

## A gymkhana to discover and generate curves in a cooperative work

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### Abstract

The student who access the University understands a curve as a function represented by an equality involving the Cartesian variables  $x$  and  $y$ . But he does not know another great set of curves such as those described by parametric equations and polar coordinates.

We show an experience developed in two consecutive courses in the first course of a bachelor's degree, which allows the students to discover and create many curves in parametric or polar representation, working in group. Also the experience facilitate the students to achieve several generic skills.

### Keywords

Mathematics, parametric curves, polar curves, general competencies.



## 1. Introduction

In this work we describe the planning, development and results obtained from computer practices for studying parametric and polar curves. This experience has been developed during the academic courses 2011-2012 and 2012-2013 in the Mathematics 2 subject corresponding to the Bachelor's Degree in Telecommunications Systems, Sound and Image Engineering (GISTSI) in the Gandia Campus of the Universitat Politècnica de València (UPV). This practices has permitted to add to the subject some general and transversal skills like teamwork, information management, use of information and communication technologies (TICs) and time management (including responsibility and work planning).

### 1.1. The subject Mathematics 2

Mathematics 2 is a first year annual subject with 9 ECTS, distributed in such a way that 4.6 ECTS are devoted to theory lessons and classroom practices, 3 ECTS are devoted to seminary lessons and 1.4 ECTS are devoted to computer practices. The students can consult the teaching guide and obtain the resources of the subject in the UPV PoliformaT platform (see Figure 1).

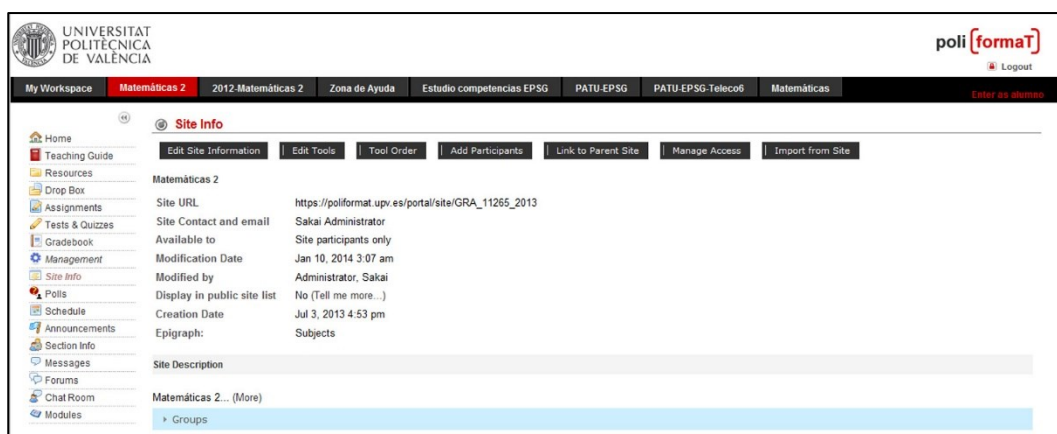


Figure 1. PoliformaT platform



The subject program has the following units: indefinite integral and applications to the resolution of ordinary differential equations; definite integrals and improper integrals; numerical series; power series and series development; study of functions of several variables; optimization of functions of several variables and double and triple integrals. Seven two hours practical computer sessions are done using Matlab<sup>®</sup> (Alamar 2008):

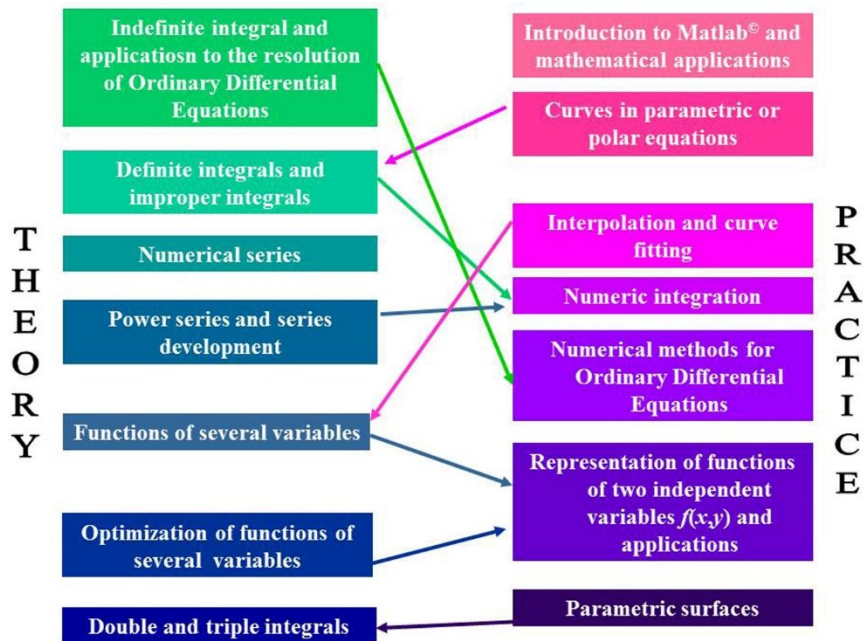
1. Introduction to Matlab<sup>®</sup> and mathematical applications.
2. Geometry of curves: parametric and polar curves.
3. Interpolation and curve fitting.
4. Numeric integration.
5. Numerical methods for Ordinary Differential Equations.
6. Graphical representation of functions of two independent variables,  $f(x,y)$ , and its applications.
7. Parametric surfaces.

Practices 3, 4, and 5 tackle another fundamental unit of the subject, the Numerical Analysis. In Figure 2 interrelation between practices and theory units is shown. It can be seen how practice 2 precedes the definite integral theory unit. Thus, thanks to the previous work on practice 2, the student will be able to calculate areas and volumes of solids of revolution generated from parametric or polar curves.

At the end of each practice the students are required to perform a project in group (2-3 students), that they have to upload in the corresponding *Assignment* application, established by the professor, in the *PoliformaT* platform. In the subject's mark, 20% corresponds to the work done on the computer practices (project practices), 70% corresponds to various partial tests of theory and problems, and 10% corresponds to the portfolio containing the works performed by the student in seminars and at home. Although the realization of the practices is voluntary, students are informed about the



advantage of its realization: a quarter of the students pass thanks to the practices valuation according to the analysis of the results of previous courses.



**Figure 2.** Interrelation between theory and computer practices.

## 1.2. General skills in Higher Education

General skills are the part of the graduate's profile that prepare them as professionals and citizens, and have to be developed along their university studies (Blanco 2009). In a study done in some universities in the United Kingdom and Australia (Fallows 2000) is concluded that there are five of general skills which have to be developed at the university: general communication abilities; information management (search, selection, analysis and evaluation of the information coming from various sources); ability for the

use of new technologies (TIC utilization); teamwork, ethics and diversity recognition; personal competencies (time management, responsibility, planning and others).

In section 3 of CIN/352/2009 decree, which establishes the requirements for the verification of official university degree that enables to work as Technical Telecommunications Engineer in Spain, appear the competencies that the students have to acquire. We highlight here three of them: knowledge of basic matters and technologies (which enables them for the learning of new methods and technologies, as well as giving them a big versatility to adapt to new situations); problem solving skills (initiative for decision making, creativity and capability for the communication and transmission of knowledge); and finally, ability to work in a multidisciplinary group in a multilingual environment and to communicate, written and orally, knowledge, procedures, results and ideas related with telecommunication and electronics.

There is no doubt that the general skills will help the university students, particularly the future technical telecoms engineers, for getting the specific skills required to practice.

### 1.2.1. Teamwork

In the computational practice described, the students have done a project in groups of 2-3 people. Further each group collaborates with the others as a part of a whole for achieving a result or a final product, obtained from the aggregation of each part.

The final product is a PowerPoint<sup>®</sup> that includes a big number of graphics. The job of each team is a part of a global PowerPoint. Moreover the student needs to share learning experiences both from the partners and the professors, which requires the development of teamwork ability (Blanco 2009). Teamwork favors learning and helps the student in acquiring other competencies that he could not get alone. Taking into account the concept of interaction, there are three types of groups: group with competitive interaction (the



student only achieves the goals if his group achieves them), group with individualist interaction (the students are rewarded for the quality of their individual work) and group with cooperative interaction (the aims of the student body are interconnected, and he assumes a positive interrelation and the consecution of the team goals). The benefits of cooperation are the improvement of the students' results, much more than those coming from individual work and higher productivity. The learning results improve when sharing objectives and working together. And this has been our bet: a cooperative interaction between teams to improve individual learning outcomes.

### 1.2.2. Information search

Previously to the beginning of the academic course, in the Welcome Days (Jornadas de Acogida, JJAA), the new students have a workshop in the computer room. The workshop content includes the library, its resources and the information search. But when the course is started, the student tend to focus only in the lesson notes or in the slides that the professor have uploaded to the PoliformaT platform. Although the teaching guide proposes a complete and varied literature, at most, only few students use the notebook of the subject.

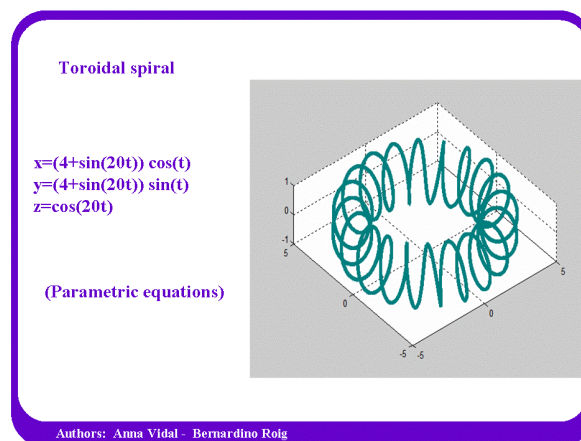
To develop skills in information search, in advance of the realization of the practice, the student has to find three equations of curves in books located in the university library. This is intended to force the student to use what he learned in the JJAA workshop localizing some texts from the teaching guide, visiting the library, manipulating texts and thus, checking that there are other books to complete the subject learning. As we indicate at the end of this paper, some students have referenced this fact in the assessment of the practice.



### 1.2.3. Use of TIC

In the JJAA, another workshop related with the UPV intranet and the platform PoliformaT is performed. Few weeks later students generally are familiar with PoliformaT and with the university e-mail but little more. However the use of TIC includes also file management, document editing, surfing the internet, making presentations,...

As part of the work, each team is required to provide the Matlab<sup>®</sup> commands used and the PowerPoint slides, in a specific format (see Figure 3) available for the students in PoliformaT.



**Figure 3.** Power Point<sup>®</sup> pattern.

The students have to capture the graphic representation from Matlab<sup>®</sup> and adapt it to the required format, respecting the dimensions and indicating the equations and the name of the authors. Later, the teacher generates a presentation with all the slides provided by the different teams, deleting the incorrect ones, classifying the rest and leaving it available in PoliformaT, so that every participant can see the end-product. Moreover, during the practice session web resources have also been used.

During the course 2012-2013 the web tool Issuu has been used. It is an online service that allows displaying digitalized material as if it were a printed publication, with a format that allows visualizing two pages at the same time (like an open book or magazine) and an animated page turning (Vidal 2013).

#### 1.2.4. Time management and planning

The PoliformaT Assignments, corresponding to the practice project, have a prefixed time delivery. Students have two weeks from the realization of the practice for uploading the work to PoliformaT. Only works uploaded in time are taken into account. So, each group has to plan and manage the time for completing and delivering the practice project.

## 2. Experience development

Math class students of High School have studied functions  $f(x)$  and their graphs as the locus of the points  $(x,y)$  of the plane in which  $x$  belongs to the domain of definition of the function and  $y=f(x)$ . They have studied straights, parabolas, conics, trigonometric functions, exponential and logarithmic functions and some others, but always through their Cartesian expressions,  $y = f(x)$ , and they have calculated areas determined by the graphics of the curves. However, for describing the trajectory in the plane (or in the 3D-space) of an object in many physical applications classical Cartesian coordinates are not usual and parametric equations are used, where the parameter is the time  $t$ . For example, for the computation of integrals on a curve which lead to know the movement of a vector field along the curve, or for the determination of the mass taking into account density, the parametric expression of the curve is required. On the other side there are curves that are difficult to be described by Cartesian equations and the utilization of polar coordinates facilitate much their representation and study.





## 2.1. Initial planning: a mathematical gymkhana

Before the realization of the practice, and in order that the students search the equations of the curves to be studied in the library, it was raised to the students a kind of gymkhana (see Figure 4). In the gymkhana the students had to write down the equations of three different curves, some of them in parametric equations and others in polar form, which they must find in the proposed bibliography. Preparation of the gymkhana is laborious for the teacher that needs to consult and localize in the library the books where the equations appear (Bradley 1998; Edwards 1996; Larson 2006, 2010; Stewart 1999, 2003; 2005; Thomas 1999). Another work of the teacher is to select and organize all the curves in groups or gymkhanas of 3 curves. Each course new curves are added to the collection. Now 40 gymkhanas are available and the curve selection is expanded to the following web pages:

<http://temasmaticos.uniandes.edu.co/Trocoides>

<http://departamento.us.es/edan/php/asig/LICFIS/LFIPC/Tema3FISPC0809.pdf>

<http://elmisantropodigital.blogspot.com.es/2012/09/mas-composiciones.html>

<http://rectas.wordpress.com/category/pov-ray/>

The gymkhana distribution among students was done in the last minutes of the previous practice.



**Gymkhana G1**

Group practice computing .....

Name of the participants:

- 1.
- 2.

You must look for and write the equations of three curves (in parametric or polar coordinates) that can be located in the following texts:

1. Book: CÁLCULO CON GEOMETRÍA ANALÍTICA. Edwards y Penney. Ed. Prentice Hall  
Curve in parametric form on page 658: epitrochoid in Figure 12.1.13
2. Book: CÁLCULO CON GEOMETRÍA ANALÍTICA. Edwards y Penney. Ed. Prentice Hall  
Curve in parametric form on page 658: Lissajous curve in Figure 12.1.15
3. Book: CÁLCULO DE UNA VARIABLE. Stewart. Ed. Thomson.  
Curve in polar form on page 552: five-petalled rose Exercise 57

**Figure 4.** Sample of one of the gymkhanas.

## 2.2. Methodology part: practice development

The whole set of first year students is divided in four groups of 20 students to realize the practices. The scripts of the practices (Alamar 2008) are available for the students. In the scripts, theory concepts alternate with practice called Get Down to Work (Manos a la obra, MAO). Students have to work with Matlab<sup>®</sup>. The professor explains the theory contents of the practice at the beginning of the session leaving some time so that the students work on the MAO sections.



### 2.2.1. Script of practice: curves geometry (curves in parametric and polar equations)

The structure of the script is as follows:

1. Operations step by step with Matlab<sup>®</sup>: Operations with vectors and/or matrices which do the same computation on each of its components.
2. Curves in parametric equations: The notation used is described

$$\begin{cases} x = x(t) \\ y = y(t) , \quad t \in [a, b] \\ z = z(t) \end{cases}$$

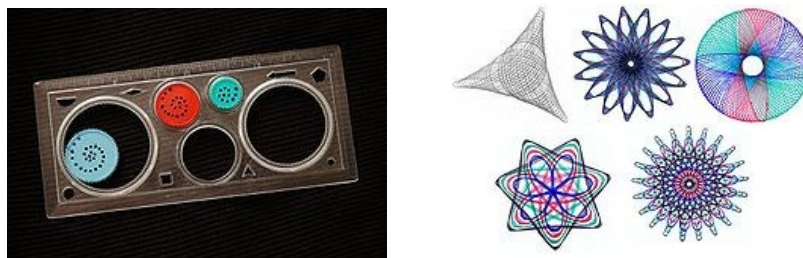
The Bezier's curves used in shape design, automotive industry and also to identify letters' shapes and other symbols in laser printers are cited as a curiosity.

Some differential geometry concepts are included: velocity vector, acceleration vector,  $T(t)$ ,  $N(t)$  and  $B(t)$  vectors of the movable reference system known as Frénet-Serret Trihedral and the normal plane determined by  $N(t)$  and  $B(t)$ . The last ones are essential for generating tube-shaped surfaces around a curve. Finally, it is defined the arc length function, and the curve's length.

3. Some parametrizations of interest: Lines, circumferences, ellipses.
4. Representing graphs of curves given in parametric coordinates in Matlab<sup>®</sup>: In the corresponding MAO a circumference is represented, and the student is asked to modify the commands to represent a spiral, a cylindrical helix and a conical helix.
5. Plane curves in polar coordinates and their representation with Matlab<sup>®</sup>.
6. Some curious parametric curves:



6.1. Cycloids, epicycloids, hypocycloids, trochoids, epitrochoids and hypotrochoids: through the web page <http://temasmaticos.uniandes.edu.co/Trocoides> directed by Aquiles Páramo Fonseca, professor in the Mathematical Department of the University of Los Andes. The student “knows what these wonderful curves are, which are their parametric equations and their most notable properties”, as there is indicated. The web page includes figures and animated programs developed with Java<sup>®</sup> by the author, so that the almost inexhaustible universe of these curves can be explored. Finally the student would be able to have fun with the spirograph in the same web page, a program to make attractive graphic designs (see Figure 5).



**Figure 5.** Spirograph for the geometric drawing of some hypotrochoids.

6.2. Lissajous' curves: Appear commonly in oscilloscopes.

7. Some curious polar curves:

7.1. Spirals: Spiral of Archimedes, logarithmic spiral.

7.2. Cardioid: It can show interference patterns and waves congruency which irradiate concentrically from a punctual source. Thanks to this property, sensibility areas of microphones and antennas can be identified. A cardioid microphone is sensible to frontal sound and minimizes sounds coming from the rear zone. The

polar pattern or directionality determines in which directions is more sensible a microphone and, therefore, where from it will take more sound.

### 7.3. Limaçons and roses.

In the last part of the practice, the representation of gymkhana's curves starts.

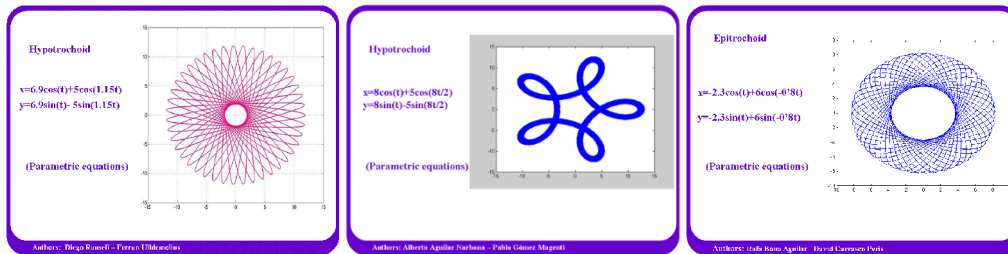
## 3. Results

During the academic year 2011-2012, a total of 26 groups participated generating a total of 36 different graphs (after debugging the wrong graphics or those that do not respect the required format), 15 in polar and 21 in parametric. Two of the students, voluntarily, located two more graphics that were also added. In the academic year 2012-2013, 34 teams participated, and have been generated 53 graphics, 21 in parametric curves and 32 polar curves. The complete publication is available in Issuu (Vidal 2013).

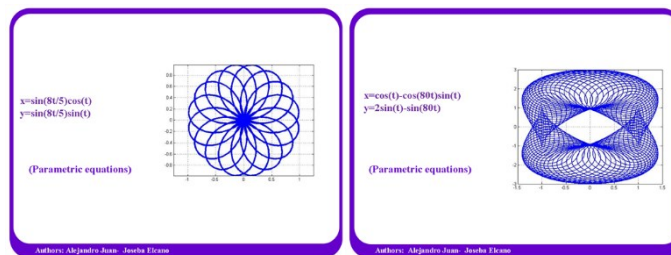


Figure 6. Publication in Issuu.

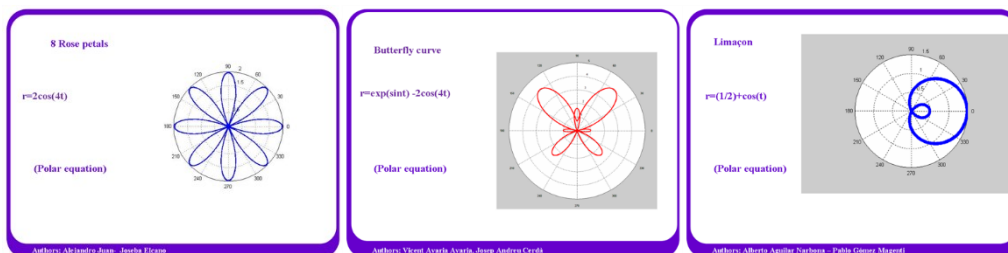




**Figure 7.** Trochoid figures.



**Figure 8.** Other parametric curves.



**Figure 9.** Curves in polar coordinates.

Figures 7, 8 and 9 show some of the curves made by the students.

#### 4. Conclusions

In the academic year 2012-2013, in addition to the memory to be submitted by each group, we asked for students a review of practice. 38 students answered some interesting questions:

Did you find this project interesting? 81.6% answered that it has been interesting or very interesting.

About the time spent making the work, 76.3% indicates that, at most they spent three hours (this time is in the provided interval).

Taking into account the observations and ideas given by the students in the assessment, we highlight the following ones:

1. 'It is a very interesting practice in which we can appreciate graphically much of the expressions that we use many times and we can find a big variety of possibilities in ways that normally cannot be appreciated in a theory lesson. It would be interesting to propose the construction of a roller coaster or a capricious bridge using these types of curves'.
2. 'It has been a very nice practice and not so much complicated'.
3. 'This is an entertaining practice. It is nice the way used for looking for our own equations in the library books; by one side because many of us may have discovered the library thanks to the practice and, on the other, because it is proved that you can learn math in another way, not only theory and problems'.
4. 'This practice is very good, because not only new things are learnt with Matlab<sup>®</sup>, it also helps us to know the campus library and how to find the material that we need at each moment'.
5. 'In our opinion this practice has been interesting due to the curiosity of the geometric shapes produced by the equations, like we have seen at class about the skating ramp that allows reaching the maximum velocity'.



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[http://issuu.com/anna\\_vidal/docs/curves\\_of\\_mathematical\\_gymkhana](http://issuu.com/anna_vidal/docs/curves_of_mathematical_gymkhana)

