

# Abstract

More and more challenging designs are required everyday in today's industries. The traditional trial and error procedure commonly used for mechanical parts design is not valid any more since it slows down the design process and yields suboptimal designs. For structural components, one alternative consists in using shape optimization processes which provide optimal solutions. However, these techniques require a high computational effort and require extremely efficient and robust Finite Element (FE) programs. FE software companies are aware that their current commercial products must improve in this sense and devote considerable resources to improve their codes. In this work we propose to use the Cartesian Grid Finite Element Method, cgFEM as a tool for efficient and robust numerical analysis. The cgFEM methodology developed in this thesis uses the synergy of a variety of techniques to achieve this purpose, but the two main ingredients are the use of Cartesian FE grids independent of the geometry of the component to be analyzed and an efficient hierarchical data structure. These two features provide to the cgFEM technology the necessary requirements to increase the efficiency of the cgFEM code with respect to commercial FE codes. As indicated in [1, 2], in order to guarantee the convergence of a structural shape optimization process we need to control the error of each geometry analyzed. In this sense the cgFEM code also incorporates the appropriate error estimators. These error estimators are specifically adapted to the cgFEM framework to further increase its efficiency. This work introduces a solution recovery technique, denoted as SPR-CD, that

in combination with the Zienkiewicz and Zhu error estimator [3] provides very accurate error measures of the FE solution. Additionally, we have also developed error estimators and numerical bounds in Quantities of Interest based on the SPR-CD technique to allow for an efficient control of the quality of the numerical solution. Regarding error estimation, we also present three new upper error bounding techniques for the error in energy norm of the FE solution, based on recovery processes. Furthermore, this work also presents an error estimation procedure to control the quality of the recovered solution in stresses provided by the SPR-CD technique. Since the recovered stress field is commonly more accurate and has a higher convergence rate than the FE solution, we propose to substitute the raw FE solution by the recovered solution to decrease the computational cost of the numerical analysis. All these improvements are reflected by the numerical examples of structural shape optimization problems presented in this thesis. These numerical analysis clearly show the improved behavior of the cgFEM technology over the classical FE implementations commonly used in industry.