

Summary

The Ph.D. Thesis “Operators on weighted spaces of holomorphic functions” presented here treats different areas of functional analysis such as spaces of holomorphic functions, infinite dimensional holomorphy and dynamics of operators.

After a first chapter that introduces the notation, definitions and the basic results we will use throughout the thesis, the text is divided into two parts. A first one, consisting of Chapters 1 and 2, focused on a study of weighted (LB)-spaces of entire functions on Banach spaces, and a second one, corresponding to Chapters 3 and 4, where we consider differentiation and integration operators acting on different classes of weighted spaces of entire functions to study its dynamical behaviour. In what follows, we give a brief description of the different chapters:

In Chapter 1, given a decreasing sequence of continuous radial weights on a Banach space X , we consider the weighted inductive limits of spaces of entire functions $VH(X)$ and $VH_0(X)$. Weighted spaces of holomorphic functions appear naturally in the study of growth conditions of holomorphic functions and have been investigated by many authors since the work of Williams in 1967, Rubel and Shields in 1970 and Shields and Williams in 1971. We determine conditions on the family of weights to ensure that the corresponding weighted space is an algebra or has polynomial Schauder decompositions. We study Hörmander algebras of entire functions defined on a Banach space and we give a description of them in terms of sequence spaces. We also focus on algebra homomorphisms between these spaces and obtain a Banach-Stone type theorem for a particular decreasing family of weights. Finally, we study the spectra of these weighted algebras, endowing them with an analytic structure, and we prove that each function $f \in VH(X)$ extends naturally to an analytic function defined on the spectrum. Given an algebra homomorphism, we also investigate how the mapping induced between the spectra acts on the corresponding analytic structures and we show how in this setting composition operators have a different behavior from that for holomorphic functions of bounded type. This research is related to recent work by Carando, García, Maestre and Sevilla-Peris. The results included in this chapter are published by Beltrán in [14].

Chapter 2 is devoted to study the predual of $VH(X)$ in order to linearize this space of entire functions. We apply Mujica's completeness theorem for (LB)-spaces to find a predual and to prove that $VH(X)$ is regular and complete. We also study conditions to ensure that the equality $VH_0(X)'' = VH(X)$ holds. At this point, we will see some differences between the finite and the infinite dimensional cases. Finally, we give conditions which ensure that a function f defined in a subset A of X , with values in another Banach space E , and admitting certain weak extensions in a space of holomorphic functions can be holomorphically extended in the corresponding space of vector-valued functions. Most of the results obtained have been published by the author in [13].

The rest of the thesis is devoted to study the dynamical behaviour of the following three operators on weighted spaces of entire functions: the differentiation operator $Df(z) = f'(z)$, the integration operator $Jf(z) = \int_0^z f(\zeta)d\zeta$ and the Hardy operator $Hf(z) = \frac{1}{z} \int_0^z f(\zeta)d\zeta$, $z \in \mathbb{C}$.

In Chapter 3 we focus on the dynamics of these operators on a wide class of weighted Banach spaces of entire functions defined by means of integrals and supremum norms: the weighted spaces of entire functions $B_{p,q}(v)$, $1 \leq p \leq \infty$, and $1 \leq q \leq \infty$. For $q = \infty$ they are known as generalized weighted Bergman spaces of entire functions, denoted by $H_v(\mathbb{C})$ and $H_v^0(\mathbb{C})$ if, in addition, $p = \infty$. We analyze when they are hypercyclic, chaotic, power bounded, mean ergodic or uniformly mean ergodic; thus complementing also work by Bonet and Ricker about mean ergodic multiplication operators. Moreover, for weights satisfying some conditions, we estimate the norm of the operators and study their spectrum. Special emphasis is made on exponential weights. The content of this chapter is published in [17] and [15].

For differential operators $\phi(D) : B_{p,q}(v) \rightarrow B_{p,q}(v)$, whenever $D : B_{p,q}(v) \rightarrow B_{p,q}(v)$ is continuous and ϕ is an entire function, we study hypercyclicity and chaos. The chapter ends with an example provided by A. Peris of a hypercyclic and uniformly mean ergodic operator. To our knowledge, this is the first example of an operator with these two properties. We thank him for giving us permission to include it in our thesis.

The last chapter is devoted to the study of the dynamics of the differentiation and the integration operators on weighted inductive and projective limits of spaces of entire functions. We give sufficient conditions so that D and J are continuous on these spaces and we characterize when the differentiation operator is hypercyclic, topologically mixing or chaotic on projective limits. Finally, the dynamics of these operators is investigated in the Hörmander algebras $A_p(\mathbb{C})$ and $A_p^0(\mathbb{C})$. The results concerning this topic are included by Bonet, Fernández and the author in [16].