Summary

The Ph.D. Thesis "Strong mixing measures and invariant sets in linear dynamics" has three differenced parts. Chapter 0 introduces the notation, definitions and the basic results that will be needed troughout the thesis. There is a first part consisting of Chapters 1 and 2, where we study the relation between the Frequent Hypercyclicity Criterion and the existence of strongly-mixing Borel probability measures. A third chapter, where we focus our attention on frequent hypercyclicity for translation C_0-semigroups, and the last part corresponding to Chapters 4 and 5, where we study dynamical properties satisfied by autonomous and non-autonomous linear dynamical systems on certain invariant sets. In what follows, we give a brief description of each chapter:

In Chapter 1, we construct strongly mixing Borel probability T-invariant measures with full support for operators on F-spaces which satisfy the Frequent Hypercyclicity Criterion. Moreover, we provide examples of operators that verify this criterion and we also show that this result can be improved in the case of chaotic unilateral backward shifts. The contents of this chapter have been published in [88] and [12].

In Chapter 2, we show that the Frequent Hypercyclicity Criterion for C_0-semigroups, which was given by Mangino and Peris in [82], ensures the existence of invariant strongly mixing measures with full support. We will provide several examples, that range from birth-and-death models to the Black-Scholes equation, which illustrate these results. All the results of this chapter have been published in [86].

In Chapter 3, we focus our attention on one of the most important tests C_0-semigroups, the translation semigroup. Inspired in the work of Bayart and Ruzsa in [22], where they characterize frequent hypercyclicity of

weighted backward shifts we characterize frequently hypercyclic translation C_0-semigroups on $C_0^\rho(\mathbb{R})$ and $L_p^\rho(\mathbb{R})$. Moreover, we first review some known results on the dynamics of the translation C_0-semigroups. Later we state and prove a characterization of frequent hypercyclicity for weighted pseudo shifts in terms of the weights that will be used later to obtain a characterization of frequent hypercyclicity for translation C_0-semigroups on $C_0^\rho(\mathbb{R})$. Finally we study the case of $L_p^\rho(\mathbb{R})$. We will also establish an analogy between the study of frequent hypercyclicity for the translation C_0-semigroup in $L_p^\rho(\mathbb{R})$ and the corresponding one for backward shifts on weighted sequence spaces. The contents of this chapter have been included in [81].

Chapter 4 is devoted to study hypercyclicity, Devaney chaos, topological mixing properties and strong mixing in the measure-theoretic sense for operators on topological vector spaces with invariant sets. More precisely, we establish links between the fact of satisfying any of our dynamical properties on certain invariant sets, and the corresponding property on the closed linear span of the invariant set, or on the union of the invariant sets. Viceversa, we give conditions on the operator (or C_0-semigroup) to ensure that, when restricted to the invariant set, it satisfies certain dynamical property. Particular attention is given to the case of positive operators and semigroups on lattices, and the (invariant) positive cone. The contents of this chapter have been published in [85].

In the last chapter, motivated by the work of Balibrea and Oprocha [4], where they obtained several results about weak mixing and chaos for nonautonomous discrete systems on compact sets, we study mixing properties for nonautonomous linear dynamical systems that are induced by the corresponding dynamics on certain invariant sets. All the results of this chapter have been published in [87].