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RECENT ADVANCES IN STRUCTURAL OPTIMIZATION

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Abstract: Structural Optimization has been widely studied issue during the last 50 years. Although Mathematical Programming initially the most-used technique, it has been replaced by other metaheuristic techniques. Among them, Genetic Algorithms is the most remarkable. This paper will cover a little description of each technique as well as the main reports and drawbacks. Finally, the most-used structure for benchmarking will be depicted, and the best reported results shown and commented.

1. INTRODUCTION

Optimal design of structures has been an active research field from ancient times. The first remarkable work about optimal design of structures was done by Galileo Galilei in the work entitled "*Discorsi e dimonstrazioni matematiche, intorno, a due nuove scienze attenenti alla mecanica et i movimenti local*" [1]. Later, Maxwell [2] and Michell [3] established the main principles for optimal design in trusses.

During the first three-quarters of the XX century, there are only a few remarkable works about this issue [4-9], and most of them were variations of Michell's studies. Later, the development of mathematical programing and computers, led this field to a higher step.

Most of the reported works until then can be classified in three groups:

- Size optimization (Figure 1): where weight reduction is achieved by changing the sectional areas of trusses while nodal coordinates and connectivity are fixed.
- Shape optimization (Figure 2): where weight reduction is achieved by changing the connectivity of nodes while the nodal coordinates and sectional areas are fixed.
- Topological optimization (Figure 3): where weight reduction is achieved by changing the nodal coordinates and connectivity while the sectional areas are fixed.

Those optimization methods can be combined in two ways:

- Nested Analysis And DesigN (NAND): where size or topology are optimized in nested loops, so that while optimizing size, topological variables remain fixed and vice-versa.
- Simultaneous Analysis and Design (SAND): where both size and topology are optimized simultaneously.

Although in the beginning, mathematical programing the most-used technique it has been relegated because its drawbacks:

- It cannot handle with mixed continuous and discrete design variables.
- It cannot handle nonlinear constraints.

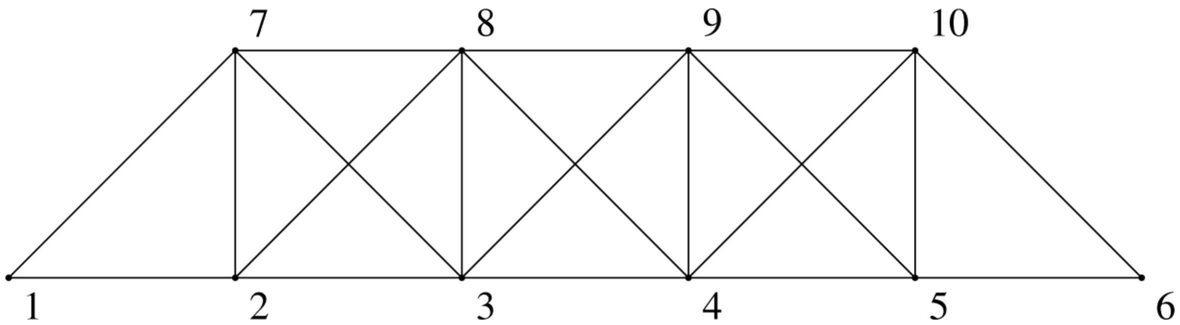

Figure 1. Size optimization

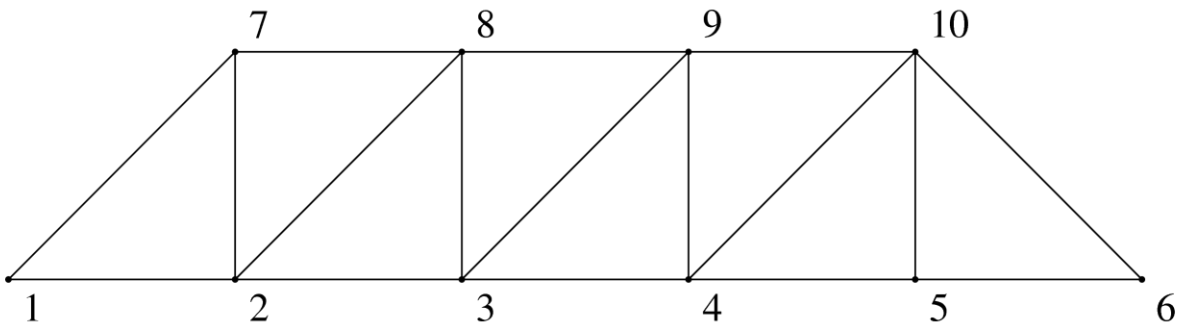

Figure 2. Shape optimization

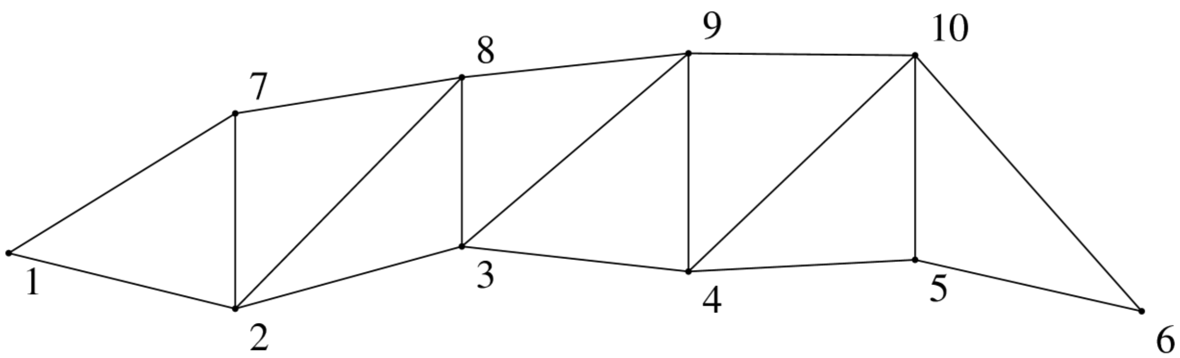

Figure 3. Topological optimization

So, other metaheuristic algorithms, most of them based on Evolutionary Computation, have been developed during the last twenty years.

Figure 4 shows that the two most important techniques in the number of papers are mathematical programming and genetic algorithms. However, the trends shown at Figure 5 reveal that genetic algorithms are the most-used technique. At present, the rest of metaheuristic algorithms are even most important than mathematical programing.

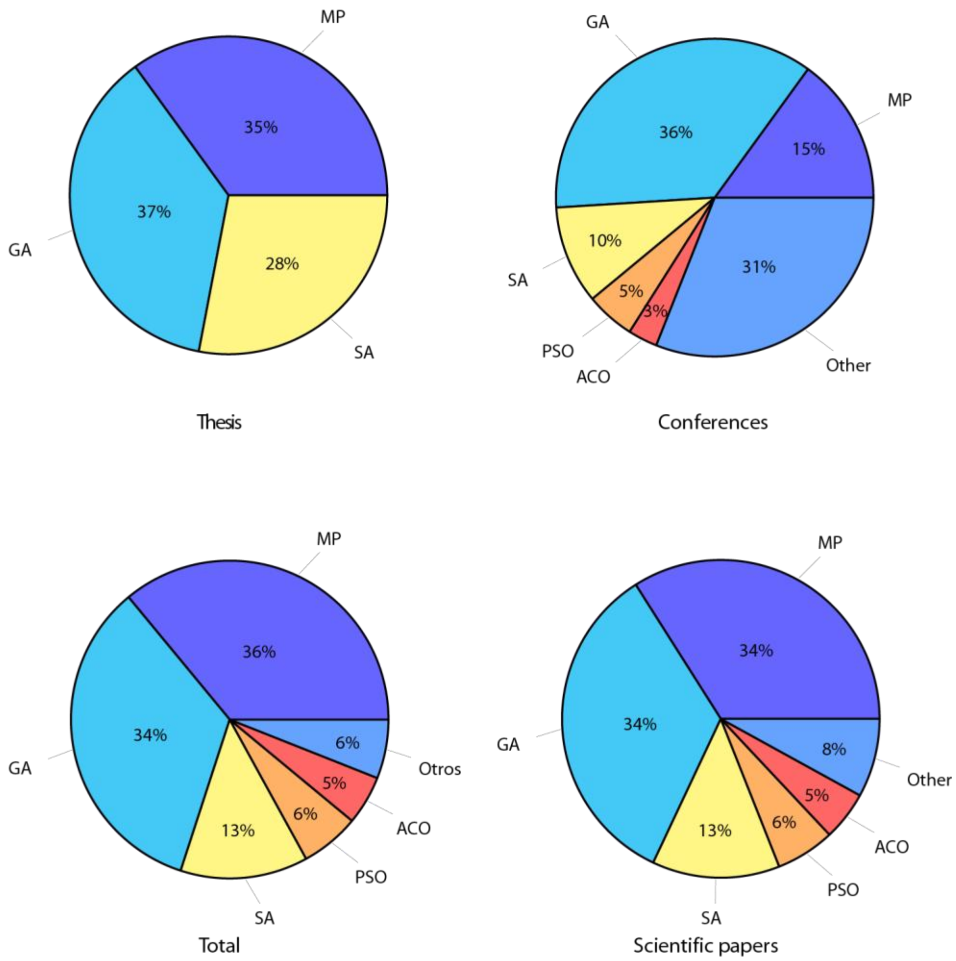

Figure 4. Overall number of published papers related to structural optimization in time.

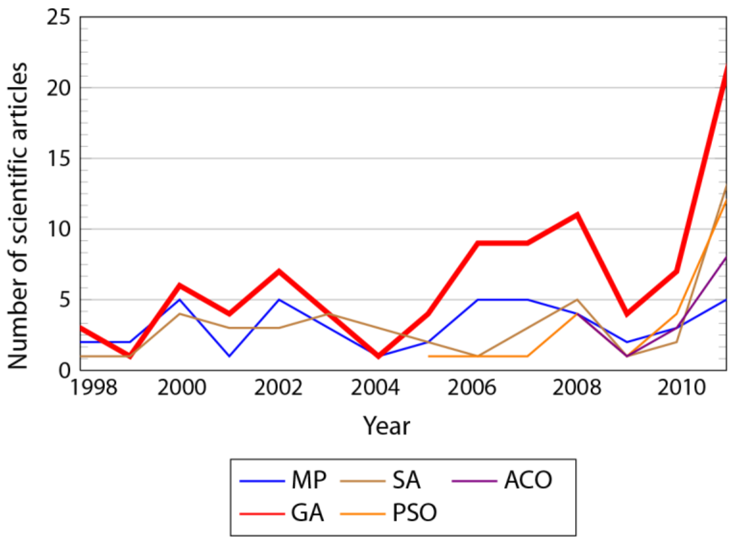

Figure 5. Number of scientific articles during the last 13 years related to structural optimization

2. METAHEURISTIC TECHNIQUES

Unlike traditional techniques, metaheuristic techniques do not follow predefined methods or rules in search. Regardless not using a direct deductive method, they are able to find good solutions in a reasonable time.

Its main drawback is that do not guarantee finding the optimum, so to ensure it; several runs are needed. Therefore, they are used if a specific algorithm does not exist.

During last decades, it has appeared countless techniques. Among them, the following must be remarked:

2.1. SIMULATED ANNEALING (SA)

This technique tries to imitate the annealing process in melted metals, freezing slowly until reaches its solid state. If freezing is slow enough, the molecules are reorganized so that the energy function achieves a global minimum. Otherwise, the energy function achieves a local minimum.

The working scheme of this algorithm is very simple. First, a trial design is chosen, usually randomly. Next this design is evaluated by the objective function. If this design is unfeasible, it is rejected. Otherwise, if the objective function returns a better result than the local optimum at this moment, then the design is accepted as new local optimum. If the objective function returns a worse result than the local optimum, the design will be accepted or rejected depending on a probabilistic criterion. The acceptance probability is tuned by a parameter called *Temperature* in analogy with the annealing process. The *Temperature* establishes the acceptance threshold and usually takes a high value initially, and it is lowered in each iteration following the metal freezing rule:

$$p = e^{-\frac{S_2 - S_1}{T}} \tag{1.1}$$

Where S_2 is the infra optimum value, $S1$ is the optimum value, and T is the *Temperature*.

Simulated Annealing has been widely studied in structural optimization problems [10-19] due to its simplicity and capacity to find local optima even with a large number of design variables. However, its main drawbacks are the difficulty of tuning the Temperature parameter, and the inability to ensure the global optimum.

2.2. GENETIC ALGORITHMS (GA)

This technique was developed by John Henry Holland and its collaborators [20-22] at the end of the .60s. Among them must be highlited De Jong [23] and Goldberg, which popularized GAs trought his seminal work: *Genetic algorithms in search, optimization, and machine learning* [24] .

GAs are based on the Natural Selection Theory principles:

- Survival of the fittest.
- Evolution is generated during reproduction.
- Sons are generated by parent’s chromosome crossover.
- Mutation allows generating sons far different from their parents.

According to Golberg, GAs are different to other techniques because:

- They use fitness functions instead of derivatives
- They work with coded variables.

- They work with a series of design points instead of a lonely point.
- They work with stochastic rules.
- Without time restriction, it is mathematically possible to achieve the global maximum

Figure 6 depicts a Simple GA flowchart. First, the individuals of the initial population are initialized by the initialization operator, which usually fills with random data the genotypes. Following, the new created individuals are evaluated. Next, the convergence is checked. If it is achieved, the algorithm stops, otherwise some individuals are selected for reproduction or mating. Which or how individuals are chosen is managed by the selection operator. Later, the genes of the selected parents are crossed to create new individuals. Crossover is handled by the crossover operator who determines where and how the genes are crossed. Following the new individuals are mutated according to the mutation operator. Finally the mutated individuals are evaluated, repeating the loop until the convergence is achieved.

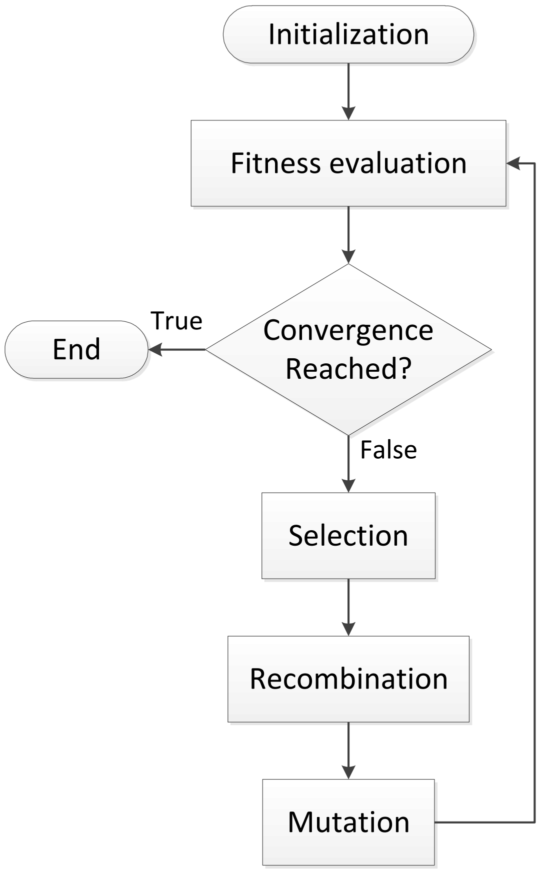

Figure 6. Simple GA Flowchart

Genetic Algorithms is the most-used technique in structural optimization problems. Among them [15, 25-55] must be remarked.

The main drawbacks of this technique are: difficult operator parameter tuning and strong problem dependence for operator and tuning.

2.3. EVOLUTIVE ESTRATEGIES (ES)

It is a variant of GAs developed by Ingo Rechenberg and Hans-Paul Schwefel where the crossover operator is odd and the mutation rate is very high. The search process is undertaken by the mutation operator alone.

Although it has not been so extensively studied like GAs, it has given some remarkable works [56-58].

This technique holds the same drawbacks than GAs but handles less operators and parameters.

2.4. PARTICLE SWARM OPTIMIZATION (PSO)

It was developed in 1995 by Kennedy [59], Eberhart andShi [60] . At present is one of the most promising techniques in structural optimization.

It is based on the social behavior of animals like shoals, insect swarms or bird flocks. This behavior is related with groups and social forces, which depend on the individual and social memory and intelligence.

The population is constituted by a series of particles, which form the swarm. These are randomly initialized into the design space. Each particle represents a feasible solution. Particles move inside the search space attracted by the fittest position of the particle along time (local optimum), as well as the fittest particle, in the same way than a swarm.

This technique has been widely researched during the last ten years. Among these the following [46, 61-69] must be highlighted.

Compared with GA this technique is very simple and does not require too many tuning. As drawback, PSO biases to local minima and its difficulty to avoid it.

2.5. ANT COLONY OPTIMIZATION (ACO)

This technique was developed by Marco Dorigo in his doctoral thesis [70]. It tries to imitate the natural behavior of ant or bee colonies. These are formed by individuals who develop several tasks like food search, food transportation, nest building and defense. Each member of the colony does its own task interacting with the other individuals. If one individual is not able to do his task, the colony does.

The first ACO was inspired in the way an obstacle is surrounded by the ants when transporting food. If an ant finds the way, it is quickly reported to the other members by the marker pheromones used.

Owed to its simplicity, it has been widely researched, as structural optimization method, during last years. Among them the following [71-74] should be remarked.

Like PSO, ACO biases to local optima.

3. TECNIQUE COMPARISON

Since the beginning of GA, many researchers tried to establish a series of functions for verification. Nevertheless, no free lunch theorem establishes that none algorithm is better in all cases than random walk. So, for structural optimization, some benchmark problems have been used. Figure 7 shows the most popular structure for benchmarking is the ten bar and six nodes one, subjected to displacement and strength constraints, proposed by Venkayya et al. [75].

Table 1 shows the best 24 reported values for that structure.

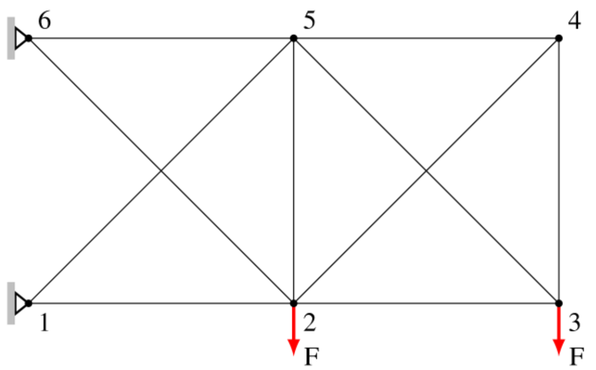

Figure 7. Ten bar and six node structure for benchmarking.

Table 1. Best reported results for the ten bars and six nodes structure.

Author	Year	Weight (kN)	δ_{max} (mm)	σ_{max} (Mpa)	Algorithm
Ebenau et al. [56]	2005	12,04	50,800	131,5	ES
Balling et al. [28]	2006	12,17	50,800	131,9	GA
Tang et al. [52]	2005	12,52	50,795	127,1	GA
Rajan [48]	1995	14,27	50,546	107,2	GA
Ai and Wang [76]	2011	14,31	-	-	GA
Groenwold et al. [36]	1999	18,66	-	-	GA
Kaveh and Shahrouzi [77]	2006	19,27	-	-	GA
Schutte et al. [69]	2003	20,50	-	-	PSO
Lee and Geem [78]	2005	20,80	-	-	HS
Li et al.[65]	2007	20,81	-	-	PSO
Wu and Tseng [79]	2010	21,05	50,800	128,5	DE
Kaveh and Shahrouzi [80]	2008	22,06	-	-	GA+ACO
Deb and Gulati [33]	2001	21,06	50,800	131,6	GA
Nanakorn et al. [81]	2001	22,08	-	-	GA
Isaacs et al. [58]	2008	22,10	-	-	ES
Ruy et al. [82]	2001	21,10	-	-	MOGA
Memari and Fuladgar [83]	1994	22,17	52,068	-	MP
Galante [84]	1992	22,19	51,511	-	GA
Camp and Bichon [71]	2004	22,22	-	-	ACO
El-Sayed and Jang [85]	1994	22,31	51,133	-	MP
Camp [86]	2007	22,36	-	-	BBBC
Perez and Behdinan [68]	2007	22,36	-	-	PSO
Adeli and Kamal [87]	1991	22,48	51,295	-	GA
Sonmez [74]	2011	22,50	50,800	-	ACO

4. CONCLUDING REMARKS

As shown in this paper, the research in structural optimization is now focused in a series of metaheuristic techniques. Among them, Genetic Algorithms stands out significantly. According to the reported results for the most used benchmark structure, Genetic Algorithms are the most suitable technique in the first seven positions.

Particle Swarm Optimization is the second best technique, no so far to Ant Colony Optimization.

The best value obtained for mathematical programming is located in position eighteen.

There is no reported paper for Simulated Annealing in the first 24 scientific papers.

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