines and methodologies for estimating the required emitters per plant in drip irrigation, focussing on the approach of Montalvo (2003).

Based on a thorough theoretical geometric analysis of the overlapping between wet bulbs of contiguous emitters, the deduction of the overlapping coefficients proposed by Montalvo (2003) was presented here. This author assumed identical net wetted areas for all emitters in the laterals. However, the overlapping areas or volumes between emitters differ in extreme emitters and interior emitters, as well as in configurations with one lateral per plant row and two laterals per plant row. Based on these findings, new formulations were proposed for the computation of the overlapping coefficient. Its calculation needs to incorporate the number of emitters as an additional variable, as well as to distinguish between the presence of one or two laterals per plant row, and between grouped and non-grouped emitters. Further, the overlapping ratios between emitters of the same lateral, and between laterals might be different.

In one lateral per plant row, the original overlapping coefficient (Montalvo 2003) overestimates the theoretical number of required emitters. In the case of two laterals per plant row, the original overlapping coefficient underestimates the theoretical number of required emitters per plant. The presented formulations were applied in different practical examples covering a wide range of scenarios. The results allowed a general overview of the influence of the soil type, the emitter flow rate, and the selected overlapping ratio in the number of required emitters per plant based on the considered theoretical approach. As was expected, the required number of emitters increased from heavy (clay) to light (sandy) soil types, and for decreasing flow rates, independently of Kp. An accurate on-site measurement of De might be highly recommended. The differences between the proposed Kp approaches increase for an increasing number of emitters per plant. In particular, the approach omitting Kp provides around 1–2 fewer emitters per plant, except with 8 l/h and 15% of overlapping, where few emitters are required in all cases. For low flow rates and high overlaps, between 1 and 3 emitters (medium soil type), and between 2 and 5 emitters (light soil type) less are provided if Kp is omitted. Among approaches considering Kp, for a single lateral per plant row, KpM and Kp'_{1L} tend to provide similar results, but KpM_{2LA}, KpM_{2LB}, Kp'_{2LA}, and Kp'_{2LB} tend to provide 1 additional emitter, or even 2 if the rounded solution is an odd number, for flow rates of 4 l/h and 2 l/h. Reducing the overlapping between laterals from 50 to 15% is translated into a decrease of emitters, which can be masked by the need for rounding to an even number. These differences will increase if the overlap between laterals is neglected.

In landscape irrigation, groups of different species with different consumptions and canopies might be irrigated within the same subunit, i.e. with identical irrigation duration. So, it is highly probable that the general criterion might lead to inconsistencies between the assigned number of emitters and the required doses per plant. The revision of guidelines and methods presented here, complemented with other experimental results and models of soil water dynamics under drip irrigation, might contribute to a better decision making of designers and field engineers.

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Declarations

Conflict of interest The authors declare no competing interests.

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