Summary

Distributional chaos was introduced by Schweizer and Smítal in [SS94] from the notion of Li-Yorke chaos in order to imply positive topological entropy for the mappings from the compact interval into itself. Distributional chaos for linear operators was considered for the first time in [Opr06] and firstly studied in the infinite-dimensional linear setting in [MGOP09].

The concept of distributional chaos for an operator (semigroup) consists on the existence of an uncountable subset and a positive real number δ such that for every pair of distinct elements of the uncountable set, both the upper density of the set of iterations (times) in which the difference of the images by the corresponding operator is greater than δ, and the upper density of the set of iterations (times) in which that difference is as small as we want, are equal to one.

This thesis is divided into six chapters. In the first one, we do a summary of the state of the art about chaotic dynamics for C_0-semigroups of linear operators.

In the second chapter, we show the equivalence between the distributional chaos of a C_0-semigroup and the distributional chaos of each one of its non-trivial operators. We also characterize the distributional chaos of a C_0-semigroup in terms of the existence of a distributionally irregular vector.

The notion of hypercyclicity for an operator (semigroup) consists on the existence of an element with dense orbit by the operator (semigroup). If, in addition, the set of periodic points is dense, we say that the operator (semigroup) is Devaney chaotic. One of the most useful tools to check whether an operator is hypercyclic is the Hypercyclicity Criterion, first stated by Kitai in 1982. In [BBMGP11], Bermúdez, Bonilla, Martínez-Giménez and Peris introduce the Criterion for Distributional Chaos (CDC) for operators. We state and prove a version of the CDC for semigroups.

In addition, in the semigroup setting, Desch, Schappacher and Webb studied in [DSW97] hypercyclicity and Devaney chaos for C_0-semigroups, giving a criterion for Devaney chaos based on the spectrum of the infinitesimal generator of the C_0-semigroup. In the third chapter, we establish a criterion for the existence of a dense distributionally irregular manifold (DDIM) in terms of the spectrum of the infinitesimal generator of the C_0-semigroup.

In Chapter 4, some sufficient conditions for distributional chaos for the translation C_0-semigroup on weighted L^p-spaces are given in terms of the admissible weight function. Moreover, we establish a complete analogy between the study of distributional chaos for the translation C_0-semigroup and for backward shift operators on weighted sequence spaces.

The fifth chapter is devoted to the study of the existence of C_0-semigroups for which every non-zero vector is a distributionally irregular vector. We also give an example of such C_0-semigroups that is not hypercyclic.

In Chapter 6, the DDIM criterion is applied to several examples of C_0-semigroups. Some of them are the solution semigroup of a partial differential equation, like the hyperbolic heat transfer equation or the von Foerster-Lasota equation, and others are the solution of an infinite system of ordinary differential equations used to modelize the dynamics of a population of cells under simultaneous proliferation and maturation.