Summary

Title

High Performance Algorithms for Computing the Singular Value Decomposition and Its Application to Model Reduction of Linear Control Systems

Abstract

TO calculate the singular value decomposition (SVD) of a real dense matrix, traditional methods start by reducing the initial matrix to a bidiagonal form, and then, calculate the SVD of the bidiagonal matrix.

The process of reducing the initial matrix to bidiagonal form is known as the bidiagonalization method, which generally consists of applying successive Householder transformations from the left and right of the matrix. Since the reflectors are applied on both sides of the matrix, this has a negative impact on the communication overheads of a parallel implementation for distributed memory systems.

Ralha and Barlow have proposed two new methods for the reduction of dense matrices to bidiagonal form in which the Householder transformations are applied only on the right side of the matrix. This allows to define all operations in terms of full columns of the matrix under transformation. Therefore, these methods are more attractive for distributed memory systems than the standard

methods and they reduce the communication overheads.

In this thesis we present a comparative study between our sequential and parallel implementations of Ralha's and Barlow's method, developed under LAPACK's and SCALAPACK's environment and its corresponding routines.

As a novel study, some changes were introduced in the Barlow's method with the aim of reducing the number of communications in the parallel implementation.

Following this line of research, the next step is to compute the SVD of the upper bidiagonal form. However, the standard problem of computing the SVD of an upper bidiagonal matrix can be seen as the problem of computing the eigenvalue decomposition (ED) of a symmetric tridiagonal matrix. Once we obtain the ED of a symmetric tridiagonal matrix, we can compute the SVD of the corresponding upper bidiagonal matrix, which is an intermediate step to obtain the SVD of the initial matrix.

The main motivation for this strategy is the development of a parallel implementation, without communication, of `zeroinNR` method proposed by Ralha in his doctoral thesis, for the resolution of the ED problem and its corresponding comparative study between our implementation and the standard one, with communications.

The model reduction of linear control systems has been studied as the scope of SVD application, and our approach was directed at the reduction to upper bidiagonal form the top of the matrix product without explicitly calculate this product, and to do this, sequential and parallel implementations of the method proposed by Golub, by Sølna and by van Dooren have been developed.

The results presented in this thesis have been obtained in the computational resources offered by the Networks and High Performance Computing Group (GRyCAP) from the Universitat Politècnica de València (UPV) and the consortium SEARCH[2] from the University of Minho (UM).

[2] SEARCH: *Services and Advanced Research Computing with HTC/HPC clusters.*