The mixed capacitated general routing problem with turn penalties

Olli Bräysy[1], Eulalia Martínez[2], Yuichi Nagata[3], David Soler[2*]

(1) *Agora Innoroad Laboratory, Agora Center, P.O.Box 35,*

FI-40014 University of Jyväskylä, Finland

(2) *Instituto Universitario de Matemática Pura y Aplicada. Universidad Politécnica de Valencia*

Camino de Vera s/n, 46022, Valencia, Spain

(3) *Interdisciplinary Graduate School of Science and Engineering, Tokyo Institute of Technology,*

4259 Nagatsuta Midori-ku Yokohama, Kanagawa 226-8502, Japan

Abstract

In this paper we deal with the Mixed Capacitated General Routing Problem with Turn Penalties. This problem generalizes many important arc and node routing problems, and it takes into account turn penalties and forbidden turns, which are crucial in many real-life applications, such as mail delivery, waste collection and street maintenance operations. Through a polynomial transformation of the considered problem into a Generalized Vehicle Routing Problem, we suggest a new approach for solving this new problem by transforming it into an Asymmetric Capacitated Vehicle Routing Problem. In this way, we can solve the new problem both optimally and heuristically using existing algorithms. A powerful memetic algorithm and a set of 336 new benchmark instances are also

*Corresponding author: Address: Instituto Universitario de Matemática Pura y Aplicada. Universidad Politécnica de Valencia. Camino de Vera s/n, 46022, Valencia, Spain. Tel:+34 963877007-76667; fax:+34 963877669. E-mail addresses: olli.m.p.braysy@jyu.fi (O. Bräysy), eumarti@mat.upv.es (E. Martínez), nagata@fe.dis.titech.ac.jp (Y. Nagata), dsoler@mat.upv.es (D. Soler).

suggested. The experimental results show that the average deviation of the suggested solution method is less than 0.05 per cent with respect to optimum.

Keywords: Vehicle routing, capacitated general routing problem, turn penalties, transformation.

1 Introduction

In many real-life vehicle routing problems it is important to consider the risk and time associated with turns, i.e., the cost of the turn. Moreover, some turns, especially U-turns, can be forbidden. This last implies that a vehicle route made with a classical graph route generator may be illegal if it does not respect the traffic signals. For example, Figure 1 shows a traffic signal located in an arterial avenue from Valencia (Spain). It indicates that the next two left turns are forbidden and the way to dodge them. These forbidden turns cannot be taken into account if we model the city map with a classical graph.

Considering turn penalties and forbidden turns is particularly important in downtown areas and for large-size vehicles. But also turn penalties are important in order to save time in tours on foot. From Figure 2 it is easy to see that going on foot, right turn $a \to b$ can be considered with zero cost, while turn $a \to c$ is much more time-consuming, because it implies to cross two streets, with up to two traffic lights.

Usually the real-life vehicle routing software are based on separate modules for shortest path calculation and vehicle route optimization. The latter is based on given time and distance matrices, calculated typically in the beginning with the shortest path procedure. In this context, the distance and time calculation are based on fixed and known stopping points (usually addresses) for the vehicles and the possible turn penalties and forbidden turns are taken into account during the shortest path calculation. However, in practice, there are several applications where the exact stopping points are not known a priori and are part of the optimization problem, such as mail collection and delivery, waste collection and street maintenance operations. In these cases, including turn penalties and forbidden turns in the vehicle routing model is very important.

Figure 1. Traffic signal indicating two forbidden left turns.

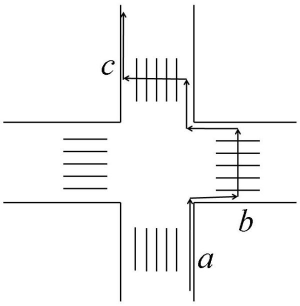

Figure 2. A street crossing.

So far research on extended vehicle routing models with turn penalties and forbidden turns has been scarce. For previous research, see Benavent and Soler (1999), Clossey *et al* (2001), Corberán *et al* (2002) and Soler *et al* (2008), that generalize several well-known single vehicle routing problems to the existence of turn penalties. They provide theoretical results about complexity and resolution and/or computational results on these extensions. The last cited work studies the most general problem, the Mixed General Routing Problem (MGRP) with turn penalties, that includes all the cases studied in the previous cited papers. MGRP consists of finding a minimal cost closed walk on the links of a mixed graph G which traverses a given subset of "required" links and a given subset of "required" vertices.

With respect to multivehicle routing problems, early papers that considered turn penalties focused on real-life applications and heuristic solution methods. See for example Bodin *et al* (1989) and Roy and Rousseau (1989). Later, turn penalties have been considered in the context of the Mixed Capacitated Arc Routing Problem (MCARP), see for example Bautista and Pereira (2004) and Belenguer *et al* (2006). The MCARP is an arc routing problem, in which a fleet of vehicles (with a known capacity) is based on a specified vertex (the depot) and must service a subset of the links of a mixed graph, with minimum total cost and such that the load assigned to each vehicle does not exceed its capacity and each link is serviced by exactly one vehicle.

Bautista *et al* (2008) present two ant colony metaheuristics for a real urban waste collection problem. This real problem is modeled as a particular case of the MCARP with turn penalties, in which they only consider two kind of turns: forbidden or allowed with zero cost. Finally, Perrier *et al* (2008) heuristically solve a real vehicle routing problem in the context of snow plowing operations that also takes into account the existence of forbidden turns.

The multivehicle extension (with capacity constraints) of the MGRP is called the Mixed Capacitated General Routing Problem (MCGRP). Due to its complexity, there are only few works on MCGRP, see e.g. Jansen (1993) who studied the undirected case and Pandit and Muralidharan (1995). However, particular cases of the MCGRP, such as the capacitated arc routing problem (CARP) and the capacitated vehicle routing problem (CVRP) have attracted a huge amount of research.

In this paper we present a generalization of the MCGRP that considers turn penalties and forbidden turns. The objective is to minimize the sum of the costs of the traversed arcs and edges together with the penalties associated with the turns made. We call the new problem the Mixed Capacitated General Routing Problem with Turn Penalties (MCGRPTP).

As far as we know, this is the first time that MCGRPTP is presented in the literature. Moreover, there is no previous research on multivehicle node routing problems with turn penalties. In this paper we also present for the first time an approach for solving capacitated routing problems with turn penalties, through suggesting a new polynomial transformation from the MCGRPTP to an asymmetric CVRP (ACVRP). To be more precise, the transformation is done in two steps: we first transform the MCGRPTP into a generalized VRP (GVRP), using a new approach suggested in this paper. The key idea of GVRP, compared to CVRP is that in GVRP each customer has several alternative service locations, and only one of them has to be selected for service. For more details on GVRP, see e.g. Ghiani and Improta (2000). In the second step we transform the GVRP into an ACVRP, using a model presented in Soler *et al* (2009).

Finally, we present a set of new benchmark problems and a very powerful memetic algorithm for the ACVRP, based on a previous study by Nagata and Bräysy (2009). The experimental results show an average deviation equal to 0.05% for instances with known optimal solution and that large-size problems can be solved with the suggested MA.

The suggested transformation makes it possible to use also any other powerful algorithm developed for ACVRP, see e.g. Fischetti *et al* (1994), Vigo (1996) or the more recent heuristic by De Franceschi *et al* (2006).

The remainder of this paper is organized as follows. Section 2 introduces some definitions and notations in order to formally define and solve the MCGRPTP. In Section 3, through two transformations, we prove that the MCGRPTP can be transformed in polynomial time into a Generalized Vehicle Routing Problem (GVRP). It is known (Soler *et al* (2009)) that the GVRP can be transformed into an ACVRP, so in Section 4 we show computational results for several sets of ACVRP benchmarks. Finally, in Section 5 we present our conclusions.

2 Definitions and notations

First, to our aim, we formally define two known problems cited before: the Asymmetric Capacitated Vehicle Routing Problem (ACVRP) and the Generalized Vehicle Routing Problem (GVRP). The second one is an extension of the ACVRP, introduced by Ghiani and Improta (2000), that can model several real-world situations and that will be the "cornerstone" to solve our problem:

The ACVRP is defined as follows:

Let $G = (V, A)$ *be a complete digraph,* $V = \{v_i\}_{i=0}^{n}$ *being its set of vertices, where* v_0 *is the depot vertex. Each vertex* v_i *with* $i > 0$ *has an associated demand* $d_i > 0$ *and each arc* $(v_i, v_j) \in A$ *has an associated cost* $c_{i,j} \geq 0$*. Moreover, a fleet of* k *vehicles with the same capacity* W *is available at the depot.*

Find a set of k *shortest routes, each starting and ending at the depot, such that each vertex* v_i $(\forall\ i \in \{1, ..., n\})$ *must be visited by one and only one vehicle and the sum of the demands of the vertices visited by each vehicle does not exceed its capacity* W.

As in other papers cited in the introduction, in all the capacitated routing problems discussed here, we will consider k to be equal to the minimum number of vehicles needed to serve all demands.

The GVRP is defined as follows:

Let $G = (V, A)$ be a directed graph where the vertex set V is partitioned into $m+1$ nonempty subsets $S_0, S_1, ..., S_m$ such that S_0 has only one vertex v_0 (the depot), S_h $(h = 1, ..., m)$ represents $l(h)$ possible locations of the same vertex which has associated a positive demand d_h, and each arc $(v_i, v_j) \in A$ has associated a cost $c_{i,j} \geq 0$. Moreover, a fleet of k homogeneous vehicles having the same capacity W is available at the depot.

Find a set of k shortest routes, each starting and ending at the depot, such that each subset S_h $(h = 1, ..., m)$ is visited exactly once and the sum of the demands of every route does not exceed the capacity W of the vehicle.

Next, we need to show some concepts and notations that have been used in previous works on turn penalties:

Given a mixed graph $G = (V, E, A)$, each pair of links $a = (u, v), b = (v, w) \in E \cup A$ has an associated turn at v, based on going from a to b, denoted as $[ab]$. Moreover, if $a, b \in E$, the same pair has another associated turn at v, based on going from b to a, denoted as $[ba]$. Each edge e incident with v has an associated U-turn at v that, if necessary, will be denoted by $[eve]$. Each link in G has associated a nonnegative cost and each allowed turn in G has associated a nonnegative penalty.

Given $a = (u, v), b = (s, t) \in E \cup A$, a *$v$-$s$ feasible chain from a to b* is an alternating sequence of links and allowed turns $\{a_1, [a_1a_2], a_2, \ldots, [a_{r-1}a_r], a_r, [a_rb]\}$, where $a_1 = a$. The cost of a feasible chain is defined as the sum of the costs of the arcs it traverses plus the sum of the penalties of the turns it makes. A v-s feasible chain from a to b is *closed* if $a = b$ and $s \neq v$.

Given $a = (u, v), b = (s, t) \in E \cup A$, a shortest (minimum cost) v-s feasible chain from a to b will be denoted by $s.f.c.(v^a, s^b)$.

Note that a feasible chain is defined such that it begins at a link and ends at a turn. This is very important in the context of forbidden or penalized turns. In classical routing problems, if we have to go from a vertex u to a vertex v and then to a vertex w, we only have to connect the shortest path from u to v with the shortest path from v

to w. But even if these shortest paths have been constructed taking into account turn conditions, the connection of both paths at v can give rise to an unavoidable forbidden turn (U-turn for example). In our case, the connection between two feasible chains at a vertex v is possible only if the first one ends at a turn $[(t,v)(v,s)]$ and the second one begins at the link (v,s), which avoids the existence of forbidden turns. Therefore, we cannot use paths between vertices as in the classical way, and this increases the difficulty of modeling how to serve demands at vertices, specially if these vertices have undesirable turns.

With these previous concepts we can formally define the problem that we study in this paper. The Mixed Capacitated General Routing Problem with Turn Penalties (MCGRPTP) is defined as follows:

Let $G = (V, E, A)$ be a mixed graph where each link $(i,j) \in E \cup A$ has an associated cost $c_{ij} \geq 0$ and each turn $[ab]$ has an associated penalty $p_{[ab]} \geq 0$ ($p_{[ab]} = +\infty$ if turn $[ab]$ is forbidden). One of the vertices, say v_0, represents the depot where there are k vehicles of an identical capacity $W > 0$ and in it all the turns are allowed with zero penalty. Let $R \subseteq E \cup A$ be a set of required links such that each $(i,j) \in R$ has an associated positive demand $q_{ij} \leq W$, and let $V_R \subseteq V - \{v_0\}$ be a set of required vertices such that each $v \in V_R$ has an associated positive demand $q_v \leq W$.

Find k closed feasible chains in G, one for each vehicle, that minimize the total cost and such that each chain passes through the depot, each demand is served by only one vehicle and the total demand served by each vehicle does not exceed its capacity W.

Note that allowing all the turns with zero penalties at the depot is due to the fact that in real-world situations, the depot normally represents a warehouse from which the vehicles begin their journey and to which they return. It makes no sense considering forbidden/penalty turns in the warehouse as the truck leaves from depot independently of the route the truck made before. Moreover, these warehouses are usually placed outside the cities with good road communications and of easy access.

Hereinafter and as in similar papers, each non-required edge will be replaced by two arcs of the same cost and opposite direction; then we assume that all edges in the graph are required ($E_R = E$, with $R = E_R \cup A_R$). Finally, for simplicity, we will not write the middle turns of a feasible chain. For example, the feasible chain $\{a, [ab], b, [b, c]\}$ will be written as $\{a, b, [b, c]\}$.

In the particular case of the MCGRPTP in which $V_R = \emptyset$, we have the MCARP with turn penalties, and in the particular case of the MCGRPTP in which $k = 1$ we have the MGRP with turn penalties. Therefore, the problem presented here generalizes both the single vehicle and the multivehicle routing problems with turn penalties studied in the literature.

3 Solving the MCGRPTP

To solve the MCGRPTP, we will first transform it into a GVRP, which in turn can be transformed into an ACVRP.

3.1 Transformation of the MCGRPTP into a GVRP

Let $G = (V, E, A)$ be a mixed graph where a MCGRPTP is defined, $E \cup A_R$ being the required link set and V_R the required vertex set. Due to the fact that in the GVRP the demand is at the vertices, we will transform graph G in which we have defined the MCGRPTP into a directed graph $G^* = (V^*, A^*)$ such that the vertices in G^* are related with the required links and required vertices in G.

To this aim, we first construct an intermediate directed graph $G' = (V', A')$ from G as follows:

First, each required edge is replaced in G by two opposite required arcs, both with the edge cost and the edge demand.

Second, subset V_R is partitioned into two subsets, V_{R_1} and V_{R_2}, such that vertices containing all allowed zero-penalty turns belong to V_{R_1}, and vertices containing forbidden or positive-penalty turns belong to V_{R_2}.

For each $v \in V_{R_1} \cup \{v_0\}$, replace vertex v in G by two vertices v^e and v^l, so that v^e has only entering arcs (the arcs entering at v) and v^l has only leaving arcs (the arcs leaving from v). Add a required arc $a_v = (v^e, v^l)$ to G such that all turns at v^e and v^l are allowed with zero penalty and the demand of this arc in G' is the one of the required vertex v in G (the demand corresponding to the arc from the depot vertex is obviously zero). Note that traversing arc a_v in G' is then equivalent to passing through vertex v in the original graph G.

For each $v \in V_{R_2}$ do:

- Replace vertex v in G by the same number of vertices v_{ij} as those of allowed turns $[a_i b_j]$ at v, so that each of these copies has only one entering arc (a_i) and one leaving arc (b_j), with its corresponding allowed turn. Note that if a_i is an entering arc at v, G' will contain at least as many copies of the entering arc a_i as there are allowed turns involving a_i at v. The same applies to the leaving arc b_j from v.

- Then, replace each vertex v_{ij} by two vertices, v_{ij}^e and v_{ij}^l, and add a "required" arc $a_{v_{ij}} = (v_{ij}^e, v_{ij}^l)$ between them with cost zero, such that $p_{[a_i a_{v_{ij}}]} = p_{[a_i b_j]}$ and $p_{[a_{v_{ij}} b_j]} = 0$, i.e. the penalty that was in the turn at v_{ij} is moved to vertex v_{ij}^e, and all these arcs will have the same demand, the one of the required vertex v. Note that traversing only one of these required arcs $a_{v_{ij}}$ in G' involves passing through vertex v in G.

Note that in the last paragraph we have written required in quotes because for each $v \in V_{R_2}$, only one of the generated arcs must be served.

After this transformation we have a directed graph $G' = (V', A')$ such that the subset A'_R comes from the required arcs, required edges and required vertices in G. A'_R will give rise to a partitioned set of vertices V^* in the graph G^* in which we will define the GVRP. Each arc between two of these vertices that do not form part of the same subset, will have associated the cost of the shortest feasible chain between the two corresponding links in G'.

From G' we then construct the graph $G^* = (V^*, A^*)$ as follows:

- For each arc $a_v \in A'_R$ with $v \in V_{R_1} \cup \{v_0\}$, associate a vertex set $S_v = \{x_{a_v}\}$ in G^*, with its corresponding demand in x_{a_v}.

- For each $v \in V_{R_2}$, associate a vertex set S_v in G^* with as many vertices $x_{a_{v_{ij}}}$ as arcs $a_{v_{ij}}$ are in A'_R, all of them with the same corresponding demand.

- For each arc $a \in A'_R$ that comes from a required arc in G, associate a vertex set S_a in G^* with as many copies of a vertex x_a as copies of arc a appear in G', all of them with the same corresponding demand.

- For each pair of opposite required arcs $e_1, e_2 \in A'_R$ that come from a required edge e in G, associate a vertex set S_e in G^* with as many copies of vertices x_{e_1} and x_{e_2} as copies of arcs e_1 and e_2 respectively appear in G', all of them with the same corresponding demand.

- For each pair of vertices $x_a, x_b \in V^*$ with $x_a \in S_i$, $x_b \in S_j$, $i \neq j$ being $a = (u, v)$ and $b = (s, t)$, add arcs (x_a, x_b) and (x_b, x_a) to A^*, with the cost of the $s.f.c.(v^a, s^b)$ and of the $s.f.c.(t^b, u^a)$ respectively in G' (if they exist).

- There is no arc between vertices belonging to the same S_i.

Given an MCGRPTP in G, we define a GVRP in G^* where the vertex set V^* is partitioned into the following subsets: S_v for all $v \in V_{R_1} \cup \{v_0\}$ (hereinafter we will denote the subset corresponding to the depot by $S_0 = \{v_D\}$), S_v for all $v \in V_{R_2}$, S_a for all $a \in A_R$ and S_e for all $e \in E_R$.

Let us see an example of the construction of the auxiliary graph G^*. Consider the mixed graph given in Figure 3 corresponding to an MCGRPTP in which vertex 1 represents the depot, there are two vehicles both with capacity 55, and there are three required arcs a, b and c, with demands 25, 25 and 20 respectively, a required edge e with demand 20 and a required vertex 5 with demand 10. Then the total demand in the graph is 100 units. Link costs and demands appear in Figure 3 with different size (demands appear in bold and small size).

We will suppose in this graph that all U-turns are forbidden except at vertex 1 (the depot) at which all turns are allowed with penalty zero, and in the rest of the vertices, right turns (according to the drawing of the graph) have penalty 1, left turns have penalty 3 except for the turn from arc b to arc $(2, 1)$ that is considered forbidden, and going straight ahead, as it occurs in the turn from arc a to edge e, has penalty zero.

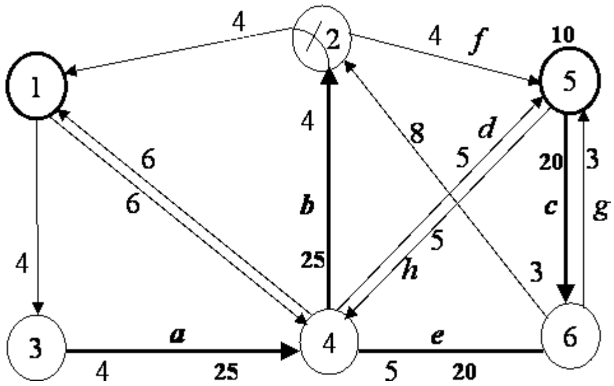

Figure 3. Graph G with $A_R = \{a, b, c\}$, $E_R = \{e\}$ and $V_R = \{5\}$.

Starting from the information given by the initial graph $G = (V, E, A)$, where $A_R = \{(3,4),(4,2),((5,6)\}$, $E_R = \{(4,6)\}$, $V_R = \{5\}$ and the depot is vertex 1, we construct the intermediate graph G' (see Figure 4):

First, we replace the required edge $e = (4,6)$ with demand $q_e = 20$ by two opposite arcs e_1, e_2 with the same cost 5 and the same demand 20.

Second, we replace vertex 1 by the sequence $\{1^e, a_1, 1^l\}$, a_1 with cost zero.

Finally, we transform vertex 5, which belongs to V_{R_2} (here $V_{R_1} = \emptyset$), into four arcs, as many as allowed turns in it, all of them with cost zero and demand 10.

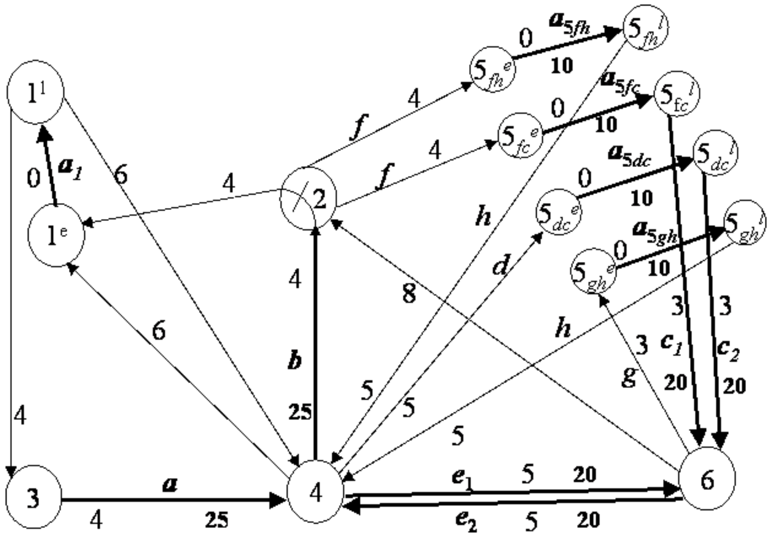

Figure 4. Intermediate graph G'.

From the intermediate graph $G' = (V', A')$ we can already construct the directed graph $G^* = (V^*, A^*)$.

The vertex set V^* is given by the following partition:

- A subset S_0 with only one vertex x_D representing the depot and corresponding to the required arc a_1.

- Subsets S_a and S_b, each one with only one vertex x_a and x_b respectively, for the required arcs $a = (3, 4)$ and $b = (4, 2)$, both with demand 25, and subset S_c with two vertices x_{c_1} and x_{c_2}, both with demands 20, for the required arc $c = (5, 6)$ which have two copies in G'.

- Subset S_e with two vertices x_{e_1} and x_{e_2} both with demand 20, for the required edge $e = (4, 6)$.

- Subset S_5 with four vertices $x_{5_{fh}}$, $x_{5_{fc}}$, $x_{5_{dc}}$ and $x_{5_{gh}}$ all of them with demand 10, for the required vertex 5 in V_{R_2}.

Figure 5 shows graph G^* in which, for simplicity, each pair of arcs (x_r, x_t) and (x_t, x_r) with $x_r \in S_i$, $x_t \in S_j$ and $i \neq j$ has been drawn as a line with two arrow heads, one at each end, and the arc costs (normally different for each direction) have been omitted.

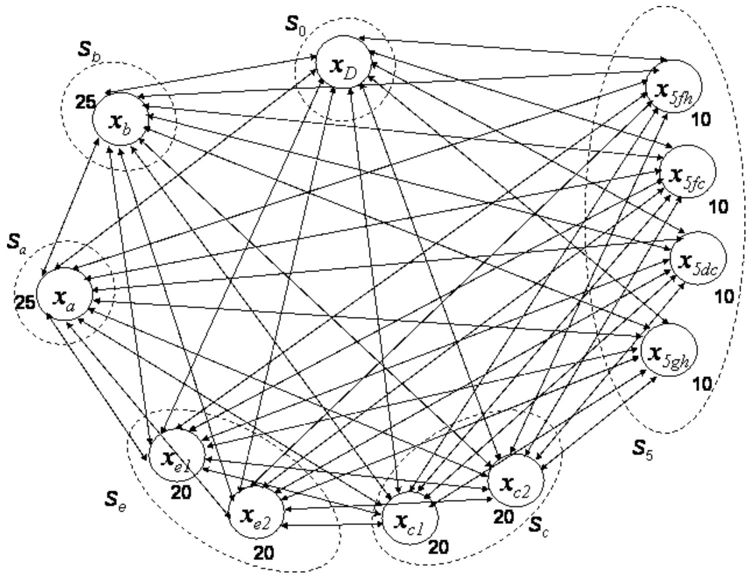

Figure 5. Directed graph G^* associated with G.

Theorem 1 *An MCGRPTP defined in G can be transformed in polynomial time into the corresponding GVRP defined in G^*.*

Proof. By the construction of G', given $B = \{T_i\}_{i=1}^k$ a set of k feasible closed chains in G corresponding to a solution to the MCGRPTP, we can associate with B a set $B' = \{T'_i\}_{i=1}^k$ of k feasible closed chains in G' such that for all $i \in \{1, \ldots, k\}$ we have:

- T'_i traverses arc a_1 (corresponding to the depot node 1 in G).
- T_i passes through a vertex $v \in V_{R_1}$ iff T'_i traverses arc a_v in G'.
- T_i passes through a vertex $v \in V_{R_2}$ iff T'_i traverses an arc $a_{v_{ij}}$.
- T_i traverses arc $a \in A_R$ iff T'_i traverses a copy of arc a in G'.
- T_i traverses edge $e \in E$ iff T'_i traverses a copy of arc e_1 or a copy of arc e_2 in G'.
- T'_i has the same cost as T_i.
- Moreover, we will suppose that if T_i satisfies demand at $v \in V_1$ ($v \in V_2$) ($a \in A_R$) ($e \in E$), then T'_i satisfies the demand located at a_v (one and only one arc $a_{v_{ij}}$) (one and only one copy of arc a in G') (one and only one copy of e_1 or e_2 in G').

Summarizing, for all $i \in \{1, \ldots, k\}$ T'_i is a feasible closed chain in G' that traverses a_1, satisfies the same demands as T_i and has the same cost as T_i.

Once we have constructed B' from B, for each $i \in \{1, \ldots, k\}$, from T'_i we construct a cycle C_i^B in G^* as follows:

Suppose that T'_i satisfies, in this chronological order, the demands $q_{j_1}, q_{j_2}, \ldots, q_{j_{m_i}}$. Then C_i^B is a cycle in G^* that visits, in this order, the sets of vertices $S_0, S_{j_1}, S_{j_2}, \ldots,$ $S_{j_{m_i}}, S_0$, and for all $t \in \{1, \ldots, m_i\}$, C_i visits only the vertex at S_t coming from the arc in G' at which demand has been satisfied by T'_i.

It is evident that the set of cycles $L^B = \{C_i^B\}_{i=1}^k$ is a solution to the GVRP in G^* its cost $c^*(L^B)$ being less than or equal to $c(B)$, this last due to the fact that the cost of the route segment of one feasible closed chain T'_i in G' between two consecutive serviced arcs (including the depot arc a_1) is greater than or equal to the cost of the shortest feasible chain in G' between them (from the first serviced one to the second serviced one), while the cost of the arc in C_i^B in G^* from the vertex coming from a serviced arc in G' to the vertex coming from the next serviced arc in G' is equal to the one of the shortest feasible chain in G' between them.

On the other hand, let $L = \{C_i\}_{i=1}^k$ be a set of k cycles corresponding to a solution to the GVRP in G^*. For each $i \in \{1, \ldots, k\}$, from C_i we construct a feasible closed chain $T_i'^L$ in G' as follows:

Let (x_a, x_b) be a generic arc of C_i, $a = (u, v)$ and $b = (w, r)$ being the arcs in A'_R from which vertices x_a and x_b come, respectively. Arc (x_a, x_b) will give rise in $T_i'^L$ to the $s.f.c.(v^a, w^b)$, and such that $T_i'^L$ assumes the demand at a (the same as the one in x_a) and b (the same as the one in x_b). Note that in this way, $T_i'^L$ has the same cost as C_i and satisfies the same demands as C_i.

From the set $B'^L = \{T_i'^L\}_{i=1}^k$ of k feasible closed chains in G', we construct now a set $B^L = \{T_i^L\}_{i=1}^k$ of k feasible closed chains in G as follows:

- "Contract" each sequence in $T_i'^L$ of the form $(u, v^e)(v^e, v^l)(v^l, w)$ with $a_v = (v^e, v^l)$ if $v \in V_{R_1} \cup \{1\}$ by $(u, v)(v, w)$ in T_i^L.

- "Contract" each sequence in $T_i'^L$ of the form $(u, v_{ij}^e)(v_{ij}^e, v_{ij}^l)(v_{ij}^l, w)$ with $a_{v_{ij}} = (v_{ij}^e, v_{ij}^l)$ if $v \in V_{R_2}$ by $(u, v)(v, w)$ in T_i^L.

- If $T_i'^L$ traverses a copy of arc e_1 or a copy of arc e_2 in G', with $e \in E$, replace this copy in T_i^L by edge e.

- Any other link or turn in $T_i'^L$ is replaced by itself in T_i^L.

- Demand at $v \in V_1$ ($v \in V_2$) ($a \in A_R$) ($e \in E$) is assigned to route T_i^L iff $T_i'^L$ satisfies the demand located at a_v (one and only one arc $a_{v_{ij}}$) (one and only one copy of arc a in G') (one and only one copy of e_1 or e_2 in G').

It is evident that $B^L = \{T_i^L\}_{i=1}^k$ is a solution to the MCGRPTP in G with $c(B^L) = c^*(L)$, this last due to the fact that for all i, $c(T_i^L) = c'(T_i'^L)$.

Let then L^{opt} be an optimal GVRP solution in G^* and let $B^{L^{opt}}$ be the MCGRPTP solution in G obtained from L^{opt} as described above. For each MCGRPTP solution B in G we have:

$$c(B) \geq c^*(L^B) \geq c^*(L^{opt}) = c(B^{L^{opt}}) \quad (1)$$

Therefore, $B^{L^{opt}}$ is an optimal MCGRPTP in G. ▌

3.2 Solving the GVRP in G^* through an ACVRP

Once we have transformed our MCGRPTP into a GVRP defined in $G^* = (V^*, A^*)$, from G^* we construct a digraph $\hat{G} = (\hat{V}, \hat{A})$ as follows:

- $\hat{V} = V^*$.

- For each S_i with $i \in \{0, 1, \ldots, m\}$ and $|S_i| > 1$, order its vertices consecutively in an arbitrary way $\{v_1^i, \ldots, v_{l(i)}^i\}$; then, for $j = 1, \ldots, l(i) - 1$, define the cost of arc $(v_j^i, v_{j+1}^i) \in \hat{A}$ as zero; also define the cost of arc $(v_{l(i)}^i, v_1^i)$ as zero.

- For every $v_j^i \in S_i$ and every $w \notin S_i$ define the cost of arc (v_j^i, w) in $\hat{A}$ equal to the cost in G^* of the arc (v_{j+1}^i, w) ((v_1^i, w) if $j = l(i)$) plus a fixed positive large quantity M if $|S_i| > 1$, and equal to the cost in G^* of arc (v_j^i, w) plus M if $|S_i| = 1$.

- Any other arc in $\hat{A}$ has infinite cost.

- Assign positive demands having sum equal to d_i to the vertices in S_i $\forall\, i$, except for the depot subset.

As it was proved by Soler *et al* (2009), the GVRP can be transformed into an ACVRP. Following their proof, we can see that to solve the GVRP in G^* we can solve the ACVRP in the digraph $\hat{G}$, and to obtain a GVRP solution from an ACVRP one, we just have to identify each path in the ACVRP solution $(v_j^i, v_{j+1}^i, \ldots, v_{l(i)}^i, v_1^i, \ldots, v_{j-1}^i, w)$ $w \notin S_i$ (w can be the depot) if $j \neq 1$ and $|S_i| > 1$, or $(v_1^i, \ldots, v_{l(i)}^i, w)$ if $j = 1$ and $|S_i| > 1$, or (v_j^i, w) if $|S_i| = 1$ (in this last case v_j^i can be the depot), with the arc (v_j^i, w) in G^*. An optimal ACVRP solution H_{opt} in $\hat{G}$, will give rise to an optimal GVRP solution L_{opt} in G^* with cost $c^*(L_{opt}) = \hat{c}(H_{opt}) - M(m + k)$.

Going on with our example, from the graph G^* where the GVRP is defined (Figure 5) we define the ACVRP in the digraph $\hat{G}$ (see Figure 6) where, for simplicity again, the pairs of arcs (x_r, x_t) and (x_t, x_r) with $x_r \in S_i$, $x_t \in S_j$ and $i \neq j$ have been drawn as lines with two arrow heads, one at each end, and the arc costs (normally different for each direction) have been omitted. Figure 6 shows the cost zero "intraset" arcs and the demand assigned to each vertex. For example, vertex 5, belonging to V_{R_2} and with demand 10 in G, has associated the set S_5 in G^* with four vertices that in $\hat{G}$ have demands 2, 2, 3 and 3, respectively.

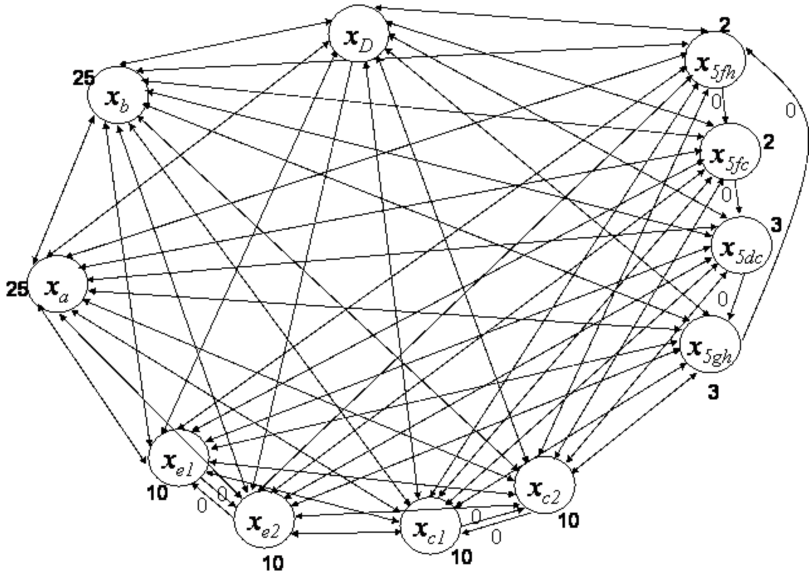

Figure 6. Directed graph $\hat{G}$ associated with G^*.

Figure 7 shows the optimal solution H to the ACVRP in $\hat{G}$ given in Figure 6 corresponding to our example; it consists of two cycles:

- $H_1 = (x_D, x_a, x_b, x_D)$ with associated cost $7 + 7 + 22 + 3M = 36 + 3M$ (these arc costs will be deduced above when explaining the solution to the MCGRPTP in G) and the demand served by this cycle is $q_a + q_b = 25 + 25 = 50$.

- $H_2 = (x_D, x_{5dc}, x_{5gh}, x_{5fh}, x_{5fc}, x_{c_2}, x_{c_1}, x_{e_2}, x_{e_1}, x_D)$ with associated cost:

$15 + 0 + 0 + 0 + 0 + 0 + 4 + 0 + 12 + 4M = 31 + 4M$ and the demand serviced by this cycle is $q_c + q_e + q_{v_5} = 20 + 20 + 2 + 2 + 3 + 3 = 50$. Note that the total cost $\hat{c}(H) = 67 + 7M = 36 + 31 + (5 + 2)M$ since $m = 5$ is the number of vertex subsets in G^* (the depot not included) and $k = 2$ is the number of cycles in the solution.

This optimal solution H in $\hat{G}$ gives rise to the optimal solution L in G^* shown in Figure 8, with total cost $c^*(L) = \hat{c}(H) - 7M = 67$, and that consists of two cycles: $C_1 = (x_D, x_a, x_b, x_D)$ with cost 36 and $C_2 = (x_D, x_{5dc}, x_{c_2}, x_{e_2}, x_D)$ with cost 31.

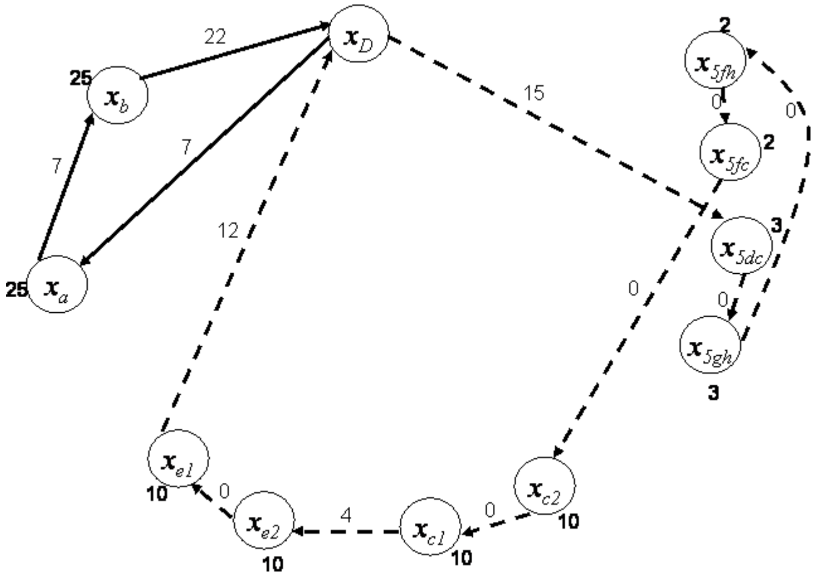

Figure 7. Optimal solution to the ACVRP in $\hat{G}$.

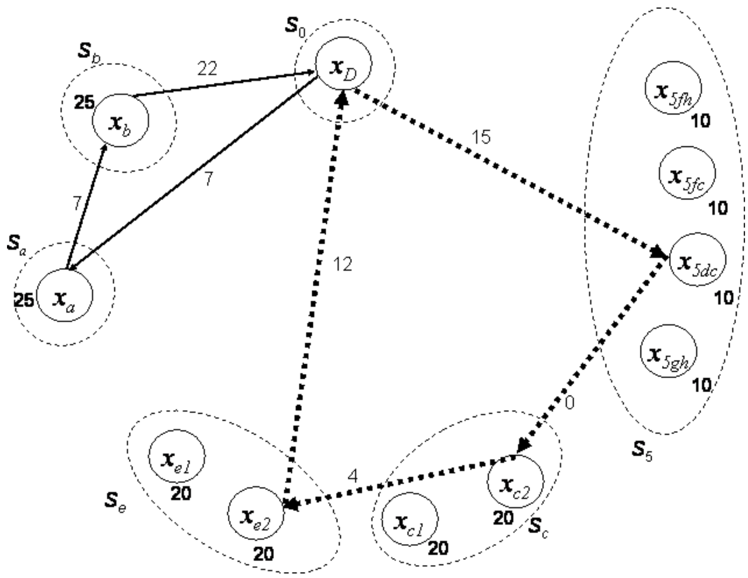

Figure 8. Optimal solution to the GVRP in G^*.

Let us find the optimal solution to the MCGRPTP in G given by these cycles, according to the proof of Theorem 1.

From $C_1 = (x_D, x_a, x_b, x_D)$ we have:

- (x_D, x_a) corresponds to the $s.f.c.(1^{l^{a_1}}, 3^a)$ in G': $(1^e, 1^l)(1^l, 3)[(1^l, 3), (3, 4)]$ with cost 0+**0**+4+**3**=7 (in the addition, normal-font numbers indicate arc costs while bold numbers indicate turn penalties). See Figure 4 to follow the chains in G'.

- (x_a, x_b) corresponds to the $s.f.c.(4^a, 4^b)$ in G': $(3, 4)[(3, 4), (4, 2)]$ with cost 4+**3**=7.

- (x_b, x_D) corresponds to the $s.f.c.(2^b, 1^{e^{a_1}})$ in G': $(4,2)(2,5^e_{fh})(5^e_{fh}5^l_{fh})(5^l_{fh},4)(4,1^e)$ $[(4,1^e),(1^e,1^l)]$ with cost 4+**1**+4+**1**+0+**0**+5+**1**+6+**0**=22.

Therefore, joining the chains we have the following feasible closed chain in G'

$T'_1 = \{(1^e,1^l),(1^l,3),(3,4),(4,2),(2,5^e_{fh}),(5^e_{fh},5^l_{fh}),(5^l_{fh},4),(4,1^e),[(4,1^e),(1^e,1^l)]\}$ with cost 36, and assuming the demands of arcs $(3,4)$ and $(4,2)$ (a and b), with a total demand of 50.

From T'_1, following again with the proof of Theorem 1, we construct the feasible closed chain with the same cost as T'_1 in the original graph G,

$T_1 = \{(1,3),(3,4),(4,2),(2,5)(5,4),(4,1),[(4,1),(1,3)]\}$ that also assumes the demands of arcs a and b (25+25).

Similarly, from $C_2 = (x_D, x_{5dc}, x_{c_2}, x_{e_2}, x_D)$ we have:

- (x_D, x_{5dc}) corresponds to the $s.f.c.(1^{l^{a_1}}, 5^{ea_{5dc}}_{dc})$ in G': $(1^e,1^l)(1^l,4)(4,5^e_{dc})[(4,5^e_{dc}),(5^e_{dc},5^l_{dc})]$ with cost 0+**0**+6+**3**+5+**1**=15.

- (x_{5dc}, x_{c_2}) corresponds to the $s.f.c.(5^{la_{5dc}}_{dc}, 5^{lc_2}_{dc})$ in G': $(5^e_{dc},5^l_{dc})[(5^e_{dc},5^l_{dc}),(5^l_{dc},6)]$ with cost 0+**0**=0.

- (x_{c_2}, x_{e_2}) corresponds to the $s.f.c.(6^{c_2}, 6^{e_2})$ in G': $(5^l_{dc},6)[(5^l_{dc},6),(6,4)]$ with cost 3+**1**=4.

- (x_{e_2}, x_D) corresponds to the $s.f.c.(4^{e_2}, 1^{e^{a1}})$ in G': $(6,4)(4,1^e)[(4,1^e),(1^e,1^l)]$ with cost 5+**1**+6+**0**=12.

Therefore, joining the chains we have the following feasible tour in G',

$T'_2 = \{(1^e,1^l),(1^l,4),(4,5^e_{dc}),(5^e_{dc},5^l_{dc}),(5^l_{dc},6),(6,4),(4,1^e),[(4,1^e),(1^e,1^l)]\}$ with cost 31, and assuming the demands of arcs $(5^e_{dc},5^l_{dc})$, $(5^l_{dc},6)$ and $(6,4)$, with a total demand of 50.

From T'_2, we construct the feasible closed chain with the same cost as T'_2 in G,

$T_2 = \{(1,4)(4,5)(5,6)(6,4)(4,1)[(4,1),(1,4)]\}$, that assumes the demands of vertex 5 belonging to V_{R_2}, of arc $(5,6)$ and of edge $(4,6)$ (10+20+20).

Figure 9 shows the optimal solution to the MCGRPTP in the original graph G: T_1 with solid bold line and T_2 with broken bold line.

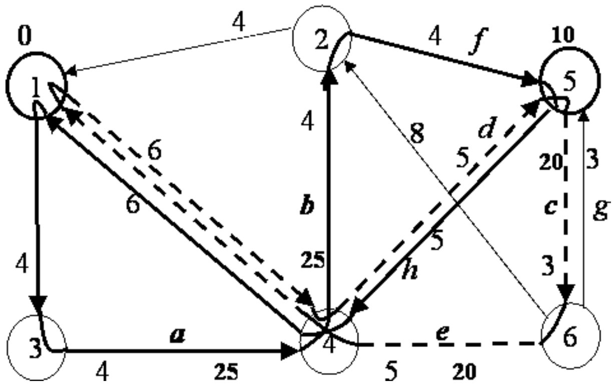

Figure 9. Optimal solution to the MCGRPTP in G.

4 Computational experiments

The aim of this section is to show that the transformation presented here can be considered as a good tool to solve MCGRPTP instances, at least heuristically due to the complexity of the problem. That is, if there exists any competitive procedure to solve the ACVRP, we can solve MCGRPTP instances within a reasonable running time. To do this, we first present a powerful heuristic algorithm for the ACVRP based on the memetic algorithm (MA) for the (symmetric) CVRP proposed by Nagata and Bräysy (2009). MA is a population-based heuristic search approach that combines evolutionary algorithm with local search algorithm. Although there are other heuristic approaches for the ACVRP, as those cited in the introduction, we selected the above mentioned MA because it is shown to be currently the most powerful heuristic method for the CVRP, and it can be applied to the ACVRP by a straightforward extension. Here the suggested MA has been tested on a set of 32 ACVRP instances by Pessoa *et al* (2007). We have also applied the MA to a set of 126 single vehicle instances by Soler *et al* (2008) because the optimal solutions are known to these instances. Finally, the MA has been applied to a set of 336 ACVRP instances with up to 623 vertices that come from the transformation of MCGRPTP instances.

4.1 The MA for the ACVRP

The main feature of the MA by Nagata and Bräysy (2009) is that the edge assembly crossover (EAX) operator generates offspring solutions by combining edges of two solutions selected as parents from the population. The generated offspring solutions may violate the capacity constraint. In this case, a subsequent local search-based repair procedure is used to restore the feasibility of the temporarily infeasible solutions. Moreover, a simple local search is applied to the obtained feasible solutions according to a standard MA procedure.

Note that the EAX was adapted to the ACVRP by defining it on the directed graph whereas the original EAX for the symmetric CVRP was defined on the undirected graph. The MA by Nagata and Bräysy minimized the total travel distance without putting any constraint on the number of vehicles and the number of vehicle was also a decision variable. However, according to the literature and therefore to the definition of the ACVRP given here, in the ACVRP the travel distance must be minimized with a given number of vehicles. So we have made a new version of the MA that minimizes the total travel distance for a fixed number of vehicles. The suggested MA has been implemented in C++ and has been executed on a ADM Opteron 2.4 GHz computer. For each instance, the MA has been executed five times.

4.2 Results for the ACVRP instances

We first analyze the efficiency of the two MA versions (fixed or variable number of vehicles) on a set of 32 ACVRP instances with known upper bounds that appear in the work by Pessoa *et al* (2007), which are variants of the 8 benchmark ACVRP instances given in http://or.ingce.unibo.it/research/cvrp-and-dcvrp. As far as we know, the work by Pessoa *et al* is the most recent paper with computational results on the ACVRP.

The results for the 32 ACVRP instances without constraint on the number of vehicles are presented in Table 1. The columns in the table list instance names (Instance), the capacity of the vehicles (C), the number of vehicles and the total travel distance of the best-known upper bound solutions (k and UB), the number of vehicles and the total travel distance of the best result in five runs (best-k and best-d.) with our MA, the average number of vehicles and the average total travel distance in these five runs (ave-k and ave-d.), and the average computation time in seconds for a run (Time).

Table 1. Computational results on known ACVRP instances.

Instance	*C*	*k*	*Best UB*	*Best − k*	*Best − d*	*Ave − k*	*Ave − d*	*Time*
a034-14f	150	14	4046	14	4046	14.0	4046.0	1.37
a036-18f	150	18	5296	19	5224	19.0	5224.0	1.21
a039-20f	150	20	5903	20	5903	20.0	5903.0	1.47
a045-18f	150	18	6399	19	5564	19.0	5568.0	4.59
a048-16f	150	16	4955	16	4955	16.6	4960.0	6.35
a056-17f	150	17	4998	17	4998	17.6	5020.0	8.82
a065-19f	150	19	6014	20	5862	20.0	5862.0	12.52
a071-17f	150	17	5006	17	5006	17.0	5014.0	13.86
a034-08f	250	8	2672	8	2672	8.0	2672.0	1.94
a036-10f	250	10	3338	11	3294	11.0	3294.0	1.67
a039-12f	250	12	3705	12	3705	12.0	3705.0	2.63
a045-11f	250	11	3544	11	3544	11.2	3555.6	5.41
a048-10f	250	10	3325	10	3325	10.0	3325.6	4.32
a056-10f	250	10	3263	10	3263	10.0	3263.0	7.46
a065-12f	250	12	3902	12	3902	12.0	3902.0	10.64
a071-10f	250	10	3486	10	3486	10.0	3486.6	12.73
a034-04f	500	4	1773	4	1773	4.0	1773.0	1.76
a036-05f	500	5	2110	5	2110	5.0	2110.0	1.95
a039-06f	500	6	2289	6	2289	6.0	2289.0	2.00
a045-06f	500	6	2303	6	2303	6.0	2303.0	3.41
a048-05f	500	5	2283	5	2283	5.0	2283.0	3.26
a056-05f	500	5	2165	5	2165	5.0	2165.0	4.70
a065-06f	500	6	2567	6	2567	6.0	2568.0	7.82
a071-05f	500	5	2475	5	2457	5.0	2457.0	7.62
a034-02f	1000	2	1406	2	1406	2.0	1406.0	1.31
a036-03f	1000	3	1644	3	1644	3.0	1644.0	1.13
a039-03f	1000	3	1654	3	1654	3.0	1654.0	1.17
a045-03f	1000	3	1740	3	1740	3.0	1740.0	1.97
a048-03f	1000	3	1891	3	1891	3.0	1891.0	1.69
a056-03f	1000	3	1739	3	1739	3.0	1739.0	3.23
a065-03f	1000	3	1974	3	1974	3.0	1974.0	4.17
a071-03f	1000	3	2054	3	2054	3.0	2054.0	4.24

We can see that in four instances out of the 32 instances the MA has improved the best known upper bound by using one more vehicle, and in the other 28 instances the solution given by the MA coincides with the best known solution both in the total travel distance and in the number of vehicles.

We also have run the MA with fixing a priori the number of vehicles equal to the one given in the best known solution (see third column in Table 1). In this case, in all of the 32 instances the total travel distance obtained with the MA coincides with the best known upper bound, with running time similar to the one given in Table 1. Therefore, the table corresponding to these results has been omitted.

In Table 2 we show the results to the set of 126 single vehicle instances given by Soler *et al* (2008) for which the optimal solution is known. In this table, results are averaged over selected groups of the instances where the 126 instances are partitioned into 24 groups according to features of the instances. The columns in the table lists the number of instances in each groups (Ins), the average number of required arcs in the subset (ARA), the number of (required) edges in all the instances in the subset ($|E|$), the average number of required vertices in the subset (ARV), the average number of vertices in the asymmetric TSP instances obtained from the original instances (AV_{ATSP}), the average time in seconds to obtain the optimal solution with the exact procedure (ATO), the average time in seconds to obtain the heuristic solution with the MA ($ATMA$), the number of instances optimally solved with the MA in the subset (OPT), and the average deviation of the MA solutions in the subset ($ADEV$). By deviation we mean $(UB - LB)/LB$.

Table 2. Computational results on known single vehicle instances.

$Group$	Ins	ARA	$\|E\|$	ARV	AV_{ATSP}	ATO	$ATMA$	OPT	$ADEV$
1	6	51	0	6	82	14.48	5,76	6	0
2	4	84	0	8	127	65.04	11,04	4	0
3	6	108	0	8	149	392,81	16,20	6	0
4	5	125	0	12	195	348,34	27,89	4	0,00010
5	7	145	0	14	212	932,43	26,87	6	0,00050
6	6	164	0	16	248	1269,71	41,29	5	0,00025
7	5	54	0	3	86	42.65	7,53	4	0,00078
8	4	84	10	8	137	478,8	12,73	4	0
9	7	107	10	9	175	490,38	21,33	6	0,00071
10	6	126	10	11	201	819,55	29,76	1	0,00059
11	5	141	10	13	231	1273,89	41,44	4	0,00052
12	6	156	10	14	245	1037,18	50,19	5	0,00007
13	6	172	10	15	265	3211,59	61,19	2	0,00093
14	5	61	10	3	119	139,92	7,18	5	0
15	6	105	20	5	167	1464,62	21,52	3	0,00049
16	4	126	20	9	208	516,69	40,62	1	0,00141
17	6	142	20	11	240	3095,38	43,53	2	0,00142
18	4	156	20	9	236	3539,15	42,90	3	0,00089
19	3	168	20	12	236	3646,25	66,26	1	0,00130
20	4	58	20	3	129	846,88	16,77	2	0,00088
21	6	100	20	5	187	1218,35	22,66	4	0,00029
22	6	65	30	3	156	2437,78	22,63	3	0,00034
23	6	113	30	6	218	2931	37,87	1	0,00087
24	3	140	30	6	191	$> 2h.$	64,47	3	0

We can see from Table 2 that the results obtained with the MA are very good. The MA was able to optimally solve 86 instances out of the 126 instances, with 0.05% average deviation.

4.3 Solving MCGRPTP instances with the MA

We applied the MA to solve also three sets of random MCGRPTP instances with up to 700 arcs, 160 (required) edges, 160 vertices, 225 required arcs, and 28 required vertices, which are transformed into ACVRP instances with up to 623 vertices. As far as we know, the largest ACVRP instance described in the literature until now has 300 vertices.

Next we describe the data generation procedure for each set.

The first set was generated from the 128 single vehicle instances given by Soler *et al* (2008), and we obtain two MCGRPTP instance sets (one with vehicle capacity 1000 and the other with vehicle capacity 2000) as follows:

> We choose the depot node as the first required vertex in the numerical order. In this node all turns are changed to be allowed with zero cost. Each required arc, required edge and required vertex will have a randomly generated integer demand in range [13,120], [7,60], and [50,120] respectively. Note that when we transform one of these MCGRPTP instances into an ACVRP instance, if an original required vertex v has demand d_v and it gives rise to a vertex subset in the ACVRP with t vertices, let r be the largest integer positive number such that $d_v = rt + c$, with $c \geq 0$, then $t - 1$ vertices in this subset will have demand r, and the last one will have demand $r + c$. These 256 generated MCGRPTP instances act as a basis for ACVRP instances with number of vertices in the interval [61,290].

The second set contains 32 MCGRPTP instances generated from 16 of the biggest single vehicle instances by Corberán *et al* (2002). Each original instance is first transformed into an MGRP with turn penalties instance with the procedure explained in Soler *et al* (2008) and then transformed into two MCGRPTP instances (one with capacity 1000 and the other with capacity 2000) with the procedure given above for the first set. These 32 instances form the ACVRP instances with number of vertices in the interval [235,406].

Finally, the third set contains 48 instances that have been obtained from 24 new large single vehicle instances randomly generated with the same instance generator used by Corberán *et al* (2002). These 48 MCGRPTP instances are transformed into ACVRP instances with the number of vertices in the interval [382,623].

In these three sets we have used the MA version with variable number of vehicles in order to obtain best upper bounds with respect to the total travel distance. The appendix shows a table containing all data and results corresponding to each individual MCGRPTP instances. In this table, each instance is named as Ixxy, where xx indicates the subset to which the instance belongs and y indicates the number of the instance inside that subset. The first 24 subsets correspond to the first set of (128) instances, subsets 25 to 27 correspond to the second set of (16) instances and subsets 28 to 31 correspond to the third set of (24) instances. The columns in the table list the following data: the number of vertices in the ACVRP instance obtained from the corresponding MCGRPTP instance ($|V_{ACVRP}|$), the total demand in the ACVRP instance (T.D.), and for $i \in \{1, 2\}$, the total travel distance of the best result in five runs obtained for a vehicle capacity of $C_i = i \cdot 1000$ units (d_i), the number of vehicles corresponding to this best result (K_i) and the average computing time in seconds for a run corresponding to the capacity C_i (Time$_i$). Note that d_i is the total distance in the original MCGRPTP instance, not in the auxiliary ACVRP instance.

The MA was able to find feasible solutions for all 336 instances including the large-size instances with up to 600 vertices, within a reasonable computation time, as reported in the appendix. The computation times vary from a few seconds to more than one hour depending on the problem size. We consider the reported computing times reasonable, given the size and complexity of the considered ACVRP instances.

Based on these results, one can conclude that at least medium-size real-world MCGRPTPs can be solved by a state-of-the-art heuristic method for the ACVRP through their transformation into an ACVRP as explained here. By the way, we have generated a large number of instances to the, until now, limited set of ACVRP benchmarck instances. Of course these new instances will be available to any researcher interested on them.

5 Conclusions

In this paper we have studied a generalization of the MCGRP including turn penalties and forbidden turns. Through an intermediate transformation into a GVRP, we have provided a procedure to transform it into an ACVRP. Then, at least from a theoretical point of view, this generalization can be solved both optimally and heuristically with existing algorithms. We have also introduced a set of new benchmark problems and adapted a recent and powerful memetic algorithm to ACVRP. The experimental results show an average deviation equal to 0.05% for instances with known optimal solution and that large-size problems can be solved with the memetic algorithm.

We are convinced that research on turn penalties will increase and be of important value in the future to reduce the gap between theoretical research and real-life applications. We hope that the theoretical and experimental results presented here can be used in the future as ideas or tools to test the efficiency of specific procedures to solve capacitated routing problems with turn penalties.

Acknowledgements

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Appendix

Inst.	$\|V\|$-$\|A\|$-$\|E\|$	$\|A_R\|$	$\|V_R\|$	$\|V_{ACVRP}\|$	T.D.	d_1	K_1	Time_1	d_2	K_2	Time_2
I11	40-110-0	44	3	61	3819	4255	4	13.11	3966	2	4.42
I12	60-140-0	56	5	75	6462	6848	5	25.63	6145	3	17.9
I13	40-90-0	36	3	46	3009	4103	4	11.04	3547	2	19.67
I14	80-170-0	68	6	92	5415	7578	6	25.15	6806	3	20.9
I15	40-90-0	36	6	54	3142	4126	4	7.56	3625	2	8.38
I16	80-170-0	68	11	118	5772	7651	6	65.18	6798	3	37.18
I21	80-200-0	80	7	107	6936	7464	7	53.7	6962	3	35.13
I22	100-220-0	88	7	121	7067	10152	8	41.1	8999	4	28.14
I23	80-200-0	80	4	95	5815	7300	6	53.97	7026	4	28.3
I24	100-220-0	88	14	158	8907	10643	9	79.62	9439	5	54.04
I31	100-260-0	104	3	112	6704	11831	7	63.81	11272	4	24.74
I32	120-260-0	104	6	127	8055	13666	11	256.67	12364	6	112.48
I33	120-290-0	116	7	147	8946	13493	9	228.25	12332	5	59.84
I34	100-260-0	104	6	136	8154	12352	9	62.52	11466	5	47.86
I35	120-260-0	104	11	147	8387	12264	9	43	11130	5	41.33
I36	120-290-0	116	14	195	10603	14253	11	248.94	12728	6	102.05
I41	140-300-0	120	8	151	9108	15107	10	93.41	13086	5	69.21
I42	140-330-0	132	12	216	10775	16681	11	320.16	15176	6	114.81
I43	140-330-0	132	6	164	10232	16542	11	139.86	15126	6	65.39
I44	140-300-0	120	20	220	11314	16130	12	214.2	13732	6	156.12
I45	140-300-0	120	14	185	9746	15101	10	204.5	13184	5	122.74
I51	160-340-0	136	8	166	10460	14698	11	156.37	13806	7	224.51
I52	160-340-0	136	19	219	8387	15947	14	284.86	13405	6	69
I53	160-340-0	136	13	191	11192	14987	12	183.21	13413	6	117.01
I54	180-380-0	152	8	183	10750	18322	11	219.97	16710	6	92.44
I55	160-370-0	148	14	218	12729	17818	13	387.24	15959	7	151.67
I56	180-380-0	152	21	250	13252	19431	14	422.96	17000	7	236.26
I57	180-380-0	152	15	217	12690	18806	13	369.88	16687	7	117.98
I61	200-420-0	168	10	210	12868	21039	13	431.41	18289	7	131.69
I62	180-400-0	160	8	193	12364	19634	13	227.68	17122	7	91.19
I63	200-420-0	168	17	289	15961	25569	16	1608.23	19197	9	348.06
I64	180-400-0	160	20	255	14227	20806	15	522.98	17808	8	214.22
I65	180-400-0	160	14	217	13445	20295	14	392.63	17340	7	180.87
I66	200-420-0	168	18	254	14903	22681	15	669.9	18661	8	235.08
I71	40-110-10	44	1	65	3612	4830	4	12.93	4528	2	11.29
I72	40-90-10	36	3	67	3501	4353	4	14.94	4151	2	16.83
I73	60-140-10	56	2	81	4290	6101	5	16.09	5808	3	16.85
I74	60-160-10	64	2	89	5011	7201	6	28.32	6859	3	30.66
I75	80-170-10	68	4	107	6462	7924	7	45.76	7248	4	31.74
I81	80-200-10	80	4	113	6703	7403	7	62.7	6993	4	30.44
I82	100-220-10	88	5	113	6824	7409	7	86.16	6993	4	34.07
I83	80-200-10	80	7	133	7279	7654	8	51.37	7034	4	47.82
I84	100-220-10	88	10	184	9826	9568	9	109.2	8820	5	47.64

Inst.	$\lvert V\rvert$-$\lvert A\rvert$-$\lvert E\rvert$	$\lvert A_R\rvert$	$\lvert V_R\rvert$	$\lvert V_{ACVRP}\rvert$	T.D.	d_1	K_1	Time_1	d_2	K_2	Time_2
I91	120-260-10	104	6	149	8587	13401	11	157.11	12181	6	78.18
I92	100-260-10	104	5	157	8386	12508	9	209.1	11584	5	81
I93	120-290-10	116	5	158	8700	12457	9	145.08	11757	5	47.07
I94	120-260-10	104	16	204	10556	12734	9	111.9	11894	5	60.81
I95	100-260-10	104	10	184	9826	13780	10	248.3	12055	5	129.32
I96	120-290-10	116	10	183	9539	12771	10	182.84	11934	5	106.03
I97	120-260-10	104	11	177	10132	12703	9	95.41	11877	5	61.5
I101	140-300-10	120	6	169	9675	14174	10	190.19	12609	5	121.77
I102	140-330-10	132	6	179	10520	16205	11	195.82	15124	6	64.7
I103	140-330-10	132	11	203	11670	16628	12	330.09	15468	6	172.42
I104	140-300-10	120	16	215	11522	15189	12	314.17	13089	6	169.71
I105	140-330-10	132	15	243	13154	17056	14	389.77	15618	7	177.68
I106	140-300-10	120	11	193	10832	14880	11	274.03	12822	6	102.74
I111	160-340-10	136	9	193	11457	16445	12	273.48	14640	6	172.66
I112	160-340-10	136	23	275	13274	17141	14	484.04	14918	7	330.28
I113	160-340-10	136	16	230	12651	16616	13	403.95	14690	7	152.03
I114	160-370-10	148	6	195	12448	18102	13	247.19	16335	7	113.22
I115	160-370-10	148	12	230	13856	19240	14	586	16446	7	323.42
I121	180-380-10	152	8	203	11981	20136	12	782.32	17250	8	397.97
I122	180-380-10	152	20	277	15076	20133	16	530.39	16547	6	406.85
I123	180-380-10	152	14	234	12662	18966	13	500.68	16671	7	211.59
I124	180-400-10	160	13	241	13650	18237	14	411.86	16348	7	265.88
I125	180-400-10	160	8	219	12867	18404	13	431.07	16303	7	169.24
I126	180-400-10	160	19	276	15023	18594	16	415.15	16647	8	308.28
I131	200-420-10	168	8	234	14073	20264	13	454.48	18009	7	153.91
I132	200-440-10	176	8	223	12780	21636	15	308.71	19289	8	140.3
I133	200-420-10	168	22	288	15931	23868	16	1173.59	19076	8	661.17
I134	200-440-10	176	21	290	15052	21369	16	557.87	18686	8	343.22
I135	200-420-10	168	15	257	14872	21877	15	689.08	18479	8	278.81
I136	200-440-10	176	15	275	15877	24410	16	790.1	20049	8	522.07
I141	60-90-20	36	1	77	3699	4084	4	20.51	3833	2	21.02
I142	60-160-20	64	5	115	5659	8403	7	52.74	7629	4	30.23
I143	60-140-20	56	5	129	6107	7284	6	64.19	6522	3	42.57
I144	80-170-20	68	5	125	6565	8181	7	82.09	7743	4	52.77
I145	80-200-20	80	1	121	6542	7986	7	58.58	7620	4	49.68
I151	100-220-20	88	4	141	8387	10444	8	195.27	9800	4	106.95
I152	120-260-20	104	4	157	8391	12665	9	102.45	11959	5	76.46
I153	120-290-20	116	3	169	10033	13550	11	156.71	12303	6	76.88
I154	100-260-20	104	3	153	8387	13614	11	189.79	12502	6	98.08
I155	120-260-20	104	7	171	9857	13125	10	299.63	12032	5	159.41
I156	120-290-20	116	6	183	9777	13596	10	257.16	12317	5	172.63
I161	140-300-20	120	6	184	10131	15037	11	133.85	13473	6	77.48
I162	140-330-20	132	6	196	10861	16840	11	304.8	14957	6	137.01
I163	140-300-20	120	11	208	11277	15554	12	239.37	13585	6	136.5
I164	140-330-20	132	11	235	12807	17620	13	497.14	15329	7	175.13

Inst.	$\|V\|$-$\|A\|$-$\|E\|$	$\|A_R\|$	$\|V_R\|$	$\|V_{ACVRP}\|$	T.D.	d_1	K_1	Time_1	d_2	K_2	Time_2
I171	160-340-20	136	16	249	13059	16078	14	418.03	14551	7	261.08
I172	160-340-20	136	6	197	11250	15202	12	251	14189	6	154.1
I173	160-370-20	148	6	214	12080	18298	13	237.53	16272	7	144.29
I174	160-340-20	136	11	224	11824	15763	12	388.2	14244	6	325.71
I175	160-370-20	148	11	245	13134	18765	14	355.11	16396	7	248.01
I176	160-370-20	148	16	288	14216	19257	15	500.41	16827	8	247.63
I181	180-380-20	152	5	212	11643	18628	12	332.17	16663	6	246.78
I182	180-380-20	152	9	230	12888	19989	13	756.61	17339	7	231.97
I183	180-400-20	160	7	229	13028	18463	14	303.72	16648	7	190.11
I184	180-400-20	160	14	267	13991	18851	15	1343.62	18193	7	914.38
I191	200-420-20	168	7	237	13896	22059	14	703.29	19778	8	802.64
I192	200-420-20	168	17	289	15961	25569.00	16	1608.23	18782	7	406.29
I193	200-420-20	168	12	267	14877	23137	15	907.11	19184	8	251.88
I201	40-110-30	44	1	105	5266	6069	6	40.75	5444	3	38.24
I202	60-140-30	56	2	123	5777	6705	6	96.64	6359	3	54.63
I203	60-160-30	64	2	129	6846	8571	7	154.47	7915	4	80.08
I204	80-170-30	68	5	144	7021	8487	8	66.81	7896	4	74.44
I211	80-200-30	80	3	149	7597	8704	8	131.29	7943	4	88.27
I212	80-200-30	80	6	163	7744	8818	8	198.14	8056	4	103.63
I213	100-260-30	104	1	192	10061	12822	11	284.98	11984	6	143.23
I214	120-260-30	104	5	183	8387	12670	10	161.83	11516	5	122.43
I215	120-290-30	116	6	199	10056	13642	11	166.43	12382	6	116.95
I216	120-290-30	116	11	233	12343	14384	13	356.58	12711	7	152.71
I221	40-90-40	36	1	117	5416	5321	6	59.06	4962	3	56.52
I222	60-140-40	56	3	147	7012	7742	8	88.71	7093	4	81.97
I223	60-160-40	64	2	149	7524	8892	8	150.32	8225	4	90.57
I224	80-170-40	68	1	149	7651	8818	8	129.22	8134	4	82.15
I225	100-220-40	88	5	214	10793	11285	11	368.81	10027	6	160.24
I226	80-200-40	80	2	165	7986	9410	8	509.77	8253	4	275.54
I231	120-260-40	104	3	207	10567	14740	11	403.89	13088	6	145.29
I232	120-290-40	116	3	207	10678	14281	11	430.29	13110	6	150.35
I233	120-290-40	116	6	221	11151	14642	12	369.99	13282	6	178.76
I234	100-260-40	104	5	208	10307	12485	9	88.39	11297	5	50.78
I235	140-300-40	120	5	216	11258	15690	12	327.21	14373	6	202.45
I236	140-300-40	120	10	242	7021	16012	13	460.42	14576	7	207.78
I241	140-330-40	132	5	231	11865	17444	12	606.84	15789	6	372.98
I242	160-340-40	136	6	241	12797	17189	13	687.33	15339	7	207.94
I243	160-370-40	148	5	249	13426	18010	14	552.3	16245	7	365.67
I244	160-340-40	136	12	271	13973	20563	14	1586.52	16225	7	904.7
I245	160-370-40	148	10	275	14247	18365	15	566.35	16602	8	239.05
I251	120-260-60	104	3	235	10952	16098	11	836.73	13467	6	287.68
I252	140-330-60	132	7	283	13898	18238	14	1108.62	15406	7	799.45
I253	160-370-60	148	6	293	14768	20488	15	1189.57	18175	8	476.2
I254	180-400-60	160	8	315	16339	21517	17	1184.31	19329	9	522.9
I256	200-420-60	168	17	361	19015	24841	20	1212.79	21615	10	1023.6

Inst.	$\|V\|$-$\|A\|$-$\|E\|$	$\|A_R\|$	$\|V_R\|$	$\|V_{ACVRP}\|$	T.D.	d_1	K_1	Time$_1$	d_2	K_2	Time$_2$
I261	100-220-80	88	2	253	10764	12723	11	643.9	11708	6	424.9
I262	120-260-80	104	3	273	12387	15312	13	529.4	14127	7	285.35
I263	140-330-80	132	2	299	14801	18469	15	1370.78	16110	8	552.23
I264	160-370-80	148	11	359	16695	22098	17	1434.42	18966	9	651.6
I265	180-400-80	160	10	361	18031	22641	19	1089.58	19829	10	793.31
I266	200-420-80	168	9	363	18409	23206	19	1685.57	20366	10	1026.85
I271	100-220-100	88	2	293	12705	12708	13	912.82	11754	7	461.76
I272	120-260-100	104	3	315	14161	15851	15	784.83	14099	8	490.66
I273	140-330-100	132	5	353	16462	18388	17	1280.67	16269	9	1086.19
I274	180-400-100	160	9	406	19041	23053	20	1927.5	20180	10	1489.17
I275	200-420-100	168	8	399	19914	29504	20	4160.04	23017	10	2203.86
I281	200-600-60	180	9	382	23593	25022	18	4449.39	20696	9	1819.96
I282	200-600-80	180	13	452	19844	26271	20	5636.98	223670	10	3160.30
I283	200-600-100	180	6	440	20324	24640	21	2681.01	21966	11	1802.93
I284	200-600-120	180	8	463	20555	26060	21	3841.40	23267	11	2136.15
I285	200-600-140	180	7	497	21577	27645	22	5756.68	24330	11	3655.87
I286	200-600-160	180	7	548	25148	25999	26	5966.49	23482	13	3879.16
I291	225-650-60	195	19	439	21355	27266	22	2306.38	23901	11	2337.94
I292	225-650-80	195	15	470	22727	29510	23	4450.06	248090	12	2900.06
I293	225-650-100	195	12	494	23987	28549	25	3187.86	25247	12	5064.86
I294	225-650-120	195	11	507	23593	31508	24	4747.81	27241	12	4546.69
I295	225-650-140	195	6	502	23593	30096	24	4434.57	26147	12	5327.40
I296	225-650-160	195	5	538	24429	30333	25	5490.39	267850	13	5215.72
I301	250-700-60	210	24	484	22346	28475	23	3718.08	247730	12	1945.99
I302	250-700-80	210	19	484	22746	32558	23	4312.41	27064	12	2631.71
I303	250-700-100	210	18	521	24925	29632	26	5427.05	25985	13	3103.63
I304	250-700-120	210	7	490	23071	30786	24	3635.21	27128	12	2674.20
I305	250-700-140	210	11	563	24875	374830	25	10734.52	297940	13	4286.82
I306	250-700-160	210	11	604	27643	34824	28	9304.83	29962	14	6554.05
I311	270-750-60	225	25	503	23948	30713	25	3003.68	29216	12	5038.99
I312	270-750-80	225	28	599	27111	32940	28	4606.29	28394	14	5215.76
I313	270-750-100	225	16	543	26907	33859	28	5026.30	29348	14	3865.43
I314	270-750-120	225	13	548	25044	30692	26	4782.07	27320	13	4545.20
I315	270-750-140	225	14	602	27722	36708	28	9063.87	306540	14	7840.02
I316	270-750-160	225	9	623	27771	39925	28	11808.50	32859	14	9424.87

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