ADJUSTING THE PARAMETERS OF THE MECHANICAL IMPEDANCE FOR VELOCITY, IMPACT AND FORCE CONTROL

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Abstract: *This work is dedicated to the analysis of the application of active impedance control for the realisation of three objectives simultaneously: velocity regulation in free motion, impact attenuation and finally force tracking. At first, a brief analysis of active impedance control is made, deducing the value of each parameter in order to achieve the three objectives. It is demonstrated that the system may be made overdamped with the adequate selection of the parameters if the characteristics of the environment are known, avoiding high overshoots of force during the impact. The second and most important contribution of this work is an additional measure for impact control in the case when the characteristics of the environment are unknown. It consists in switching among different values of the parameters of the impedance in order to dissipate faster the energy of the system, limiting the peaks of force and avoiding losses of contact. The optimal switching criteria are deduced for every parameter in order to dissipate the energy of the system as fast as possible. The results are verified in simulation.*

Keywords: robot control, impact, force control, impedance control, switching.

1. INTRODUCTION

Possibly the most characteristic problem of robot force control is the abrupt change from free to constrained motion. The importance of this discontinuity is emphasized by the fact that in the typical industrial applications the environment is very stiff and the dynamics of the system is much faster during the constrained motion. The system is highly underdamped. In the transient phase (impact) the force may reach dangerous peaks.

Before achieving the contact, the magnitude to be controlled is the velocity, and after, the force. Since different magnitudes are to be controlled and the characteristics of the system have an important change, it seems logical to use one controller for each phase. In order to make the transition from free to constrained motion as smooth as possible a third controller may be introduced. It is called impact control.

The impact is the most important phase because high peaks of force may occur and cause irreversible damage to the robot, the environment or the tool. Even if that doesn't happen, smaller peaks of force deteriorate gradually the mechanics of the robot. Another potential problem of the impact phase is the possibility of bouncing. All these drawbacks could be easily avoided designing an overdamped controller if the characteristics of the environment were known. Unfortunately it is often not the case. For this reason, any a priori selected parameters of the regulator may not be adequate, and additional actions could be necessary if the system appears to be underdamped when the contact is achieved. An additional inconvenient of the impact control is the fact that this phase is extremely brief and may last just a few sampling periods. This implies that, for example, an adaptive controller may be too slow to protect the system.

The impact control has been extensively researched and very diverse solutions have been proposed. The most compilation from different sources as well as a very exhausting analysis of the impact control has been published by Brogliato[1] in 1999. It should be emphasized that there are two ways to treat the impact[1, 2]: the rigid and the flexible model. The former doesn't consider what is happening during the contact phase, just before and after it. It is assumed that the duration of the impact is infinitely short. The relation between the velocities in the moment of the contact and after the rebound is given by the coefficient of restitution. In the flexible model the impact is treated analytically considering the robot and/ or the environment as elastic bodies. This model will be used in this article.

Following will be enumerated some methods for impact control. With the rigid model, Brogliato et al.[3] proposed two methods for limiting the number of rebounds and assuring in this way the stability of the system. There are more works based on the flexible model. Volpe and Khosla[4] proposed three methods for impact control. All three are oriented to avoid contact loss rather than the protection against peaks of force. Hyde and Cutkosky[5] proposed in 1994 the modulation with pulses of the feedforward. These pulses are computed to suppress the transitory harmonics. Xu, Hollerbach and Ma[6] implemented a PD force control with feedforward is used for force reference tracking as well as for the impact control. The parameters of the regulators vary according a non linear law. They decrease if the robot is approaching the reference in order to reduce the residual energy. In the contrary case, they increase. Ferretti, Magnani and Zavala Río[7] proposed to apply during the impact a feedforward determined empirically combined with the force regulator, in order to avoid contact losses.

The switching of parameters was introduced in force control by B. Armstrong et al.[8,9], in a study similar to the present work. The authors switch the gain matrix according to the state of the system. The essential idea is the same, but a different mathematical methodology was used. In the work of B. Armstrong et al. LMI was used to demonstrate the validity of the method.

From the mentioned sources, it can be deduced that the techniques for impact are control are very heterogeneous. They don't use the same model. Some are dedicated to avoid contact losses regardless of the possible peaks of force. Others are limited to guarantee the convergence of the system after a finite number of rebounds. Some consider the characteristics of the environment are completely known.

As stated earlier, in addition to the impact controller, a position/ velocity and force regulators are also necessary. A force control task may consist of three controllers and two processes with very different dynamics. Switching among them is a potential source of bouncing, sliding regime and even stability loss.

In order to avoid the switching among controllers, impedance control may be used. This is a control strategy theoretically adequate for the three phases, proposed by Nevill Hogan[10]. Its objective is to impose a desired dynamics to the robot rather than tracking the force reference. Its main advantage is that the same controller can be used for both free and restricted motion. The main disadvantage of impedance control, as it has been stated in many sources, for example by De Schutter et al.[11], is that it is necessary to have an exact model of the environment in order to reach the force reference. This assumption may be impossible for some real applications.

The first contribution of this article is the deduction how to select the impedance parameters in order to overcome this limitation, assuming the characteristics of the environment are known. It is demonstrated that the adequate combination of parameters allows reaching the reference value of the force regardless of the environment, achieving velocity control during free motion and attenuating the impact. Nevertheless, this technique does not guarantee the behaviour of the system during the impact if the parameters of the environment are unknown.

The second and most important contribution of this paper is an impact controller based on the switching of the parameters of the impedance depending on the dissipated and the generated forms of energy in the given instant. The basic ideas of this work have been published by the authors in 2005[12]. Also, the authors of this paper published a method for simultaneous velocity, force, and impact control[13]. The approach is similar to this article, but it was applied to explicit instead of implicit force control. The parameters are different, as well as the way to adjust them.

The proposed method guarantees an improvement of the damping of the system, regardless of the characteristics of the environment. It needs just a few sampling periods to be effective.

2. SOME CONSIDERATIONS REGARDING IMPEDANCE CONTROL

The dynamics equation of a robot arm subject to an external force is well known[14, 15, 16]:

$$\tau - J^T(q)F = D(q)\ddot{q} + H(q,\dot{q}) + G(q) \qquad (1)$$

Where τ is the vector of motor torques, $J(q)$ the Jacobian matrix of the robot, F the vector of external forces acting on the robot's end effector, $D(q)$ the inertia matrix of the robot, $H(q,\dot{q})$ the matrix of centrifugal and Coriolis torques $G(q)$ the vector of gravity torques on the motors, and q, $\dot{q}$ and $\ddot{q}$ are the vector of joint positions, velocities and accelerations, respectively.

Solving the equation (1) for the acceleration:

$$\ddot{q} = D^{-1}(q)(\tau - J^T(q)F - H(q,\dot{q}) - G(q)) \qquad (2)$$

On the other hand, the relation between the Cartesian and joint velocities is[14, 15, 16]:

$$\dot{X} = J(q)\dot{q} \qquad (3)$$

And the accelerations:

$$\ddot{X} = J(q)\ddot{q} + \dot{J}(q)\dot{q} \qquad (4)$$

Solving (4) for $\ddot{q}$:

$$\ddot{q} = J^{-1}(q)\ddot{X} - J^{-1}(q)\dot{J}(q)\dot{q} \qquad (5)$$

Replacing (2) in (5) and solving for $\ddot{X}$:

$$\ddot{X} = J(q)D^{-1}(q)(\tau - J^T(q)F - H(q,\dot{q}) - G(q)) + \\ + \dot{J}(q)\dot{q} \quad (6)$$

This system is highly non-linear. The acceleration depends not only on the motor torque and the external force, but also on the inertia matrix, centrifugal, Corilois and gravity forces and the robot Jacobian. These magnitudes vary depending the joint positions and velocities.

Applying a constant force on the end effector, the acceleration vector would vary both its intensity and its direction during motion. The behaviour of the robot when in contact with the environment would be complicated to predict for the robot operator. Impedance control allows an intuitive reaction of the robot to external forces. It may be obtained manipulating the input variable τ. Some ways to achieve it are enumerated and briefly described in subsection 2.1.

Assuming that the torque vector τ is set to the exact value that compensated the centrifugal, coriolis and gravity forces as well as the term $J(q)\dot{q}$, then equation 6 becomes:

$$\begin{aligned} &\ddot{X} = -J(q)D^{-1}(q)J^T(q)F \Leftrightarrow \\ &\Leftrightarrow F = -(J(q)D^{-1}(q)J^T(q))^{-1}\ddot{X} = \\ &= -J(q)^{-T}D(q)J^{-1}(q)\ddot{X} = M(q)\ddot{X} \end{aligned} \quad (7)$$

Where:

$$M(q) = -J(q)^{-T}D(q)J^{-1}(q) \quad (8)$$

Represents the real Cartesian inertia matrix of the system, i.e. the relation between the force and the acceleration. It should be noted that it varies with the configuration of the robot. Also, it is non diagonal (in the general case). For these reasons, the inertia of a robot changes during motion.

The mechanical impedance is the relation between the force and the velocity of the system:

$$Z(s) = \frac{f(s)}{v(s)} = \frac{Ms^2X(s) + BsX(s) + KX(s)}{sX(s)} = \\ = Ms + B + \frac{K}{s} \quad (9)$$

Where Z represents the impedance, f the external force, v the velocity, x the position, M the mass, B the damping and K the stiffness of the system.

The impedance control consists in imposing to the system the desired mass, damping and stiffness (M_d, B_d and K_d respectively) instead of the real ones.

Several formulations can be found for the mechanical impedance. Some of them are:

$$\begin{aligned} &F = M_d(\ddot{X} - \ddot{X}_{ref}) + B_d(\dot{X} - \dot{X}_{ref}) + K_d(X - X_{ref}) \\ &F = M_d\ddot{X} + B_d(\dot{X} - \dot{X}_{ref}) + K_d(X - X_{ref}) \\ &F = M_d\ddot{X} + B_d\dot{X} + K_d(X - X_{ref}) \\ &F = B_d\dot{X} + K_d(X - X_{ref}) \\ &F = B_d(\dot{X} - \dot{X}_{ref}) + K_d(X - X_{ref}) \\ &F = M_d\ddot{X} + B_d\dot{X} \end{aligned} \quad (10)$$

The first one is most common[14, 15, 16]. The second one has been used in the original paper of Hogan[10]. The first three can be found in the work of Seraji and Colbaugh[17]. The fourth formulation is also called stiffness or compliance control[16]. It has the advantage that it needs neither acceleration nor force sensor to be implemented. Nevertheless, the inertia is not controlled and thus the behaviour of the robot varies in different configurations. The same formulation has been adopted by Christian Ott el al.[18, 19] for the control of elastic robots. It should be noticed that the real robot is a second order system, while the fourth formulation is first order. That is due to the fact that that the formulation represents the stationary behaviour of the system while the inertia, present only in the transient phase, is not included in the equation. The fifth formulation has been used by Lu et al[20]. In this case the order of the system is reduced by using a sliding mode controller. A first order system is obtained and thus oscillations are avoided, and neither peaks of force nor contact losses may occur. Finally the sixth formulation may be used for human friendly robots that execute tasks in cooperation with humans[21] or for emulation of human muscles[22]. Since the stiffness is zero, if the robot is displaced by an external force, it will not return to its initial position.

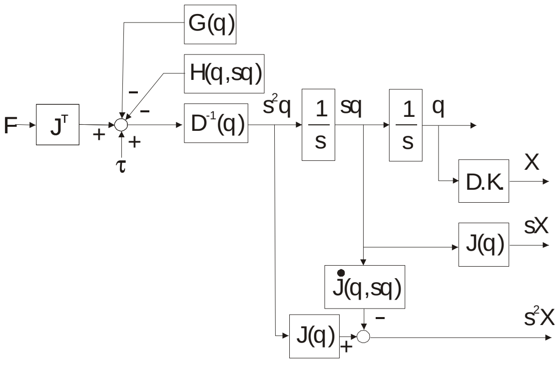
The first formulation is the most general one and all the others may be considered special cases of it. It may be written in the form:

$$F = M_d\ddot{X} - M_d\ddot{X}_{ref} + B_d\dot{X} - B_d\dot{X}_{ref} + K_dX - \\ - K_dX_{ref} \Leftrightarrow F = M_d\ddot{X} + B_d\dot{X} + K_dX - FF \quad (11)$$

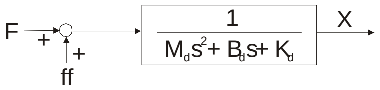
Where FF is the feedforward term.

Therefore, the impedance is defined by four parameters: the mass, the damping, the stiffness and the feedforward. Following, the effect of all of them will be analysed in constrained and free motion in order to obtain an adequate performance in all the phases of the task.

The following figure represents a schema of the real, physical, dynamics of the robot as well as the dynamics achieved by means of impedance control.



a)



b)

Figure 1. Schema of the real (a) dynamics of the robot and the dynamics obtained by impedance control (b). "s" represents the Laplace operator and the block "D.K." the direct kinetics.

As it may be observed, in the real dynamics of the robot (Figure 1, schema a)), the relation between the external force F and the Cartesian motion is complex and highly non- linear. It depends on the actual configuration of the robot, the join velocities and the motor torques.
In the impedance control (Figure 1, schema b)) the behaviour of the system is linear. Physically, the dynamics is the same as in the schema a) in figure 1, but the motor torques τ are manipulated to obtain the dynamics represented in schema b), which is also given by equation (11). Next subsection describes the way it may be achieved.

2.1 *The implementation of the impedance control*

The impedance control may be implemented in several ways. Canudas de Witt et al. [23] describe two methods: via linear state feedback and by inverse dynamics. The former is adequate for a one degree of freedom robot. The latter is better for several degrees of freedom. Another method for the implementation of the impedance control is the sliding mode control[20]. In this article the inverse dynamics method will be used because it achieves the linealization and the decoupling of the system. Following will be presented a brief description of this method.

In order to obtain the dynamics of the system describe in equation (11), the acceleration must be:

$$\ddot{X}_d = M_d^{-1}(F_{ext} + FF - B_d\dot{X} - K_d X) \quad (12)$$

According to (5), the acceleration in joint space should be:

$$\ddot{q}_d = J^{-1}(q)(\ddot{X}_d - \dot{J}(q)\dot{q}) = \quad (13)$$
$$= J^{-1}(q)(M_d^{-1}(F_{ext} + FF - B_d\dot{x} - K_d x) - \dot{J}(q)\dot{q})$$

The final expression for motor torques is obtained substituting (13) in (1):

$$\tau = D(q)J^{-1}(q)(M_d^{-1}(F_{ext} + FF - B_d\dot{X} - K_d X) -$$
$$- \dot{J}(q)\dot{q}) + H(q,\dot{q}) + G(q) + J^T(q)F \quad (14)$$

In this way, not only the system is made linear, but also it is decoupled. The behaviour of the system in direction of any Cartesian ***axis*** is independent of the other directions.
The torque from equation (14) cannot be computed when the robot goes through a singularity, i.e. a configuration where the Jacobian matrix is non-invertible. The singularities are a major problem in robotics and its solution is beyond the scope of this article. Nevertheless, following will be mentioned two solutions proposed by other researchers. In the first one[24] the controller is split in two parts. The first one controls the distance from the singularity. The second one controls the motion in the direction orthogonal to the singular direction. While this method is adequate in free motion, it may have problems during the contact. In the second solution[19], it is preferred simply to avoid the singularities. The introduction of a second controller which forces the robot to move away from singularities is proposed. It is activated only in the proximity of singularities. The final control action is obtained as a sum of the outputs of the two controllers.
Another case when the Jacobian matrix is non-invertible is when it is not square. That happens if the number of degrees of freedom of the robot is different than six.
If the robot has five or less degrees of freedom, it will be unable to control its motion/ force along all the directions of space. This may be dangerous in contact tasks since the trajectory in the non- controlled directions is unpredictable and the robot may penetrate deeply in the environment causing high peaks of force. It may be unadvisable to use a robot with less than six degrees of freedom in contact tasks.
If the robot has more than six degrees of freedom it may achieve the same end effector trajectory with different combinations of joint motions. It is called a redundant robot. Although the inverse jacobian does not exist, several solutions for the pseudo inverse may be found in the literature. One of them is the right pseudo-inverse[15]. A more general case is a pseudo-inverse that minimizes the quadratic cost function of joint velocities[16].
It should be noticed that the redundancy may help in many cases to solve the problem of the singularities by extracting all the linearly independent equations[16].

2.2 The behaviour of the system in constrained motion

Since it has been demonstrated in section 2.1. that the system may be decoupled, in the rest of the article the case of a single degree of freedom will be treated without loss of generality.
Lower case letters will be used instead of capitals for one dimensional variables like force, position, velocity, etc.

Assuming that the deformation of the environment is elastic, the reaction force of the environment will be:

$$f = -K_e(x - x_e) - B_e\dot{x} \qquad (15)$$

Where x_e is the coordinate of the environment's surface.
The dynamics of the position in constrained motion may be obtained from equations (3) and (10):

$$ff + K_e x_e = M_d\ddot{x} + (B_d + B_e)\dot{x} + (K_d + K_e)x \qquad (16)$$

The roots of the characteristic polynomial are:

$$s_{1,2} = \frac{-(B_d + B_e) \pm \sqrt{(B_d + B_e)^2 - 4(K_d + K_e)M_d}}{2M_d} \qquad (17)$$

and the discriminant:

$$(B_d + B_e)^2 - 4(K_d + K_e)M_d \qquad (18)$$

In order to make the system as damped as possible, it is convenient to assign a high value to B_d and low values to M_d and K_d.

The final value of the position will be:

$$x_\infty = \frac{ff + (K_d + K_e)x_e}{K_d + K_e} \quad if \quad K_d \neq 0 \qquad (19)$$

$$x_\infty = \frac{ff + K_e x_e}{K_e} \quad if \quad K_d = 0$$

The final value of the force main be obtained as:

$$f_\infty = K_e(x_\infty - x_e) \qquad (20)$$

Thus:

$$f_\infty = K_e\frac{ff + K_d x_e}{K_d + K_e} \quad if \quad K_d \neq 0 \qquad (21)$$

$$f_\infty = ff \quad if \quad K_d = 0$$

This means that choosing $K_d = 0$ not only damps the system, but also allows us to reach the reference value of force regardlessly of the characreristics of the environment.

2.3 The behaviour of the system in free motion

In this case $F_{ext}=0$ and the dynamics of the system is defined by the following equation:

$$ff = M_d\ddot{x} + B_d\dot{x} + K_d x \qquad (22)$$

The final values ***of*** the position and velocity will be:

$$x_\infty = \frac{ff}{K_d} \quad K_d \neq 0 \qquad (23)$$

$$\dot{x}_\infty = \frac{ff}{B_d} \quad K_d = 0 \wedge B_d \neq 0$$

Therefore, a stiffness different of zero will make the robot reach the distance given by equation (23), where it will stop. This correponds to position control. Assigning a stiffness equal to zero will make the system go to a constant speed. This corresponds to velocity control. The latter is more practical for impact control since it does not require previous knowledge about the position of the environment.
On the other hand, the damping should be chosen the way to assure the desired final velocity v_{ref}:

$$B_d = \frac{ff}{v_{ref}} \qquad (24)$$

2.4 Conclusions about the selection of the impedance parameters

This section will contain a recapitulaton of the previous conclusions for the selection of the parameters in order to achieve the desired performance both in free and constrained motion.

The stiffness K_d should be set to zero for two reasons. At first, in free motion, velocity control is obtained instead of position control, what is better suited for impact achieving a softer impact in the case when the exact position of the environment is unknown. Second, the final value of force does not depend on the characteristics of the environment and the force reference may be always reached.
The value of *ff* should be selected equal to the reference force in order to achieve tracking error zero according to (21).
The damping B_d is used to assure the system will have a velocity equivalent to the reference value during the free motion according to (24).

Regarding the mass M_d, it is the only parameter that practically has no importance during free motion.The value assigned to the mass should be low in order to damp the system during the impact. If the stiffness of the environment is known, the value of the mass that makes the system underdamped may be obtained from (18):

$$M_d < \frac{(B_d + B_e)^2}{4(K_d + K_e)} \qquad (25)$$

Briefly the values to be assigned are the following:

- $ff=F_{ref}$, where F_{ref} is the force reference.
- K_d =0.
- B_d: according to equation (24) in order to attain the reference speed.
- M_d: if the charachteristics of the system are known, in order to make the system overdamped according to (25). Otherwise as small as possible.

Usually in the applications of force control, the active damping is adjusted for smoothing the impact. In this case this is achieved by means of the mass. The damping is used for velocity control in free motion.

It should be emphasized that the selection of the parameters in the way it is described above assures that both velocity reference and force reference will be reached during free and constrained motion respectively, even if the stiffness of the environment is unknown. Nevertheless, it is not possible to guarantee that the system will be overdamped, neither its behaviour during the transition phase according to (17) and (18).

3. SWITCHING THE VALUES OF THE PARAMETERS

The previous results show how to select the impedance parameters if the characteristics of the environment are known. Nevertheless, in some cases this is not true, and the behaviour of the system in the transition cannot be controlled. Given the potential danger of the impact phase it is convenient to introduce an additional measure in order to make the transition as soft as possible.

The proposed method is based on the transformation of the energy. For example, when the robot penetrates into the environment, the kinetic energy it had in free motion is transformed in elastic potential energy of the environment. It is convenient to assign a low value of mass in order to reduce the kinetic energy and a high stiffness to avoid a deep penetration into the environment. When the robot starts rebounding, the inverse energy transformation occurs. Then the mass should be high to limit the acquired velocity and the stiffness low to reduce the elastic force.

Before starting a deeper analysis of the proposed method, some assumptions made in the article will be mentioned in this paragraph. It will be considered for simplicity that all the magnitudes are normalised, nondimensional quantities. This assumption does not have any influence on the generality of the conclusions. It will be also considered that the position of the environment is on a positive coordinate, i.e. that positive velocity means the robot is leaning towards the environment and negative velocity means it is moving away. The contrary case is completely symmetric and the conclusions obtained for one case are also valid for the other one.

3.1. Physical principle of the energy dissipation by means of the switching of the parameters

During constrained motion, the system is typically underdamped. While it oscillates around the equilibrium point, the kinetic energy is transformed to potential and vice versa. Considering an ideal case, a system without dissipation the total energy in every moment should be the sum of the potential and the kinetic energy:

$$E = E_k + E_p = \frac{1}{2}kx^2 + \frac{1}{2}m\dot{x}^2 \qquad (26)$$

Where k is the stiffness and m the mass of the system.

In the instant when velocity is zero, the position reaches its extreme point and the system has only potential energy (it will be assumed for simplicity that the origin of the coordinate system is in the equilibrium point):

$$E = E_p = \frac{1}{2}kx_{\max}^2 \qquad (27)$$

The extreme values of the position:

$$x_{extreme} = \pm\sqrt{\frac{2E}{k}} \qquad (28)$$

In the instant when the position is zero, the speed has an extreme point and there is only kinetic energy:

$$E = E_k = \frac{1}{2}m\dot{x}_{\max}^2 \qquad (29)$$

The extreme value of the velocity:

$$\dot{x}_{extreme} = \pm\sqrt{\frac{2E}{m}} \qquad (30)$$

In the phase plane the system is represented by an ellipse. It is obvious from (28) that decreasing k will make ellipse higher and from (30) that decreasing the mass will make it wider, as it may be appreciated in the following figures.

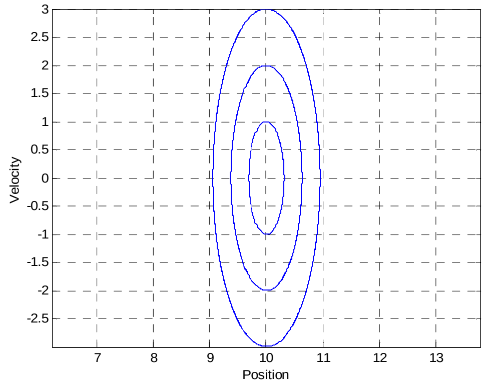


Figure 2. Phase diagrams of the system when the stiffness is higher than the mass for three values of the energy.

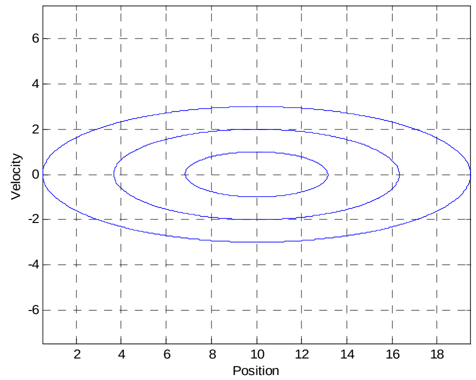


Figure 3. Phase diagrams of the system when the mass is higher than the stiffness for three values of the energy.

Assigning a high mass and low stiffness in the second and fourth quadrant, and doing the opposite in the first and the third one, the system will be closer to the equilibrium point every time it intersects any of the axes. This is represented in the following figure.

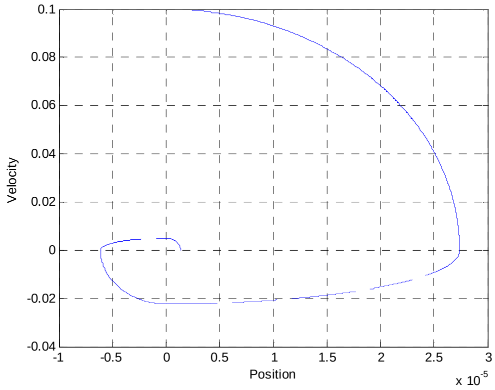


Figure 4. Phase diagram of the system when the mass is higher than the stiffness in the even quadrants (dashed line) and the opposite in the odd ones (full line). The initial state is (0,0.1).

Switching the parameters in the instants of changes of quadrant a conservative system is made dissipative. It is a form of energy dissipation, and thus it may be used for impact control.
It should be emphasized that the previous reasoning has been made for an idealized system rather than a real one. The main difference is that the equilibrium point changes when the stiffness is switched, according to (19). Also, the damping has not been taken into account. The analysis made in this section is more descriptive than precise. The exact one is left for the next section.

3.2. Switching criteria and sliding regimes

In order to deduce the optimal switching criteria the following energy Lyapunov-like function is used:

$$V = \frac{1}{2}\dot{x}^2 + \frac{1}{2}(x^2 - x_\infty)^2 \tag{31}$$

It represents the Euclidian distance from the equilibrium point x_∞ in the phase plane. It is evident that faster convergence means faster energy dissipation. On the other hand, the term $\frac{1}{2}(x - x_\infty)^2$ is equivalent to the elastic potential energy of the environment, scaled by a factor that may depend on the units. In the same way, the term $\frac{1}{2}\dot{x}^2$ is proportional to the kinetic energy. Therefore, it may be stated that *V* equivalent to the total energy of the system. Any quadratic function of velocity and distance of the origin would have the same effect.

It should be emphasized that in the typical applications of Lyapunov functions, the origin of the coordinate system is located at the equilibrium point, and hence it is not taken into account. Nevertheless, when a parameter is switched, the equilibrium point may also change, which may influence the stability and generally the behaviour of the system. For this reason, x_∞ is included in the considerations.

The derivative of *V*:

$$\dot{V} = \ddot{x}\dot{x} + \dot{x}(x - x_\infty) \tag{32}$$

Assuming the dynamics of the system:

$$M_d\ddot{x} + (B_d + B_e)\dot{x} + (K_d + K_e)x = ff + K_e x_e \tag{33}$$

And

$$\ddot{x} = \frac{ff + K_e x_e}{M_d} - \frac{B_d + B_e}{M_d}\dot{x} - \frac{K_d + K_e}{M_d}x \tag{34}$$

The substitution of the expression (34) in the expression (32) gives:

$$\dot{V} = \frac{ff + K_e x_e}{M_d}\dot{x} - \frac{B_d + B_e}{M_d}\dot{x}^2 - \frac{K_d + K_e}{M_d}x\dot{x} + \dot{x}(x - x_\infty) \tag{35}$$

Introducing the expression for x_∞ from (19):

$$\dot{V} = \frac{ff + K_e x_e}{M_d}\dot{x} - \frac{B_d + B_e}{M_d}\dot{x}^2 - \frac{K_d + K_e}{M_d}x\dot{x} + \dot{x}\left(x - \frac{ff + K_e x_e}{K_d + K_e}\right) \tag{36}$$

This formula will used to deduce the switching criteria for the parameters. They will be analysed one by one in the following subsections.

3.2.1. The mass

This subsection is dedicated to the deduction of the switching criteria of the mass in function of the state of the system in order to dissipate the energy and thus to soften the impact. Also, the possibility of sliding regimes provoked by the switching will be analyzed. Finally, a study of the effect of the noise of the acceleration sensor will be made.
It will be assumed that the mass is switched between two values: the minimal and the maximal one. It will also be assumed that only non- negative value may be assigned to the mass although the contrary would be possible in impedance control.
For the achievement of a soft impact it is necessary to dissipate the energy of the system very fast. The derivative of the energy should be always as small as possible.
In order to appreciate the effect of the mass on the behaviour of the system, the partial derivative of $\dot{V}$ with respect to the mass is deduced:

$$\frac{\delta\dot{V}}{\delta M_d} = -\frac{ff + K_e x_e}{M_d^{\,2}}\dot{x} + \frac{B_d + B_e}{M_d^{\,2}}\dot{x}^2 + \frac{K_d + K_e}{M_d^{\,2}}x\dot{x} \tag{37}$$

Substituting (34) in (37):

$$\frac{\delta\dot{V}}{\delta M_d} = -\frac{\ddot{x}\dot{x}}{M_d} \tag{38}$$

When this expression is positive the energy is dissipated slower as the mass increases. When it is negative the energy is dissipated faster as the mass increases.

It seems logical to assign a high value to the mass when (38) is negative and a low one in the contrary case. For this reason the following switching law is proposed:

$$M_d = \begin{cases} m_{\min} & if \quad \dot{x}\ddot{x} < 0 \\ m_{\max} & if \quad \dot{x}\ddot{x} > 0 \end{cases} \qquad (39)$$

If $\dot{x}\ddot{x}$ is positive, the term associated to the mass is absorbing energy from the system. In the contrary case it is delivering energy. It should be noticed that $\dot{x}\ddot{x}$ corresponds to the derivative of the square of the velocity and thus of the kinetic energy. Therefore, $\dot{x}\ddot{x} < 0$ means that the kinetic energy is decreasing, i.e. being transformed into elastic potential energy. Assigning a small value of the mass will mean reducing the amount the kinetic energy to be dissipated. In the contrary case, when $\dot{x}\ddot{x} > 0$, the kinetic energy is increasing, i.e. potential energy is transformed in kinetics.

The switching criteria (39) have been verified by means of simulations. These were made at first assigning smaller and smaller values to m_{min}, while keeping m_{max} constant. Next, the contrary was made: m_{min} was kept constant, while the values of m_{max} were increased in several successive experiments. Testing the two cases separately, the effectiveness of the switching criteria is verified. Otherwise, the positive results in one case could compensate the negative ones of the other, giving a false appearance of the validity of the method.

The adopted values for the simulations are the following: M_d =10 kg (if not switched), B_d=10 Ns/m, K_d=0, *ff*=1000 *N* (if not switched) K_e=100000 N/m, *Be*=10 Ns/m, x_e=0 (for simplicity). It is assumed that that the robot impacts with the environment in the instant *t=0*. The adopted value for the stiffness of the environment is very high and the system is highly underdamped, what corresponds to the reality in the force control applications.

The results are represented on the following graphics.

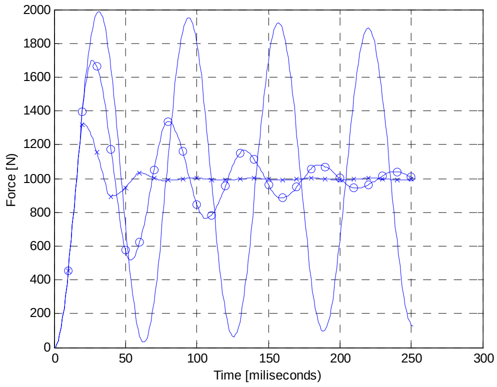


Figure 5. Force in function of the time when switching m_{min} and keeping the value m_{max}=10 in all the cases. a) m_{min}=10 (full line) b) m_{min}=5 (circles) c) m_{min}=1 (crosses).

It can be appreciated that when the value of m_{min} is decreased, the peaks of force are reduced. Also, the convergence of the system is faster.

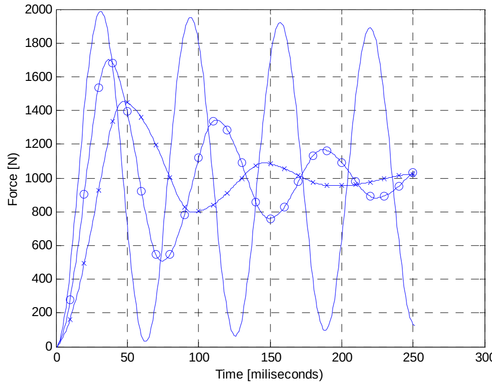


Figure 6. Force in function of the time when switching m_{max} while m_{min}=10 in all the cases. a) m_{max} =10 (fulle line) b) m_{max} =20 (circles) c) m_{max} =50 (crosses).

It can be appreciated that increasing m_{max} also reduces the peaks of force. Nevertheless, it slows the convergence of the system.

It may be concluded that the simulations results confirm the validity of the switching criteria (39) regarding the protection of the system, as both decreasing m_{min} and increasing m_{max} reduce the peaks.

3.2.1.1. Sliding regimes when switching the mass

The analysis of sliding regimes and sliding modes is a very extensive field and thus beyond the scope of this work. Nevertheless a brief explanation will be given in order to improve the clarity of the article. Further information can be found in different sources [25].

A parameter switches when the state variables satisfy a given condition that can be expressed as

$$S=0 \qquad (40)$$

When this condition is true it said that the system is on the switching surface.

If the system is not on the surface, there are four possible cases:

$$S<0 \quad and \quad \dot{S}>0 \qquad (41)$$

$$S>0 \quad and \quad \dot{S}<0 \qquad (42)$$

$$S>0 \quad and \quad \dot{S}>0 \qquad (43)$$

$$S<0 \quad and \quad \dot{S}<0 \qquad (44)$$

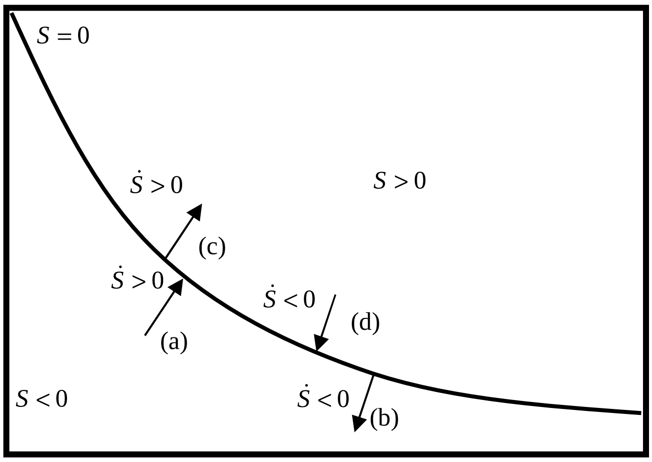


Fig. 7. The four possible cases. The thick line represents the switching surface *S=0*. Above the surface *S>0*, and below it *S<0*. The following cases are possible: a) *S<0* and $\dot{S}>0$, the system tends towards the surface. b) *S<0* and $\dot{S}<0$, the system is moving away from the surface, c) *S>0* and $\dot{S}>0$, the system is moving away from the surface, d) *S>0* and $\dot{S}<0$, the system heads for the surface.

If any of the conditions (43) or (44) is true, the system is moving away from the switching surface. The mathematical demonstration of this statement will be omitted, but its meaning is rather logical. In the first case, both the value of *S* and its derivative are positive. In the second case *S* and its derivative are negative. Thus, in both cases the distance from the surface is increasing.

In the contrary case, if any of the conditions (41) or (42) is true, the system is tending towards the switching surface.

In summary, if the values of *S* and its derivative have the same sign, the distance of the switching surface is increasing. If their signs are opposite, the distance is increasing.

If a system satisfies both conditions (41) and (42), it will remain on the surface once it has reached it. This is called a sliding regime. It is a harmful phenomenon, because the system is stuck on the surface instead of tracking the reference values.

In impedance control, the system is second order, typically underdamped. Its behaviour is oscillatory. It crosses the switching surface in every period. When this happens, it is obvious that the sign of *S* changes. In order to cross the surface, the system must head for it (conditions (41) or (42)), passes through the surface (*S=0* for an instant), and move away from it (conditions (43) or (44). Therefore, in an oscillatory system, if no switching is performed, the sign of *S* changes when crossing the surface, while the sign of remains $\dot{S}$ the same. Nevertheless, the switching of a parameter may provoke a change of the sign of $\dot{S}$. In this case, the system is pushed back to the surface whichever is the sign of *S*. The system becomes unable to leave the surface and remains on it.

In summary, a sliding regime may occur if there is a possibility the switching of a parameter to provoke the change of the sign of $\dot{S}$ when crossing the surface.

In order to analyze the conditions of the sliding regime for the concrete case of the switching of the mass, it is important to emphasize that, according to (39) the mass switches four times in every period as both the speed and the acceleration change their sign twice. Therefore, there are two switching surfaces, when velocity and acceleration go through zero:

$$
\begin{aligned}
S_1 &= \dot{x} = 0 \\
S_2 &= \ddot{x} = 0
\end{aligned}
\tag{45}
$$

Given that the acceleration is the derivative of the velocity, when the former passes through zero, the latter has an extreme point. When the acceleration passes from positive to negative, it is evident that the velocity has a maximum and therefore it is positive. The product $\dot{x}\ddot{x}$ goes from positive to negative. In the second case of change of sign acceleration, when it becomes positive, it corresponds to a minimum of the velocity, which is therefore negative. The product $\dot{x}\ddot{x}$ goes from positive to negative like in the previous case. Thus, taking into account the condition (39) in both cases of change of sign of the acceleration the mass switches to its minimal value from the maximal value.

These conclusions are represented in the following tables.

Table 1. Signs of relevant magnitudes and value of the mass when acceleration changes its sign from positive to negative.

Magnitude	Before crossing the surface	After crossing the surface
Acceleration	>0	<0
Velocity	>0	>0
$\dot{x}\ddot{x}$	>0	<0
mass	m_{max}	m_{min}

Table 2. Signs of relevant magnitudes and value of the mass when acceleration changes its sign from negative to positive.

Magnitude	Before crossing the surface	After crossing the surface
Acceleration	<0	>0
Velocity	<0	<0
$\dot{x}\ddot{x}$	>0	<0
mass	m_{max}	m_{min}

A similar analysis can be made for the changes of the sign of the velocity. In order to change its sign from positive to negative the acceleration must be negative, and therefore the product $\dot{x}\ddot{x}$ becomes positive. To change from negative to positive, the acceleration must be positive, and thus $\dot{x}\ddot{x}$ goes positive again. This is summarized in the following tables:

Table 3. Signs of relevant magnitudes and value of the mass when velocity changes its sign from positive to negative.

Magnitude	Before crossing the surface	After crossing the surface
Velocity	>0	<0
Acceleration	<0	<0
$\dot{x}\ddot{x}$	<0	>0
mass	m_{min}	m_{max}

Table 4 Signs of relevant magnitudes and value of the mass when velocity changes its sign from negative to positive.

Magnitude	Before crossing the surface	After crossing the surface
Velocity	<0	>0
Acceleration	>0	>0
$\dot{x}\ddot{x}$	<0	>0
mass	m_{min}	m_{max}

Summarizing, the mass switches to:

- m_{min} when the acceleration changes its sign.
- m_{max} when the velocity changes its sign.

Following will be realised the analysis of the possibility of occurrence of a sliding regime in both surfaces.
The derivative of S_1 is:

$$\dot{S}_1 = \ddot{x} = \frac{ff + K_e x_e}{M_d} - \frac{B_d + B_e}{M_d}\dot{x} - \frac{K_d + K_e}{M_d}x \qquad (46)$$

Given that both position and velocity are continuous and therefore don't change when the mass is switched, and that ff, K_d, B_d, K_e, B_e and x_e are constant, the change of the value of the mass (assuming it is always positive) doesn't influence directly the sign of $\dot{S}_1$. As consequence, the switching of the mass cannot change the sign of the surface nor provoke the appearance of a sliding regime.
The derivative of S_2 is:

$$\dot{S}_2 = \dddot{x} = -\frac{B_d + B_e}{M_d}\ddot{x} - \frac{K_d + K_e}{M_d}\dot{x} \qquad (47)$$

Assuming that in the proximity of the switching surface the value of the acceleration is nearly zero, the following can be assumed:

$$\ddot{x} \approx 0$$
$$\frac{B_d + B_e}{M_d}\ddot{x} << \frac{K_d + K_e}{M_d}\dot{x} \qquad (48)$$
$$\dot{S}_2 \approx -\frac{K_d + K_e}{M_d}\dot{x}$$

Therefore, the switching of the mass would not change the sign of the derivative of the surface S_2, and thus there cannot be sliding regime in this surface.

3.2.2.2. The effect of the noise of the acceleration measurement

The switching criteria of the mass (39) require the knowledge of the value of the acceleration. Nevertheless, the acceleration measurement is subject to noise. The analysis of the effect of this noise will be presented in this section.
In order to have realistic values, the data for the analysis will be taken from the datasheet of the Analog Devices ADXL330. It is a low cost, 3-axis, on-chip accelerometer. It is extended in the research as well as in the commercial applications.

The bandwidth in each axis is selected by the user by means of capacitors connected to the measured outputs. It is basically a low pass filter. With a lower bandwidth the noise filtering is improved but the resolution of the accelerometer is deteriorated. A trade-off should be found for each application. The user should limit the bandwidth to the lowest frequency needed by the application to maximize the resolution and dynamic range of the accelerometer.

According to the datasheet, the root mean square noise should be calculated by the formula:

$$N = N_0\sqrt{1.6B} \qquad (49)$$

Where N is the r.m.s. noise, N_0 the noise magnitude and B the bandwidth.

The noise magnitude N_0 for the ADXL330 is 280 in x and y, and 350 $\mu g/\sqrt{Hz}$ in the z axe.

On the other hand, the typical sampling periods used for robot control are between one and ten kilohertz. A higher bandwidth for the sensor does not make sense. For a bandwidth of 10 KHz and a noise magnitude of 350 $\mu g/\sqrt{Hz}$ (z axe, the worst case), the obtained r.m.s. noise will be:

$$N = 350X10^{-6}\sqrt{1.6X10000} = 0.04272\text{g} \qquad (50)$$

Simulating the system with this value of noise practically no difference has been observed with respect to the ideal case (no noise at all). For this reason, the case of the filter adjusted to 10 KHz has not been represented on the figure.

Simulations have been made with the filter adjusted to cut-off frequencies of 100 KHz and 1MHz. Both of these values are unrealistically high. Although such high sampling rates could be implemented, many problems would appear, like the delay of the electronics (A/D conversion, etc…) or the frequency of pulse width modulation of the power stage of the motor (seldom higher than 100 KHz). Nevertheless some simulations have been made in these unrealistically unfavourable conditions in order to estimate the effect of the noise.

According to formula (49) a value of r.m.s. noise of 0.14g has been obtained for 100 KHz, and 0.44272g for 1MHz.
The following figure represents the results of the simulations of the system without noise, with a cut-off frequency of 1MHZ as well as for the case without switching.

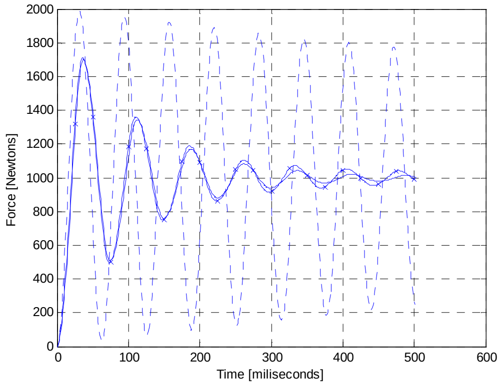


Figure 8. The effect of noise on the switching. The full line represents the force in the ideal case (no noise at all) and the line with crosses the case when the filter is adjusted to 1 MHz. The maximal mass is 20kg and the minimal one 10 kg. The dotted line represents the case when no switching at all is made (the mass is 10 kg all the time).

It may be appreciated that the noise deteriorates the effect of switching. The difference between the case with noise and the ideal one increases with each switching, its effect is accumulative. This may be explained by the fact the switching criteria (39) are chosen for the optimal dissipation of energy. Any other switching law worsens the energy dissipation. The noise affects the switchings when acceleration goes through zero. When they occur, the switching will not be performed exactly according to (39) due to the noise. Thus less than optimal energy will be dissipated in every switching when the acceleration changes its sign. So, there will be more and more energy accumulated respecting the case without the case noise.
Nevertheless, the results are much better than in the case without switching, even with the unrealistically unfavorable level of noise.
On the other, the noise makes almost no difference for the initial peaks that are the most dangerous ones.
In summary, the noise affects very slightly the energy dissipation, and almost not at all in the first, most important, peaks.

3.2.2. The stiffness

The partial derivative of (36) respecting the stiffness is:

$$\frac{\delta \dot{V}}{\delta K_d} = -\frac{1}{M_d}\dot{x}x + \dot{x}\frac{ff + K_e x_e}{(K_d + K_e)^2} \qquad (51)$$

According to (51) the following switching law is proposed in order to maximize the energy dissipation:

$$K_d = \begin{cases} k_{\max} & if \quad -\frac{1}{M_d}\dot{x}x + \dot{x}\frac{ff + K_e x_e}{(K_d + K_e)^2} < 0 \\ k_{\min} & if \quad -\frac{1}{M_d}\dot{x}x + \dot{x}\frac{ff + K_e x_e}{(K_d + K_e)^2} > 0 \end{cases} \qquad (52)$$

It may be appreciated that the criteria depend on the environment stiffness, which was assumed to be unknown initially. As consequence, the switching conditions cannot be detected. Thus, the switching of the stiffness according to (52) cannot be implemented. It could be adequate to assign the value *K_d=0* to the stiffness according to the conclusions of the second section, in order to reach the force reference. Nevertheless, if the task requires stiffness different then zero (typical peg-in-a-hole problem), its value may be assigned in the moment the contact is achieved.

3.2.3. The damping

The partial derivative of (36) respecting the damping is:

$$\frac{\delta \dot{V}}{\delta B_d} = -\frac{1}{M_d}\dot{x}^2 \qquad (53)$$

Since it is always negative, this term is always dissipative. As consequence, the switching of the damping does not improve the performance of the impact control.

3.2.4. The feedforward

The partial derivative of (36) respecting the feedforward is:

$$\frac{\delta \dot{V}}{\delta ff} = \frac{\dot{x}}{M_d} - \frac{\dot{x}}{K_d + K_e} \qquad (54)$$

Since in the practical robotic applications:

$$M_d << K_d + K_e$$

It can be assumed:

$$\frac{\delta \dot{V}}{\delta ff} \approx \frac{\dot{x}}{M_d} \qquad (55)$$

Thus the switching criteria for the feedforward should be:

$$ff = \begin{cases} ff_{\max} & if \quad \dot{x} < 0 \\ ff_{\min} & if \quad \dot{x} > 0 \end{cases} \qquad (56)$$

These criteria are rather logical. When the velocity is positive, the robot is penetrating into the environment. A low feedforward will have as consequence a lower penetrating depth. When the velocity is negative, the robot is retiring from the environment and the feedforward should be high to push him back inside and prevent the rebounding. In both cases the value of the feedforward is selected to be opposed to the motion of the robot and is acting as a sort of brake. Hence, it is a form of energy dissipation.

(37)

3.2.4.1. Sliding regimes when switching the feedforward

The only switching surface is:

$$S = \dot{x} = 0 \quad (57)$$

And its derivative:

$$\dot{S} = \ddot{x} = 0 \quad (58)$$

The system crosses the switching surface twice in every period: when velocity goes from positive to negative and vice versa. Following both cases will be analysed one by one. The switching instant will be called t_{sw}, the instant immediately before will be called t_{sw-} and the one immediately after t_{sw+}.

When velocity passes from positive to negative the following statements can be made:

1. $\dot{x}(t_{sw-}) > 0 \wedge \dot{x}(t_{sw+}) < 0$ (59)
2. $\ddot{x} < 0$, otherwise the first statement could not be satisfied.
3. According to (56):
   $ff(t_{sw-}) = ff_{\min} \wedge ff(t_{sw+}) = ff_{\max}$ (60)
4. The sliding regime occurs if $\dot{S}$ changes its sign when the feedforward is switched. Its value just before switching:
   $$\ddot{x}(t_{sw-}) = \frac{ff_{\min} + K_e x_e}{M_d} - \frac{B_d + B_e}{M_d}\dot{x} - \frac{K_d + K_e}{M_d} x < 0$$
   (61)
5. $\dot{S}$ after switching:
   $$\ddot{x}(t_{sw+}) = \frac{ff_{\max} + K_e x_e}{M_d} - \frac{B_d + B_e}{M_d}\dot{x} - \frac{K_d + K_e}{M_d} x > 0$$
   (62)
6. Assuming the velocity is zero near the surface, and that K_d is also zero, this expression becomes:
   $$\ddot{x}(t_{sw+}) = \frac{ff_{\max} + K_e x_e}{M_d} - \frac{K_e}{M_d} x > 0 \Leftrightarrow \frac{ff_{\max} - f}{M_d} > 0$$
   (63)

Therefore, a sliding regime happens if the assigned feedforward is lower than the actual force. It can be avoided simply by not performing the switching if the force is too low. This can de predicted because ff_{max} is known and the force can be measured. Another alternative is assigning ff_{max} a value lower than the actual force.
A similar reasoning may be made for the case when the velocity goes from negative to positive:

1. $\dot{x}(t_{sw-}) < 0 \wedge \dot{x}(t_{sw+}) > 0$ (64)
2. $\ddot{x} > 0$, otherwise the first statement could not be satisfied.
3. According to (56):
   $ff(t_{sw-}) = ff_{\max} \wedge ff(t_{sw+}) = ff_{\min}$ (65)
4. The sliding regime occurs if $\dot{S}$ changes its sign when the feedforward is switched. $\dot{S}$ before switching:
   $$\ddot{x}(t_{sw-}) = \frac{ff_{\max} + K_e x_e}{M_d} - \frac{B_d + B_e}{M_d}\dot{x} - \frac{K_d + K_e}{M_d} x > 0$$
   (66)
5. In order a sliding regime to occur $\dot{S}$ must become negative after the switching:
   $$\ddot{x}(t_{sw+}) = \frac{ff_{\min} + K_e x_e}{M_d} - \frac{B_d + B_e}{M_d}\dot{x} - \frac{K_d + K_e}{M_d} x < 0$$
   (67)
6. Assuming the velocity is zero on the surface, and also K_d is zero according this expression becomes:
   $$\ddot{x}(t_{sw+}) = \frac{ff_{\min} + K_e x_e}{M_d} - \frac{K_e}{M_d} x < 0 \Leftrightarrow \frac{ff_{\min} - f}{M_d} < 0$$
   (68)

Therefore, the sliding regime can be avoided by assigning to ff_{min} a value higher than the actual force.

Following will be represented the simulation results. The simulations will be realised first increasing the values of ff_{max}. Then they will be performed decreasing ff_{min}.

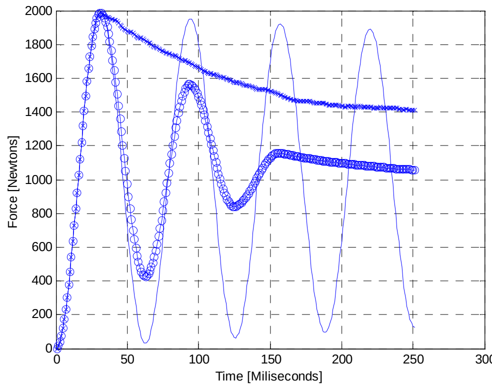


Figure 9. Diagram of the force in function of the time when ff_{max} is switched. a) $ff_{min} = ff_{max}$=1000 (Full line) b) ff_{min} =1000 and ff_{max} =1200 (Circles) c) ff_{min} = 1000 and ff_{max} =2000 (Crosses).

It can be observed that the value of ff_{max} does not have any influence on the first peak, because it is not activated until after the peak is reached. Nevertheless, the increase of ff_{max} reduces the subsequent peaks.
The switching happens in all the cases near the extreme values of the force what corresponds approximately to the value zero of the velocity. When ff_{max} is augmented to 1200, the system is damped and the extreme values are reduced until the third peak, after approximately 150 milliseconds, when the system enters into a sliding regime. It happens because the force in the instant of switching is lower then ff_{max} (1200N). For the case ff_{max} is 2000N, the sliding regime happens at the first switching, because the peak in this instant is lower than ff_{max} (2000N). The sliding regime appears according the theoretical results reflected in equation (63).
The sliding regimes may be avoided by omitting the switching or by modifying ff_{max}. The next figures represent the three possible cases. In the first one, the switching of the feedforward is made when any of the switching criteria (56) is true. As consequence, the system enters a sliding regime after the third peak (Figure 10). In the next case (Figure 11), the

possibility of sliding regime is predicted according to condition (63), and the switching is not performed. The sliding regime is avoided but the damping of the system is poor. At last, the sliding regimes are predicted according the equation (63) and the value of ff_{max} is modified in order to avoid them (Figure 12). In this example the law for compute ff_{max} has been:

$$ff_{\max} = \frac{f - ff_{\min}}{2} \tag{69}$$

In this way it is always smaller than the actual force, and condition (63) is not verified. The sliding regimes are avoided and the damping is improved.

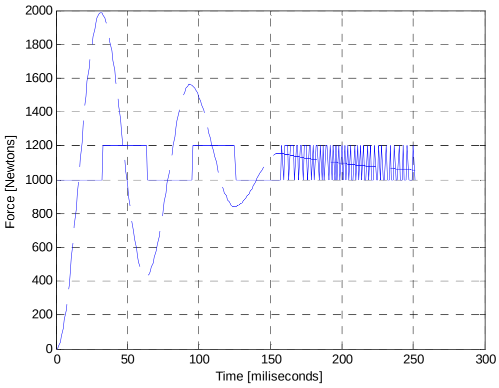


Figure 10. Diagram of the force (dashed line) and feedforward (full line) for ff_{min} =1000 and ff_{max} =1200. A sliding regime appears at the third peak of force.

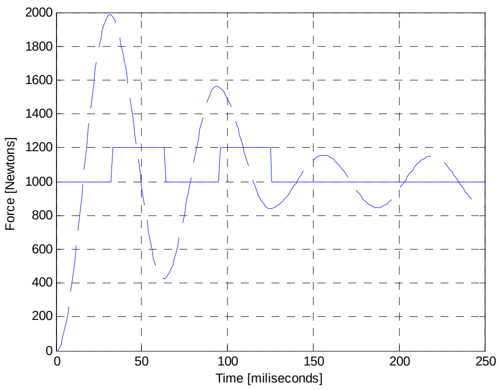


Figure 11. Diagram of the force (dashed line) and feedforward (full line) for ff_{min} =1000 and ff_{max} =1200 when switching of the feedforward is not performed at the third peak avoiding the sliding regime.

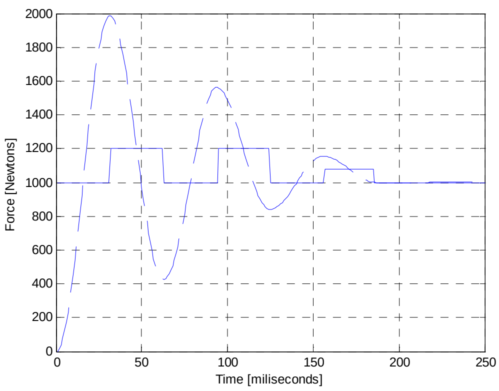


Figure 12. Diagram of the force (dashed line) and feedforward (full line) for ff_{min} =1000 and ff_{max} =1200 when ff_{max} is readjusted according to (69).

The figure 13 represents the force in function of the time when for different values of ff_{min}. It may be appreciated that as the peaks of force are smaller as ff_{min} decreases, but the also the system enters sooner a sliding regime.

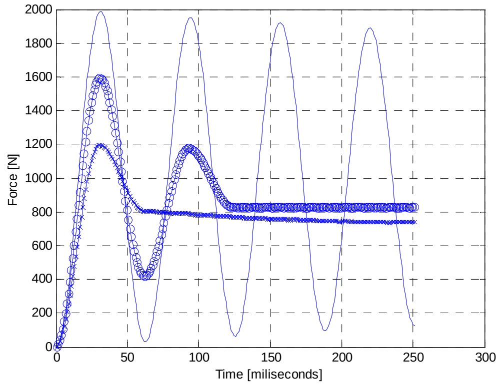


Figure 13. Diagram of the force in function of the time when ff_{min} is switched. a) ff_{min} = ff_{max} =1000 (full line) b) ff_{min} =800 (circles). c) ff_{min} = 600 (crosses).

It may be appreciated that decreasing ff_{min} reduces the peaks of force until a sliding regime happens. In this case, a solution could be assigning a higher value to ff_{min} according to (62).

In summary, in both cases the switching of ff improves the impact control until the system enters a sliding regime, what can be easily avoided.

4. THE CONTACT LOSS

All the previous analysis treated the case when the robot and the environment are in contact. It has been demonstrated that the switching of the parameters accorded to laws (39) and (56) increases the dissipation of the energy and thus softens the impact. Given that it reduces the amplitudes of the oscillations, it also reduces the probability of contact loss. Nevertheless, the contact loss may happen. This section studies the effect of the switching of the mass and the feedforward in the case when contact is lost.

In the case of contact loss, the system is in free motion and its behaviour is given by equation (22). According to previous conclusions (subsections 2.4 and 3.2.2), it will be assumed that the active stiffness is zero (K_d=0).The dynamics of the system is given by the following equation:

$$ff = M_d \ddot{x} + B_d \dot{x} \tag{70}$$

In the next subsections will be studied the effects of the switching of the mass and the feedforward respectively for the dynamics of the system given in (70).

The main criterion for the evaluation will be the velocity of the robot on a new impact, after a contact loss. It will be assumed that the switching of a parameter improves the impact control if it reduces the velocity on the instant the contact is achieved again.

On the other hand, it is empirically known that a body that rebounds on the ground, without any human intervention, is a stable system and the number of rebounds is finite. So, it will be considered that, if the switching reduces the impact velocity in each subsequent contact, the energy dissipation is improved compared with the case when no action at all has been applied. For this reason, the fact that the impact velocity is reduced in each impact will be considered as a sufficient condition to demonstrate the stability of the system, as well as a countable number of impacts.

4.1. The effect of the switching of the mass

In the instant of the loss of contact, the robot is moving away from the environment. The feedforward term is pushing it back to the surface and it slows until it stops and reverses the speed.

Therefore, it may be stated that a rebound consists of two phases. In the first one it is going in the opposite direction of the environment and thus its velocity is negative. It goes slower and slower, thus the acceleration is positive.

In the second phase, the robot is moving towards the environment so its velocity is positive. Since it is moving faster and faster, the acceleration is also positive.

If the switching criteria are made according to (39), the value of the mass M_d is given in the following table.

Table 5 Signs of relevant magnitudes and value of the mass for the two phases of a rebound.

Magnitude	First phase	Second phase
Acceleration	>0	>0
Velocity	<0	>0
$\dot{x}\ddot{x}$	<0	>0
mass	m_{min}	m_{max}

Therefore the mass is m_{min} while the robot is rebounding and m_{max} while it is returning back to the surface.

The fact the mass is low while the robot is bouncing back means that, for a given initial velocity, it will stop faster since it has less inertia. The distance it will cover will be shorter.

When the robot heads for the surface again, its mass will be higher. Thus, the acceleration will be lover and it will reach less velocity while traveling the same distance.

Therefore, the velocity of the impact will be smaller than the one the robot had in the instant of contact loss.

The following simulations confirm the previous conclusion. It will be assumed that the robot leave the environment with the velocity of -0.1 m/s. It will also be assumed that the travelled distance in both directions is equal.

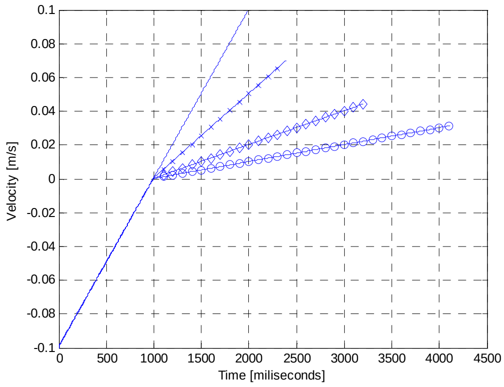


Figure 14. The change of the velocity when switching m_{max} (m_{min} =10 kg in all the cases). The full line represents the case when m_{max} =10 kg (no switching), the line marked with 'x' m_{max} =20 kg, the one with diamonds m_{max} =50 kg and the one with circles m_{max} =100 kg.

The two phases may be appreciated in the diagram: the first one lasts until the velocity reaches zero. The second phase ends when the contact is achieved again. Since the damping is relatively small compared to the mass and the feedforward, the acceleration is practically constant during each phase. For this reason the change of velocity seems linear and thus looks like a straight line in the figure.

It should be emphasized that during the first phase the mass has the value of m_{min} according to table 5 (In this simulation it corresponds to 1000 milliseconds). In the figure this line is the same for all four values and is left as a full line, without any additional elements.

Finally, it may be appreciated in the figure that increasing m_{max} the impact velocity is decreased. This seems a logical result, because the acceleration will be smaller for a greater inertia. Thus, the velocity reached for the same travelled distance will also be smaller.

In the next figure are represented the simulations when switching m_{min}.

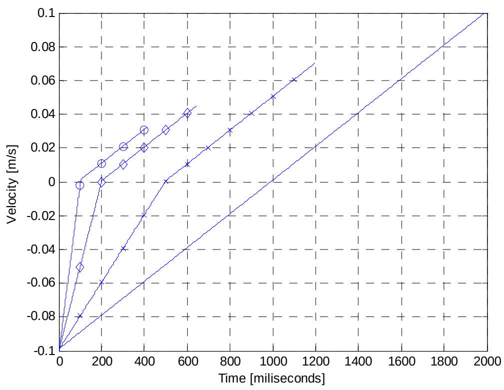


Figure 15. The change of the velocity when switching m_{min} (m_{max} =10 kg in all the cases). The full line represents the case when m_{min} =10 kg (no switching), the line marked with 'x' m_{min} =5 kg, the one with diamonds m_{min} =2 kg and the one with circles m_{min} =1 kg.

It may be concluded that decreasing m_{min} reduces impact velocity. It also makes shorter the time from contact loss to the new impact.

4.2. The effect of the switching of the feedforward

Switching the feedforward according to (56) means two things:

1. It will be high while the robot is bouncing back. Thus the robot will be stopped faster limiting the traveled distance.
2. The feedforward will be low when the robot is heading for the surface. Thus the robot will accelerate slower and reach a lower velocity in the instant of the impact.

For this reasons the switching the feedforward softens the impact: at first the traveled distance is lowered, and second, the acceleration when it returns back to the environment is decreased. Both facts are favorable for reducing the impact velocity and thus for softening the impact.
Following will be represented some simulation results.

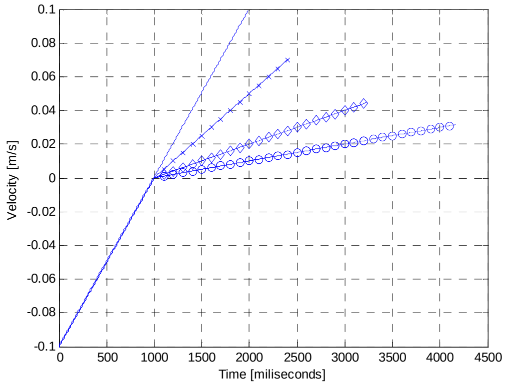


Figure 16. The change of the velocity when switching ffmin(ff_{max} =1000 N in all the cases). The full line represents the case when ff_{min} =1000 N (no switching), the line marked with 'x' ff_{min} =500 N, the one with diamonds ff_{min} =200 N and the one with circles ff_{min} =100 N.

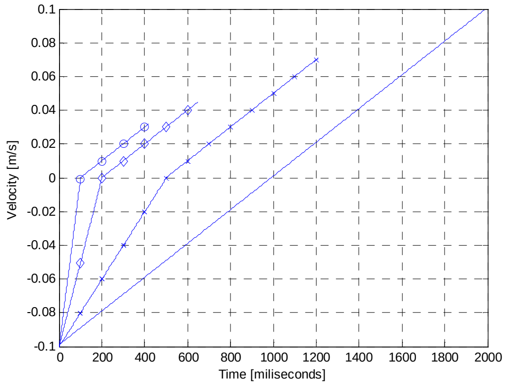


Figure 17. The change of the velocity when switching ff_{max} (ff_{min} =1000 N in all the cases). The full line represents the case when ff_{max} =1000 N (no switching), the line marked with 'x' ff_{max} =2000 N, the one with diamonds ff_{max} =5000 N and the one with circles ff_{max} =10000 N.

It may be appreciated in the figure that decreasing ff_{min}, reduces both the impact velocity and the duration the time between the contact loss and the next impact.

Concluding, the switching the mass and feedforward improves the impact control in the case of contact loss.
On the other hand, in the case of loss of contact, strategies[3] different than switching of parameters may be used.

5. CONCLUSIONS

This article is dedicated to the analysis of the application of impedance control in order to obtain velocity control in free motion, impact attenuation and force reference tracking. The emphasis is put on the impact because it is the most dangerous part of the task and possibly the most interesting from the point of view of control.

At first, a general analysis of the impedance control was made. The conclusion has been made that an impedance controller is defined by four parameters: the feedforward, the mass, the damping and the stiffness. Also, it has been demonstrated that all three phases of the task can be controlled with the same impedance controller.

The feedforward must be set to the value of the force reference in order to reach it. The value of the stiffness must be zero. Contrary to the usual trend in impedance control, the damping is used for velocity control in the free motion. If the stiffness of the environment is known, the value of the mass may be computed to make the system underdamped, avoiding force overshoots and the possibility of bouncing and thus achieving a perfect impact control.

If the stiffness of the environment is unknown, an improvement of the impact control may be made by means of switching between parameters of the mechanical impedance. This is the most important contribution of this article.

The four parameters have been analysed and it has been demonstrated that the effect of switching is different for each of them.

The switching criteria for the stiffness cannot be detected unless the characteristics of the environment are known. Therefore there is no sense to switch this parameter.

The switching of the damping does not provide any advantage, since the term associated to it always dissipates energy.

The switching of the feedforward improves the performance of the system initially, but after a few periods the system may go into a sliding regime. Nevertheless, the first periods are the most critical and thus the switching of this parameter is useful when it is most important. The sliding regimes may be predicted and avoided.

Finally, the mass is possibly the best-suited parameter for switching. The switching criteria do not depend on the characteristics of the environment and the system never enters a sliding regime.

The case of a loss of contact has also been analysed. It has been deduced that both switching the mass and the feedforward reduce the velocity of the next impact and thus improve the impact control.

In summary, the switching of the parameters, when used in the way described in this article, always improves the softening of the impact, regardless of the characteristics of the environment. Hence, it is a robust method.

The switching of the parameters also has the advantage that it needs just a few sampling periods to be effective.

ACKNOLWEDGEMENTS

The authors want to express their gratitude to the Plan Nacional de I+D, Comisión Interministerial de Ciencia y Tecnología (FEDER-CICYT) for the partial financing of this work under the projects DPI2009-13830-C02-01 and DPI2010-20814-C02-02.

REFERENCES

1. Brogliato B., Nonsmooth mechanics. (Springer Verlag, London, 1999).
2. Brach R., Mechanical Impact Dynamics- Rigid Body Collisions (John Wiley & Sons Publishers, 1991).
3. Brogliato B., Nicolescu S.I., Orhant P., "On the Control of Finite Dimensional Mechanical Systems with Unilateral Constraints," IEEE Transactions on Automatic Control. 42 (2), 200-215 (1997).
4. Volpe R., Khosla P, "A Theoretical and Experimental Investigation of Impact Control for Manipulators," International Journal of Robotics Research, 12 (4), 351-365 (1993).
5. Hyde J., Cutkosky M., "Controlling Contact Transitions," IEEE Control Systems 14 (1), 25-30 (1994).
6. Xu Y., Hollerbach J. M., Ma D., "A nonlinear PD controller for force and contact transient control," IEEE Control Systems Magazine, 15 (1): 15–21 (1995).
7. Ferretti G., Magnani G., Zavala Río A., "Impact modelling and Control for Industrial Manipulators," *IEEE Control Systems Magazine,"* **18**(4): 65-71 (1998).
8. B. Armstrong, D. Neevel and T. Kusik "New results in NPID control: Tracking, integral control, friction compensation and experimental results," *IEEE Trans. on Control Systems Technology* **9** *(2)*, 399–406 (2001).
9. B. Armstrong, J. Gutierrez, B. Wade, R. Joseph "Stability of Phase-Based Gain Modulation with Designer-Chosen Switch Functions," The International Journal of Robotics Research 25, (8), 781-796 (2006).
10. Hogan N., "Impedance control: an approach to manipulation, Part I-theory, Part II – implementation, Part III- applications," Journal of Dynamic Systems, Measurements and Control, 107, 1-24, (1985).
11. De Schutter J., Bruyninckx H., Zhu W.H., Spong M., Force control: a bird's eye view. *IEEE CSS/RAS International Workshop on Control problems in Robotics and Automation: "Future Directions"*. San Diego, California (1997).
12. R. Zotovic, A. Valera Fernández, P.J. García Gil, "Impact and force control with switching between mechanical impedance parameters," *Proceedings of the 16th IFAC World congress*, Prague, Czech Republic, (2005).
13. R. Zotovic Ranko, A. Valera Fernández, "Simultaneous velocity, impact and force control," *Robotica,* **27** (7), 1039-1047 (2009).
14. F.L. Kewis, D. M. Dawson, C. T. Abdallah, *Robot Manipulator Control - Theory and Practice.* (Marcel Dekker Inc., New York, 2004).
15. M. W. Spong, S. Hutchison, M Vidyasagar, *Robot Modeling and Control*, (John Wiley & Sons Publishers, 2006).
16. B. Siciliano, L. Sciavicco, L. Villani, G. Oriolo, *Robotics – Modeling, Planning and Control*. (Springer Verlag, London, 2009).
17. H. Seraji, R. Colabaugh, "Force Tracking in Impedance Control," *International Journal of Robotics Research*, **16** (1), 97-117 (1997).
18. Ott C., Albu-Schäffer A., Kugi A: and Hirzinger G., "On the Passivity-Based Impedance Control Of Flexible Joint Robots," IEEE transactions on robotics, **24**, (2), 416-429, (2008).
19. Ott, C.,*Cartesian impedance control of redundant and flexible joint robots.* (Springer Verlag Berlin Heidelberg, 2008).
20. Lu Z. and Goldenberg A., Robust impedance control and force regulation: theory and experiments. *International Journal of Robotics research*, 14(3): 225-254 (1995).
21. Ikeura R., Inooka H., "Variable impedance of a robot for cooperation with a human.", *IEEE international conference of robotics and automation*. Nagoya, Japan 3097-3102, (1995).
22. Tsuji_T., Tanaka Y., "Bio-mimetic impedance control of robotic manipulator for dynamic contact tasks," Robotics and Autonomous Systems 56 306–316, (2008).
23. C. Canudas de Wit, B. Siciliano, G. Bastin, *Theory of Robot Control*. (Springer Verlag Limited London, 1997).
24. K.S. Chang, O. Khatib, "Manipulator control at kinematic singularities: A dynamically consistent strategy", IEEE/ RSJ International Conference on Intelligent Robots and Systems (1995), pp 84-88.
25. C. Edwards, and S.K Spurgeon, Sliding mode control: theory and applications, (Taylor and Francis publishers, London, 1998).