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Bicriteria food packaging process optimization in double-layered upright and diagonal multihead weighers

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ABSTRACT

Double-layered multihead weighing machines contain twice the number of hoppers as present in a simple machine with the same number of heads, which enables additional objective optimization possibilities considering the increased number of combinations among hoppers. This research study deals with bicriteria optimization for double-layered upright and diagonal machines using brute force as the optimization criteria. One of the optimization objectives is related to the target weight; the target weight must be at least and as close as possible to the weight to pack. Furthermore, this study also aims to minimize the time for which a certain portion of a product remains in the hopper while waiting to be selected for package formation. This time is known as priority and is measured based on the number of iterations or the number of packages produced by the machine while the hopper waits to be discharged. For these purposes, Different strategies were tested for both machines, which simultaneously optimize the target weight and the priority of the hoppers, showing the reduction of the extraction of the process in addition to reducing the costs of excess product and its reprocessing.

1. Introduction

The ever-growing complexity of data processed and analyzed for efficient decision-making has prompted packaging companies to develop strategies and tools to ensure timely responses to meet the needs of their customers and consumers. One of the most frequent challenges companies experience is process optimization, wherein complex problems must often simultaneously meet more than one objective. In fact, different optimization techniques have evolved along with the needs of the food packaging industry. Each technique offers particular advantages based on the specifications and restrictions of their target functions associated with computational costs related to memory consumption and execution times of their implementation algorithms.

On a global scale, as stringent regulations guarantee that customers receive what they are actually buying, packaged contents must be consistent with their corresponding label, ingredients, and net content. Hence, the actual net content must not be lower than the net content reported to consumers on the label. In this context, multihead weighers are

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Symbols and abbreviations

Q	Number of packages
ℓ	Iteration in which the package is packed, $\ell \in= \{1, 2, 3 \ldots Q\}$
H_{Wi}	Set of n weighing hoppers $i = \{1, 2, 3 \ldots n\}$
H_{Bj}	Set of n booster hoppers $j = \{n + 1, n + 2, \ldots 2n\}$
$H_m = \left\{H_{Wi} \cup H_{Bj}\right\}$	Set of $2n$ hoppers $m = \{1, 2, 3, \ldots 2\mathrm{n}\}$
H'	Subset of combined hoppers
T	Target label weight
k	Number of hoppers combined
$w_{i\ell}$	Actual weight of each hopper $i = \{1, 2, 3, \ldots 2n\}$ in the ℓ interaction based on the filling strategy $(S_1, S_2,$ or $S_3)$
W_ℓ	Sum of the weights of k hoppers in the ℓth iteration
P_{max}	Maximum priority allowed for $2n$ hoppers
$p_{i\ell}$	Positive integer; priority of the $i \in \{1, 2, 3, \ldots 2n\}$ hopper in the $\ell \in= \{1, 2, 3 \ldots Q\}$ iteration
P_ℓ	Sum of priorities of k hoppers in the ℓth iteration
$p_{max\ell}$	Positive integer; highest priority of all hoppers in the $\ell \in= \{1, 2, 3 \ldots Q\}$ iteration
S	Set of hoppers that meet all problem conditions
θ	Relative weight or importance of target priority; varies at each iteration
D	The optimal point is found during each iteration using the set of k hoppers
ANOVA	Analysis of variance
$\mathrm{CV_{paq}}$	Coeficiente de variacion de los Q paquetes

important because their accuracy and reliability foster agile processes and guarantee standard compliance [1,2]. For these purposes, such machines require a certain configuration in their base code that may guarantee their content packaging accuracy as this is the first objective to be met for compilation with legal guidelines. This configuration must minimize differences between the actual package content and the net content reported on the product label, thus reducing the variability between the two values and complying with one of the quality principles [3]. By reducing the target weight variability T, the quality of the food packaging process is improved [4], costs associated with product reprocessing times are reduced, and excess product is minimized [5]. In other words, processes must be optimized to guarantee that the packaged content is as close as possible to the net content reported on the label. For quality engineering, optimization techniques, new algorithms, and experimental designs can be used to improve and optimize industrial processes [6].

When exploring the food packaging industry, many products require rapid packaging processes. In such cases, the time gap between product weighing and product packaging becomes critical. Here, the quality'sproduct is enhanced when it is packaged quickly. The time gap is the second problem that must be addressed when coupled with the label content, constituting a bicriteria optimization problem.

Double-layered multihead weighers feature one set of hoppers with another set of hoppers underneath, forming two hopper layers. The hoppers on the upper layer receive the product from the feeding hoppers (H_F) and record the weight of the received product. These hoppers are known as weighing hoppers (H_W). Once the weight is recorded, the hoppers are discharged to auxiliary hoppers known as booster hoppers (H_B). So, weigh hoppers receive a new portion of product. When the machine hoppers are full, a combinatorial analysis and the appropriate mathematical design are used to select a subset H' from k hoppers to optimize the objective(s) stated in the problem. This subset is then discharged to the package based on the consideration that in machines with an upright configuration, a weighing hopper cannot be selected without its booster hopper; alternatively, in machines with a diagonal configuration, a weighing hopper and a booster hopper cannot be selected simultaneously (Fig. 1).

In this study, a new bicriteria optimization algorithm is proposed for double-layered upright and diagonal machines. For each package, this algorithm select a H' subset from k hoppers so that the sum of their weights is less than the maximum capacity of the package but greater and as close as possible to the value reported on the label (target weight T). Furthermore, the algorithm ensures the minimum total time from product weighing to the discharge of k hoppers. This is known as hopper selection priority (p). This algorithm was validated using a case study of actual weights based on a process simulation using a software program specifically designed and developed for such purposes. The experimental tests were conducted using filling strategies, which were previously validated in other studies [7–9].

The remainder of this paper is divided as follows. Section 2 discusses the background of multiobjective optimization for multihead machines. Section 3 describes the configuration of the double-layered upright and diagonal machines used in the hopper-filling process. Section 4 describes the target process functions. Section 5 presents the packaging algorithm, and Section 6 reveals the preliminary numerical results and assessments. Section 7 describes the results obtained using the experimental designs for different scenarios assessed for the process, and Section 8 discusses reprocessing or excess material costs. Finally, Section 9 presents the conclusions from this study.

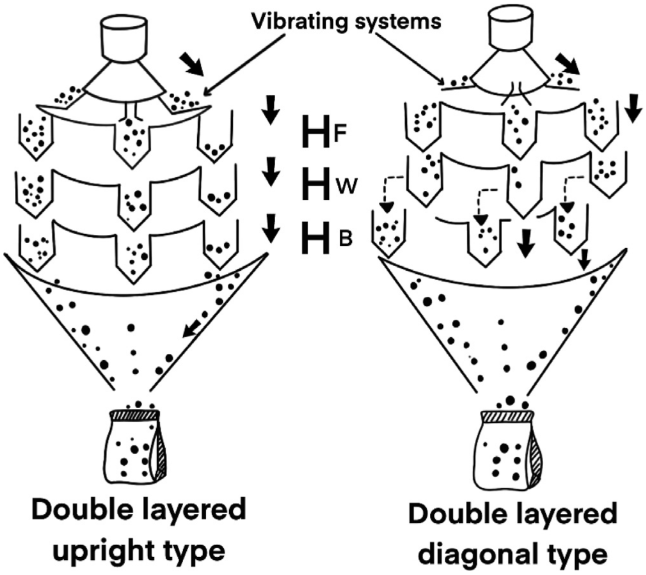

Fig. 1. Upright and diagonal multihead weigher components and system.

2. Background

Different researchers have already addressed the packaging problems associated with multihead machines. In fact, several studies have considered multiobjective process optimization based on a machine configuration. The net package content and time during which the product remains in the hoppers before being discharged are the two bicriteria optimization objectives studied in the scientific publications available. For example, an integer weight case study was conducted using a dynamic programming algorithm to minimize the maximum time that a product remains in the hopper before being discharged into a package while assuring that the total weight of each package was as close as possible to its target weight. [10]. This same principle was later extended to double-layered upright and diagonal machines from the discrete weight perspective [11]. Subsequently, an algorithm was introduced to reduce the execution times of the proposed models [12], thus increasing the efficiency of the packaging process proposed previously [10]. This new algorithm was then applied to duplex and quasiduplex machines [13]. Similarly, another study proposed the use of heuristic algorithms to achieve enhanced results in the bicriteria packaging process that targets label weights and priority orders [14,15]. Finally, new bicriteria approaches have been developed for optimizing the food packaging process for real weights [8,9]. These approaches are considered innovative because the relative importance of the objectives is ensured dynamically and adjusted for each packaging operation. Another study determined the operational process conditions [16], wherein the package weight and hopper priority were the main objectives. These approaches are very useful for the packaging of fresh or frozen products. However, no scientific research has studied diagonal and upright machines by considering real weights in hoppers based on preset *k* values. The foregoing adds a greater level of importance to the present study, wherein a new bicriteria optimization algorithm is proposed for double-layered upright machines by considering real weights and preset *k* values.

3. Machine configuration

The successful optimization of the packaging process requires two stages. The first stage corresponds to the machine configuration, which is addressed in this section and is related to the quantity of the product supplied to each hopper. The proposed algorithm is related to the number of *k* hoppers combined and is based on the average product supplied to each weighing hopper according to three strategies proposed in the literature [7,9] (Table 1 and Eqs. (1)–(3)). The three strategies known as S_1, S_2, and S_3 consider the set of *n* weighing hoppers divided into five, three, and one subgroups, respectively. In the S_3 strategy, each hopper is filled with the same average quantity of product ($\mu = T/k$) by considering the number of hoppers combined. However, in S_2 and S_3 strategies, each group receives a different average product quantity. To guarantee the variation in the average product supply, we will rely on the δ parameter, which is commonly used in statistical process control to simulate out-of-control processes. In this study, this parameter will be used to guarantee that not all hopper subgroups receive an even average product supply. The δ values used range from 0 to 3 with increments of $\delta_{min} = 0.5$.

Similarly, the coefficient of proportionality γ ($\sigma = \gamma\mu$) for the packaged product, as applied in other studies [16,17], is related to the average content captured by each hopper and its variability. For example, some studies [17] have proven

Table 1
Number of weighing hoppers (n_i) and average content of hoppers (μ_i) for each subgroup using the equal distribution strategy according the S_1, S_2, and S_3 strategies.

Equal distribution								
μ	n	S_1, $a = mod\left(\frac{n}{5}\right)$					S_2	S_3
		$a=0$	$a=1$	$a=2$	$a=3$	$a=4$		
$\mu_1 = \mu - \delta\sigma$	n_1	$\frac{n}{5}$	$[\![n/5]\!]$	$[\![n/5]\!]+1$	$[\![n/5]\!]+1$	$[\![n/5]\!]+1$	$[\![n/3]\!]$	n
$\mu_2 = \mu - (\delta - \delta_{min})\sigma$	n_2	$\frac{n}{5}$	$[\![n/5]\!]$		$[\![n/5]\!]$	$[\![n/5]\!]+1$		
$\mu_3 = \mu = \frac{T}{k}$	n_3	$\frac{n}{5}$	$[\![n/5]\!]+1$	$[\![n/5]\!]$	$[\![n/5]\!]+1$	$[\![n/5]\!]$	$n - 2 \cdot [\![n/3]\!]$	
$\mu_4 = \mu + (\delta - \delta_{min})\sigma$	n_4	$\frac{n}{5}$	$[\![n/5]\!]$	$[\![n/5]\!]$	$[\![n/5]\!]$	$[\![n/5]\!]+1$		
$\mu_5 = \mu + \delta\sigma$	n_5	$\frac{n}{5}$	$[\![n/5]\!]$	$[\![n/5]\!]$+1	$[\![n/5]\!]+1$	$[\![n/5]\!]+1$	$[\![n/3]\!]$	

that the theoretical standard deviation for a hopper with the target weight $T = 250$ g required for a product (such as ravioli) by combining $k = 5$ hoppers can be calculated as follows: $\sigma = 0.331 \cdot \frac{250\,\text{g}}{5} = 16.55$ g.

Thus, the S_1 strategy divides the n weighing hoppers into five subgroups, ($n_1, n_2, n_3, n_4,$ *and* n_5, *where* $\sum_{i=1}^{5} n_i = n$) and feeds different average product quantities to each subgroup ($\mu_1, \mu_2, \mu_3, \mu_4,$ *and* μ_5) according to Eq. (1). The S_2 strategy divides the n weighing hoppers into three subgroups ($n_1, n_3,$ *and* n_5) and feeds different average quantities to each subgroup ($\mu_1, \mu_3,$ *and* μ_5) according to Eq. (2). Finally, in the S_3 strategy, the n weighing hoppers are filled with the same amount of product $\mu = \frac{T}{k}$ for each hopper according to Eq. (3). In all three (S_1, S_2 *and* S_3) strategies, each booster hopper adopts the same supply from its corresponding weighing hopper until the complete filling of $2n$ system hoppers.

$$w_i \sim \mathrm{N}(\mu_j, \sigma = \gamma\mu_j) = \left\{ \begin{array}{c} \mu_1 = \mu - \delta\sigma \\ \mu_2 = \mu - (\delta - \delta_{min})\sigma \\ \mu_3 = \mu = \frac{T}{k} \\ \mu_4 = \mu + (\delta - \delta_{min})\sigma \\ \mu_5 = \mu + \delta\sigma \end{array} \right\} \quad (1)$$

$$w_i \sim \mathrm{N}(\mu_j, \sigma = \gamma\mu_j) = \left\{ \begin{array}{c} \mu_1 = \mu - \delta\sigma \\ \mu_3 = \mu = \frac{T}{k} \\ \mu_5 = \mu + \delta\sigma \end{array} \right\} \quad (2)$$

$$w_i \sim \mathrm{N}(\mu, \sigma = \gamma\mu) = \left\{ \mu = \frac{T}{k} \right\} \quad (3)$$

In addition to the S_1, S_2, and S_3 filling strategies (in which an average product supply is established for each subgroup), we must define a distribution of hoppers, namely, the number of hoppers assigned to each subgroup (Tables 2 and 3). In this sense, three types of distributions are proposed: equal, central, and extreme. In the equal distribution strategy, each group includes approximately the same number of hoppers. For example, in the S_1 filling strategy, the total number of hoppers ((n)) is divided among the five subgroups $\left(\frac{n}{5}\right)$. Each n_i ($i = 1, \ldots, 5$) is assigned a number of hoppers equal to the largest integer multiplied by 5 that is closest to n (integer part $\left[\!\left[\frac{n}{2}\right]\!\right]$). If the remainder of the division $\left(mod\left(\frac{n}{5}\right)\right)$ is 1, the central group (n_3) will have one more hopper; if it is 2, a hopper will be assigned to each extreme group. For $mod\left(\frac{n}{5}\right) = 3$, they are distributed among $n_1, n_3, andn_5$ and $mod\left(\frac{n}{5}\right) = 4$. In the case of n_3, the subgroup will have one less hopper than the rest. The central distribution strategy assigns as many hoppers as possible to the central set of hoppers. For example, in the S_2 filling strategy for $n \leq 8$ and $n > 8$, one and two hoppers will be assigned, respectively, to each end and the surplus is assigned to the central group. Finally, the extreme distribution strategy assigns the largest number of hoppers to the extreme subgroups n_1 *and* n_5 and the least number of hoppers to n_3.

4. Target process functions

This section presents the mathematical model for the process in double-layered upright and diagonal machines. The symbols, decision variables, target functions, and constraints for each process are also presented.

4.1. Target function

In the considered bicriteria packaging problem, two objectives are set. The first objective is related to the product weight packaged, and the second objective refers to the time the product remains in the hopper before being packaged. Each objective will be addressed in detail below.

Table 2
Results obtained using the S_1, S_2, and S_3 strategies for $k = 3, 4, 5,$ and6, $P_{max} = 10$, and $\gamma = 0.123$.

Fusilli												
	k	Upright			Diagonal			μ_1	μ_2	μ_3	μ_4	μ_5
		μ_{paq}	σ_{paq}	CV_{paq}	μ_{paq}	σ_{paq}	CV_{paq}					
	3	251.430	1.083	0.0043	251.570	1.149	0.0045	62.83	67.96	83.33	98.71	103.83
S_1	4	251.149	0.974	0.0038	251.248	0.954	0.0038	47.13	50.97	62.50	74.03	77.88
	5	250.894	0.885	0.0035	250.845	0.695	0.0027	37.70	40.78	50.00	59.23	62.30
	6	250.612	0.500	0.0019	250.587	0.489	0.0019	31.42	33.98	41.67	49.35	51.92
	3	251.653	1.219	0.0048	251.523	1.121	0.0044	62.83		83.33		103.83
S_2	4	251.211	0.941	0.0037	251.142	0.878	0.0034	47.13		62.50		77.88
	5	250.778	0.609	0.0024	250.724	0.577	0.0023	37.70		50.00		62.30
	6	250.531	0.429	0.0017	250.517	0.424	0.0016	31.42		41.67		51.92
	3	251.721	1.248	0.0049	251.290	0.992	0.0039			83.33		
S3.	4	251.316	0.995	0.0039	251.009	0.856	0.0034			62.50		
	5	250.894	0.716	0.0028	250.754	0.7641	0.0030			50.00		
	6	250.758	0.948	0.0037	250.597	0.760	0.0030			35.714		

Table 3
Factors and levels studied in the DOE for diagonal and upright machines.

Factors	Factor levels						
Strategy: Number of subgroups	$S_1 = 5$	$S_2 = 3$	$S_3 = 1$				
Hopper distribution	Central	Equal	Extreme				
Filling position delta (δ)	0	0.5	1	1.5	2	2.5	3

4.1.1. First objective

This objective basically aims to minimize the difference between the effective content of the package W_ℓ and the net content reported on the label T. Binary vectors X_i and Y_j are defined for the weighing hoppers Eq. (4) and booster hoppers Eq. (5), respectively; components x_i or y_j take the value 1 if the hopper weight H_m is selected or takes the value 0 (Eqs. (6) and (7)).

$$X_i = (x_1, x_2, x_3, x_4, \ldots, x_n) \tag{4}$$

$$Y_j = (y_{n+1}, y_{n+2}, y_{n+3}, y_{n+4} \ldots y_{2n}) \tag{5}$$

$$x_i = \begin{cases} 1, & \text{if } H_{Wi} \textit{ is the selected hopper} \\ 0, & \textit{otherwise} \end{cases} \tag{6}$$

$$y_j = \begin{cases} 1, & \textit{if } H_{Bi} \textit{ is the selected hopper} \\ 0, & \textit{otherwise} \end{cases} \tag{7}$$

To select the hoppers whose sum of the weights are the closest and greatest to the package weight, the function is minimized $f_1 = W_\ell - T$.

$$\textit{minimize } f_1(x, y) = \left[\sum_{i=1}^{n} x_i w_i + \sum_{j=n+1}^{2n} y_j w_j \right] - T \tag{8}$$

4.1.2. Second objective

In this case, the discharge priority will be given to the hoppers that have not been selected for the longest time to make up the content of a package; hence, the function is maximized $f_2 = P_\ell - P_{max} \leq 0$.

$$\textit{minimize } f_2(x, y) = P_{max} - \left[\frac{\sum_{i=1}^{n} x_i p_i + \sum_{j=n+1}^{2n} y_j p_j}{k} \right] \tag{9}$$

Restrictions: In both the objectives, the restrictions are related to hopper restrictions. For the double-layered upright machine, the operating restriction of the system is that an upper hopper cannot be selected if its corresponding booster hopper has not been selected, indicating that Eq. (10) must be used. For the double-layered diagonal machine, a weighing hopper and its booster cannot be selected simultaneously, which is guaranteed using Eq. (11).

$$x_i - y_j \leq 0 \tag{10}$$

$$x_i + y_j \leq 1 \tag{11}$$

At each ℓth iteration, the best combination of k hoppers from the available $2n$ hoppers must be selected. If Eqs. (6)–(10) for the double-layered upright machine are satisfied simultaneously for the entire set of k combined hoppers and Eqs. (6)–(9) and (11) for the double-layer diagonal machine are satisfied, the combination is valid.

4.2. Bicriteria function

A function is proposed that combines the two target functions to satisfy them simultaneously. For each iteration, our approach is to determine a combination of k hoppers that are closer to the ideal point (T, P_{max}) with $\left(W_\ell, \frac{P_\ell}{k}\right)$ coordinates, namely, the point that is closest to the ideal point. W_ℓ is some part of the first objective that must be minimized, and $\frac{P_\ell}{k}$ is some part of the second objective that must be minimized. Therefore, Eq. (12) representing the Euclidean distance integrates both the objectives, wherein we have also added θ to regulate the relative weight of the discharge priority of the k hoppers. The optimal point is found during each iteration using the set of k hoppers that minimize D.

$$D = \sqrt{(1-\theta)\left(W_\ell - T\right)^2 + \theta\left(\frac{P_\ell}{k} - P_{max}\right)^2} \tag{12}$$

θ varies at each iteration and takes values of $0 \le \theta < 1$, and it is defined according to Eq. (13).

$$\theta = \frac{p_{max\ell} - 1}{P_{max}} \tag{13}$$

Notably, in the first ($\ell = 1$) iteration, hoppers show a priority of 1. Hence, $p_{max\ell} = 1$, making $\theta = 0$. Then, the set of k hoppers closer to the target weight will be selected. Similarly, if the ℓth iteration presents more than one $k - nuple$ hoppers wherein $\forall i$, $p_{i\ell} = P_{max}$, the expression $\left(\frac{P_\ell}{k} - P_{max}\right) = 0$ and the set of k hoppers with less $|W_\ell - T|$ is selected. Alternatively, among the set of $k - nuple$ hoppers with equal weight W_ℓ, those with the highest P_ℓ are chosen.

5. Optimal weighing and packaging algorithm in double-layered upright and diagonal machines

Our food packaging problem is related to the knapsack problem [18–20], and the search algorithm used in the packaging process is brute force [21], in which all combinations of k hopper that meet the constraints are tested individually and the one closest to the optimal point is selected after evaluating it using the target function. Further, each step of the algorithm is presented.

Step 1. In the initial process configuration, the following parameters are recorded: the number of packages processed (Q); number of weighing hoppers (n); number of hoppers combined (k); target label weight (T) ; and maximum allowed priority of each hopper (P_{max}), understood as the maximum number of packages that can wait to be discharged. All hoppers start at zero priority $p_i = 0$.
Step 2. Once the initial settings are assigned, the empty weighing hoppers H_{Wi} are loaded with randomly assigned weights w_i, $i \in \{1, 2, \ldots, n\}$ according to the S_1, S_2, or S_3 strategy and by considering the number of hoppers in each subgroup. A priority of $p_{i\ell+1} = p_{i\ell} + 1$ is assigned.
Step 3. To the extent that the weighing hoppers record the amount of product received, they discharging their content to their empty booster hoppers H_{Bj}: $w_{(n+i)\ell} = w_{i\ell}$, $p_{(n+i)\ell} = p_{i\ell}$, $w_{i\ell} = 0$, and $p_{i\ell} = 0$. The weighing hoppers previously discharged are reloaded with the new content.
Step 4. Once all the hoppers are filled, the software verifies that they do not exceed the discharge time limit. If $p_{i\ell} > P_{max}$, then $w_{i\ell} = 0$ and $p_{i\ell} = 0$.
Step 5. The software also validates that all the hoppers contain the product; otherwise, it restarts the loading process for the empty hoppers.
Step 6. The highest priority for all hoppers is obtained $p_{max\ell}$ and calculated for $\theta = \frac{p_{max\ell}-1}{P_{max}}$. The hoppers that meet the criteria (reported in Section 4.2) are combined for the upright or diagonal machines by calculating the distances of each combination to the ideal point and saving the information of k hoppers resulting in the minimum distance. The product is discharged from the optimum-point hoppers and packaged once each combination has been verified.
Step 7. The process starts again with the loading of the empty hoppers until the total number of packages (Q) required has been reached.

6. Preliminary analysis

During the algorithm implementation, we used the coefficient of proportionality of $\gamma = 0.123$ for fusilli product [17]. In addition, a total of $n = 16$ weighing hoppers were assessed at $k = 3, 4, 5$ and 6, $\delta = 2$, $\delta_{min} = 0, 5$, $P_{max} = 10$, and a target weight of $T = 250$ gm. The number of hoppers in each subgroup is shown in Table 2. The calculated performance measures were the average weight of packages produced (μ_{paq}), standard deviation of the packages produced (σ_{paq}), and coefficient of variation of the packages produced $\left(CV_{paq} = \frac{\mu_{paq}}{\sigma_{paq}}\right)$ for Q = 10,000 packages.

The data produced show a low T value, with asymmetric behavior contradicting the assumption of normalcy, because weights greater than and as close as possible to the target weight are evaluated. The results for the double-layers upright

Table 4
Robust ANOVA for CV_{paq}: diagonal and upright machines.

	Diagonal machine		Upright machine	
	Value	p value	Value	p value
Subgroup	2,6203.307	0.0001	12,555.0799	0.0001
Strategy	2,710.003	0.0001	759.7305	0.0001
Delta	13,780.865	0.0010	10,456.7978	0.0010
Subgroup · Strategy	9,867.217	0.0010	4,743.2698	0.0010
Subgroup · Delta	28,433.190	0.0010	15,806.8678	0.0010
Strategy · Delta	2,441.629	0.0010	914.9166	0.0010
Subgroup · Strategy · Delta	9,077.823	0.0010	4,941.5710	0.0010

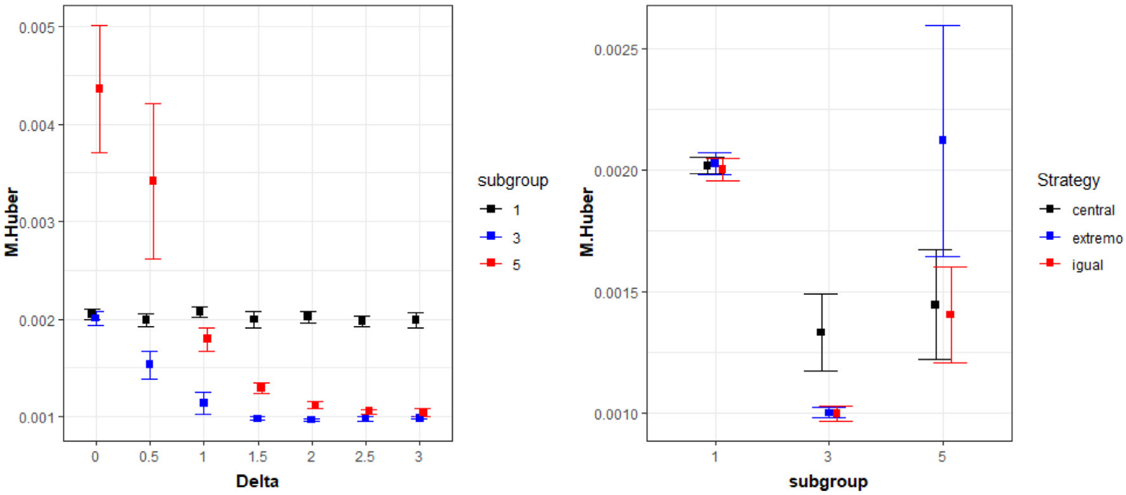

Fig. 2. Interaction plot with Huber M-estimators for diagonal machines.

and diagonal double machines are presented in Table 4. In relation to the number of hoppers combined (k), the results improve by increasing the number of hoppers to be combined k. Regardless of the machine type and strategy, the values closest to T are obtained with $k = 6$, with low coefficients of variation $\left(CV_{paq}\right)$ in the case of the fusilli and ravioli products.

Alternatively, when reviewing the strategies, by dividing the n hoppers in three groups, the S_2 strategy reduces process variability in both machines and products. Similarly, better performance is preliminarily obtained in the case of the diagonal machine during the process when comparing the strategies and the number of hoppers combined. Based on these findings, the experimental design presented in Section 7 was proposed to determine the conditions that reduce the process variability by providing values greater than and as close as possible to the target weight when prioritizing the hoppers that hold more undischarged contents as the process progresses.

7. Experimental design

Based on our preliminary results, we conducted a multifactorial design of experiments (DOE) of fixed effect factors [22] to determine the best combination of treatments that provides the least process variability and reaches its optimum point. The multifactorial design consists of three factors, as shown in Table 5 for a total of 63 treatments in each machine, each of them with three replicates (total 252 runs). The coefficient of variation (CV_{paq}) recorded in each run of 10,000 packages is the response variable used in the design.

In addition to analyzing the best factor combination, the upright and diagonal machines are compared to determine the machine that offers lower process variability. Here, the first factor encompasses three levels consisting of the S_1, S_2, and S_3 strategies that determine the hopper subgroups. The second factor is the type of hopper grouping, which will be performed using the equal, central, and extreme distribution strategies. The filling position constitutes the third factor and is set at seven levels $\delta = 0$, 0.5, 1, 1.5, 2, 2.5, and 3 with $\delta_{min} = 0.5$. The design is applied to each machine with $n = 16$ weighing hoppers, $k = 6$ hoppers combined, maximum priority of $P_{max} = 10$, a target weight of 500 g for the fusilli product, and $\gamma = 0.123$.

Given the asymmetric behavior of the data, the robust ANOVA and Huber estimators for interactions [23] are used to assess our design. Based on the results shown in Table 6, there are significant differences between the levels of each factor as well as in their interactions by obtaining significant p values. When assessing Figs. 2 and 3, the process exhibited similar behavior for both machines according to the configuration and combination of factors. In the upright and diagonal

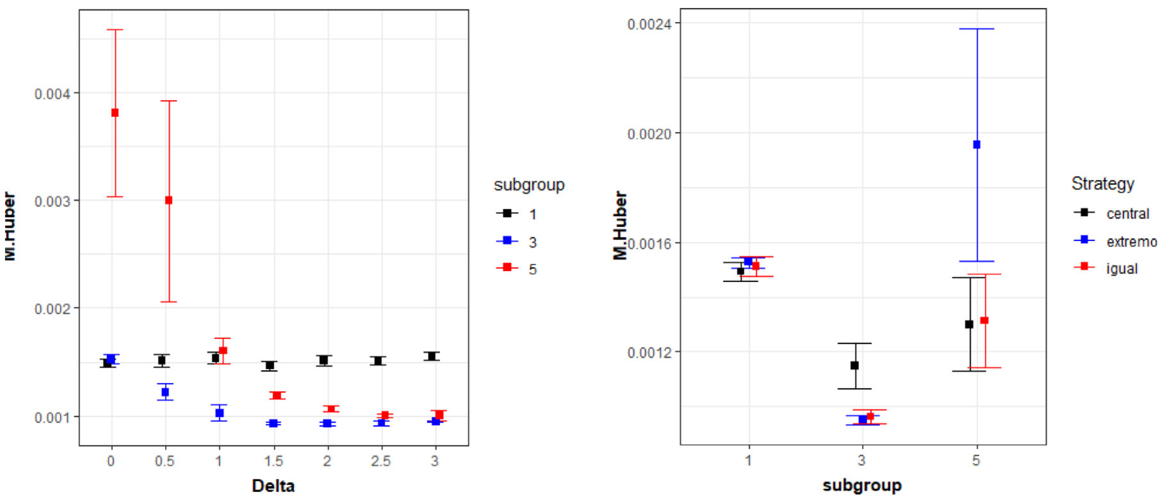

Fig. 3. Interaction plot with Huber M-estimators for upright machines.

Table 5
Fusilli costs.

	Fusilli
c: Raw material unit costs (€cent/g)	0.03
r: Reprocessing costs (€cent/package)	5.64

Table 6
Robust ANOVA for estimating costs: diagonal and upright machines.

	Diagonal machine		Upright machine	
	Value	p value	Value	p value
Subgroup	47,088.31	0.0001	27,378.398	0.0001
Strategy	10,512.32	0.0001	5,612.130	0.0001
Delta	36,705.61	0.0010	25,834.105	0.0010
Subgroup · Strategy	39,803.86	0.0010	16,350.307	0.0010
Subgroup · Delta	69,503.24	0.0010	28,580.031	0.0010
Strategy · Delta	16,704.41	0.0010	8,471.424	0.0010
Subgroup · Strategy · Delta	51,675.71	0.0010	22,678.960	0.0010

machines, considering the delta levels (δ), the coefficient of variation decreases for values between $\delta = 1.5$ and $\delta = 3$. Regarding the number of subgroups in which the set of n hoppers is divided, the strategy $S_2 = 3$ achieves the best results for the packaging process in the cases studied using the two machines. Alternatively, when considering the distribution strategy, the case of extreme distribution is not recommended when working with five hopper subgroups because it shows the largest variability. However, for both machines, this distribution generates the optimum value considering three subgroups (S_2) with $\delta = 2$, regardless of the type of machine.

8. Economic analysis

The product quality depends on how close its quality features are to their nominal value, and everything that deviates from the said nominal value is considered a loss to society [24]. Thus, any gap between the actual package contents and the content reported on the label represents additional costs for the company. Similarly, considering the hopper priority, hoppers will be discharged if they exceed the waiting time (P_{max}) and in the model proposed herein, the hoppers must enter the reprocessing cycle. To calculate the excess content costs and the costs associated with reprocessing the product discharged by the hoppers, the reference values implemented by Beretta [25] and listed in Table 5 below will be used.

Considering that r represents the value per each 200-g package, we will use $r_1 = \frac{5.64}{200} = 0.0282$ as the reprocessing cost per gram. The excess product costs for each package are estimated $(W_\ell - T)$, so the total excess product for each cycle of $Q = 10{,}000$ packages is $W_T = (\mu_w - T) \cdot 10{,}000$, where $\mu_w = \frac{\sum_1^Q W_\ell}{Q}$ is the average content of Q packages along with the cost of reprocessing the discharged product (W_D) and W_D is the quantity of product discharged by the hoppers that exceeded the waiting time (P_{max}). Then, the costs for all 10,000 packages are obtained using Eq. (14).

$$cost = W_T \cdot \mathrm{c} + W_D \cdot r_1 \tag{14}$$

where c is the unit cost per gram and r_1 is the cost per gram of the reprocessed product.

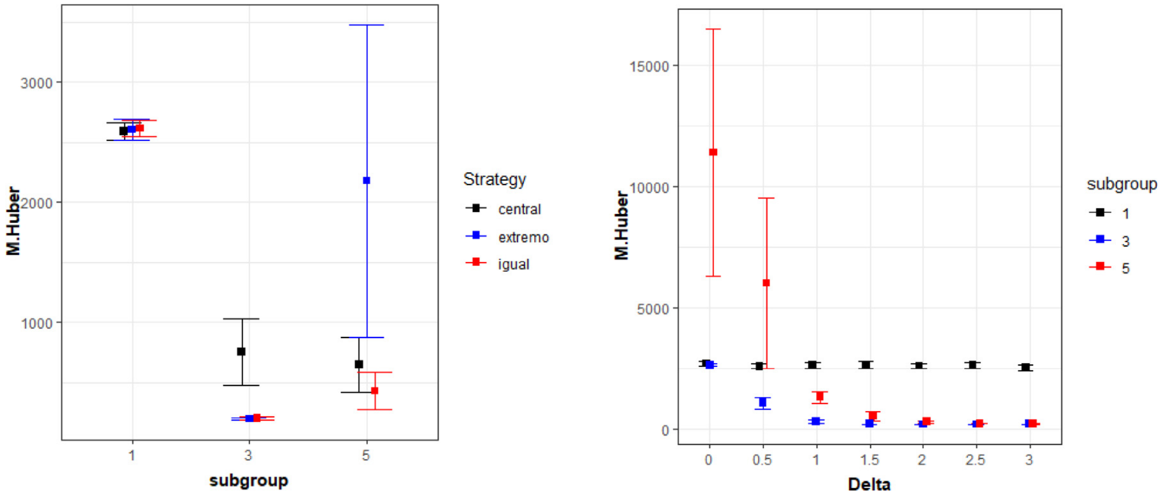

Fig. 4. Interaction plot with Huber M-estimators for diagonal machines.

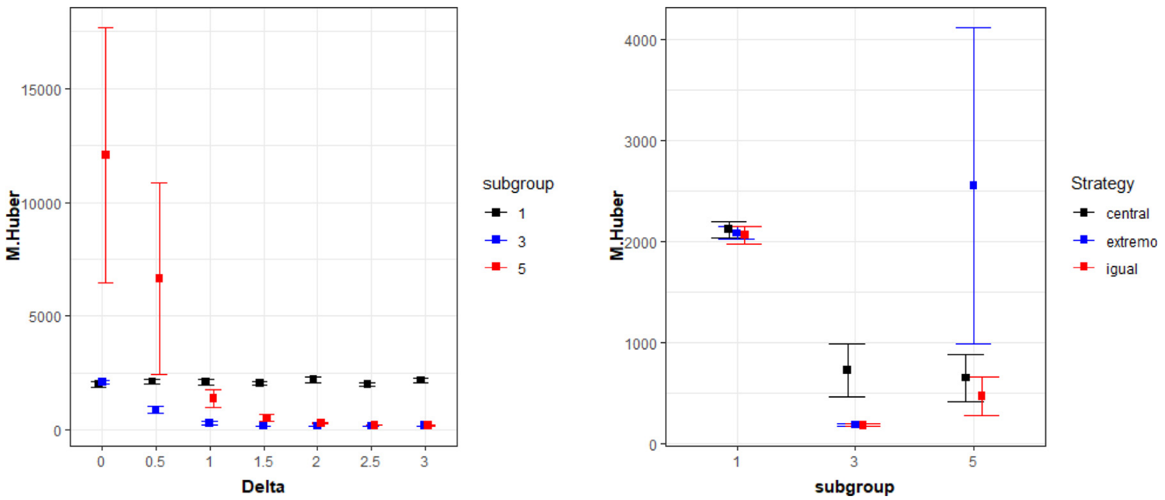

Fig. 5. Interaction plot with Huber M-estimators for upright machines.

Based on the results shown in Table 5, there are significant differences between the levels of each factor as well as in their interactions based on the p values. When assessing Figs. 4 and 5, the process exhibited similar behavior for both machines based on the configuration and combination of factors. In the upright and diagonal machines, considering the δ, the costs decrease for values between $\delta = 1.5$ and $\delta = 3$. Regarding the number of subgroups in which the set of n hoppers is divided, the strategy $S_2 = 2$ yields the best results for the packaging process in the cases studied using the two machines. Alternatively, when considering the distribution strategy, the extreme distribution is not recommended when working with five hopper subgroups as it generates higher costs. Both machines achieve the lowest costs when using the equal distribution per subgroup by considering three subgroups (S_2) with $\delta = 3$.

9. Conclusions

The food packing industry constantly requires process optimization to remain competitive in the market by reducing its operating costs while improving the quality of the final product according to the needs of its customers. The multihead weighing process is no stranger to this reality. In this study, a new food packaging algorithm and its corresponding optimization model for double-layered upright and diagonal multihead weighers are presented. The algorithm is initially validated by considering the machines with $n = 16$ weighing hoppers using three hopper-filling strategies (S_1, S_2, and S_3) and by assigning each subgroup the same number of hoppers and different combinations of hoppers k. To assess the performance of the process, two products (ravioli and fusilli) were tested at different γ values and an DOE was established to compare different distribution strategies (central, equal, or extreme), subgroup strategy ($S_1, S_2,$ and S_3), and filling position δ factor levels. These conditions were applied to in both the machines using the fusilli product as an example ($\gamma = 0.123$) with $n = 16$ feeding hoppers, $k = 6$ hoppers combined, and a maximum priority of $P_{max} = 10$. To assess the process, the mean and standard deviations of all 10,000 packages were recorded to obtain the CV_{paq} as a design response variable. The results indicated that the best filling configuration to reduce the process variability is the S_2 strategy, particularly when using three hopper subgroups, assigning the largest number of hoppers to the extreme group, and using filling position $\delta = 2$. Moreover, it is evident that the excess product and reprocessing costs decrease in the case of both machines when using an equal distribution of hoppers per group when considering three subgroups (S_2) with $\delta = 3$.

Data availability

The authors are unable or have chosen not to specify which data has been used.

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