# A view on the internal consistency of linear source identification for I.C. engine exhaust noise prediction

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## Abstract

Considerable efforts have been devoted to the development of predictive models that, from a certain set of data related to an engine, and making use of an adequate representation of the effect of the silencing elements, provide an estimate of the exhaust noise emitted. Such models should allow for the consideration of the engine and its interaction with the exhaust system. This is properly achieved by gas-dynamic models, which are becoming the standard, but linear models solved in the frequency domain and representing the engine as a linear time-invariant source may still play a role in exhaust system design, as the engine is treated as a black box. Such a representation is very attractive for engine manufacturers, since it gives the possibility to provide data on the engine without any possibility to trace back to its real characteristics. In order to provide additional criteria for the suitability of the application of a linear time-invariant representation to an engine exhaust, in this paper a multi-load method has been used to extract source characteristics from gas-dynamic simulation results. The details of the method, in which the resulting over-determined system is solved by fitting the values of the source parameters in a least-squares sense, are described, and different approaches are used in order to check the internal consistency of the source representation: the identification of pressure and velocity sources, and the application of the least-squares criterion to the modulus or to the real and imaginary parts separately. In particular, eight different determinations of the source impedance are obtained and, considering the application of the formalism to an engine exhaust, the differences observed provide a suitable criterion for the evaluation of the suitability of the representation and of the particular set of loads chosen.

*Keywords:* linear source, source strength, source impedance, identification, least squares

## 1. Introduction

Exhaust line design starts in relatively early stages of vehicle development, and so it is frequent that the engine may still be subject to changes during the exhaust line development, whose repercussions from the viewpoint of noise are not easy to predict. Additionally, even if the engine is in a sufficiently advanced development stage, engine manufacturers are more and more reluctant to supply engine prototypes of new or innovative designs. This situation has led to the development of predictive models that, from a certain set of data related to the engine, and making use of an adequate representation of the effect of the silencing elements, provide an estimation of the noise emitted in the presence of a certain exhaust line. Such models must of course include specific models for silencers, but they should also allow for the consideration of the engine, its interaction with the exhaust system and the radiation process. A common aspect for all the models with a certain practical utility is the hypothesis of one-dimensionality of the flow in the ducts of the exhaust system. This hypothesis is fully justified in the ducts themselves, given their usual dimensions in exhaust lines, but may not hold for the silencers and other types of singularities (duct junctions, etc.).

Among the different possible models, it can be stated that non-linear models solved in the time domain provide the most reasonable option. The superiority of these models lies in the fact that the representation of the engine is rather

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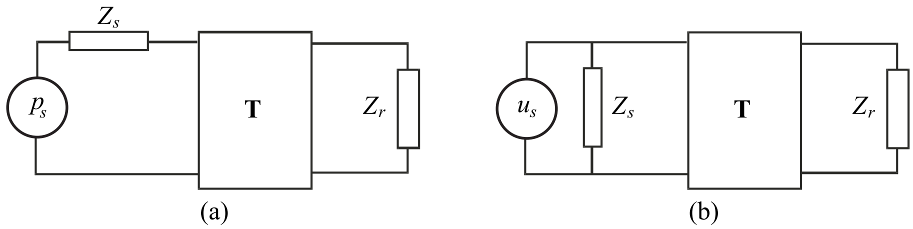


Figure 1: Linear representations of an exhaust system: (a) Pressure source; (b) velocity source.

realistic, since performing the calculation in the time domain allows describing the complex interaction between the cylinders and the exhaust system. Also, the coupling between the engine and the turbo group in turbocharged engines is best managed in the time domain. In summary, all the time-variant character of the engine as a source is accounted for, as well as any interactions between the source and the exhaust line.

When the method of characteristics or finite differences schemes are used, the limitations of these models are mainly associated with the description of the silencers [1]. However, when the finite volume method is used it is possible to consider complex silencer configurations in a relatively straightforward manner [2]. This is due to the ability of these methods to mesh the different calculation domains with the required dimensionality (one-dimensional for ducts, three-dimensional for cavities), and the subsequent analysis of the mass and momentum exchanges between the control volumes. These methods have thus become a standard tool in industrial practice; however, linear models solved in the frequency domain may still play a role in exhaust system design, as they are still better for describing sound flow interaction and losses at discontinuities such as area changes, perforated ducts and plates, or even the open end itself. Additionally, they provide a representation of the engine as a black box, what makes this representation very attractive for engine manufacturers, since it gives the possibility to provide data on the engine without being possible to trace back to its real characteristics.

In Figure 1 the representation of the engine and the exhaust system adopted in these linear models is shown. The weak point (and the actual strength) of these models lies in the representation of the engine, which is described as an equivalent acoustic source, characterized by a certain source strength ($p_s$ for a pressure source or $u_s$ for a velocity source) and an equivalent impedance $Z_s$ (in series or in parallel, respectively), which gives account of the reflective properties of the source. This source, by definition, is time-invariant, which is not the case for an engine exhaust, in which the source varies with time according to the interaction between the cylinders and the flow in the exhaust manifold. In the case of turbocharged engines, this time-invariant source should also incorporate any effects coming from the presence of the turbine. In spite of these shortcomings, the simplicity of these methods is such that an abundant literature has been published on the determination of these characteristic parameters [3], even though their physical interpretation is certainly doubtful [4]. As it can also be seen in Figure 1, the exhaust system is represented by its global transfer matrix **T** , which may represent a sensible assumption, as it is generally accepted that the linear approximation gives a reasonably approximate representation of the flow in the exhaust line (downstream of any aftertreatment devices), where the amplitudes are moderate [5]. With the notation of Figure 1, the spectrum of the exhaust orifice velocity $u_0$ is therefore obtained from

$$\begin{bmatrix} p_s' \\ u_s \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Z_s^{-1} & 1 \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} 1 & Z_R \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ u_0 \end{bmatrix} \tag{1}$$

for a velocity source, and from

$$\begin{bmatrix} p_s \\ u_s' \end{bmatrix} = \begin{bmatrix} 1 & Z_s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} 1 & Z_R \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ u_0 \end{bmatrix} \tag{2}$$

in the case of a pressure source. Different procedures have been proposed for the determination of the source magnitudes (the strength $p_s$ or $u_s$, depending on the source representation, and the impedance $Z_s$) which can be classified into two main groups: direct methods, which make use of an external source, and indirect methods without any external source. The application of direct methods to the exhaust of an engine is not straightforward, as it is difficult to find an acoustic source more powerful than the engine itself and, even if it were available, it is likely that non-linear effects could not be neglected [6]; however, they have been applied successfully to the case of the intake [7].

Indirect methods are based on the assumption that source characteristics are independent of the exhaust system considered (the "load", in the usual terminology of acoustical-electrical analogies), so that they may be obtained from measurements (or simulations) considering a certain set of "independent" loads. Since two complex parameters must be identified, one needs at least two independent loads providing two equations; therefore, the simplest of these methods is the two-load method [8]. However, problems related with the acquisition of complex magnitudes suggested the development of methods requiring more than two loads, such as the three-load method [9] and the four-load method [10].

In the case that the engine of interest is not fully developed it has become usual practice to apply any of the previously commented methods to the results of gas-dynamics simulations. Apart from providing an estimate of the source characteristics, such an application may provide additional insight into the applicability of the representation, as all the relevant flow variables are available. First reported results, making use of the two-load method, can be found in [11]. Subsequent work was published in [12], were both the two-load method and the direct method were investigated numerically, and in [13].

All the above indirect methods, regardless if applied to experimental measurements or to simulation results, share an essential limitation associated with the use of a rather limited number of loads, and related with the eventual lack of independence of the loads chosen for all the frequency range of interest. Such limitations can be overcome if some over-determination is introduced in the solution, by using more loads than those strictly required, and solving the resulting problem in a least-square sense. Different solutions have been proposed which are mostly evolutions of the four-load method, either using radiated pressure measurements [14, 15] or in-duct measurements [16, 17].

Additionally, over-determination has been used to check to what extent the underlying assumptions of linearity and time-invariance are satisfied. Earliest attempts were based on the study of the scattering in source data resulting from different load pair combinations in the two load method [18]. A more comprehensive approximation is that provided by the so-called linearity tests [19, 20], in which a certain magnitude properly normalized to unity is defined that quantifies how well the data fits a linear source model (unity representing a perfectly linear behaviour). A problem associated with these methods –and with any alternative procedure based on over-determination– is that it is not possible to discriminate between deviations from unity due to non-linearities and time-invariance and deviations due to measurement errors or, in the case that gas-dynamic simulations are used, to computational errors such as those arising from improper meshing or insufficient convergence.

In a spirit similar to that of linearity tests, in this paper a least-squares extension of the two-load method is formulated and subsequently used to investigate the internal consistency of the linear time-invariant representation when applied to the exhaust of an internal combustion engine. The approach differs from the techniques presented in [19] and [20] in the sense that, instead of defining some characteristic magnitude, different independent solutions are found for the source magnitude and impedance, both for pressure and velocity sources. As all the solutions obtained should be the same in the ideal case, the deviations observed provide a useful criterion for the evaluation of the basic assumptions of the representation, comprising the suitability of the chosen set of loads. In this way, source magnitude and impedance are obtained simultaneously with some assessment of their consistency, although the abovementioned problems related to the influence of experimental or computational errors will still be present. The proposed methodology has been applied to results of gas-dynamic simulations, and the results indicate its sensitivity to account for the dependence of the linear representation validity on engine operation conditions.

## 2. Identification procedure

Consider first the case of a velocity source. The important issue of which loads and how many of them must be used will be considered later. Now, assume that a convenient set of $N$ loads has been defined. From equation (1), by using the source admittance $Y_s = Z_s^{-1}$, for the $k$-th load one may write:

$$u_s = Y_s p_k + u_k \tag{3}$$

where $p_k$ and $u_k$ are the pressure and the velocity at the source section, respectively. The source magnitudes are then identified as those which fulfill equation (3) for all the loads considered, in a least-squares sense. Since all magnitudes in equation (3) are complex, such a condition may be imposed either on the real and imaginary parts simultaneously, or on the modulus. In the first case, the quantities to be minimized are

$$\Lambda_r^u(u_s, Y_s) = \sum_{k=1}^{N} \left(\hat{u}_k - \hat{u}_s + \hat{Y}_s\hat{p}_k - \check{Y}_s\check{p}_k\right)^2 \quad ; \quad \Lambda_i^u(u_s, Y_s) = \sum_{k=1}^{N} \left(\check{u}_k - \check{u}_s + \check{Y}_s\hat{p}_k + \hat{Y}_s\check{p}_k\right)^2 \tag{4}$$

where the following notation has been introduced:

$$u_s = \hat{u}_s + \mathrm{j}\,\check{u}_s \quad ; \quad Y_s = \hat{Y}_s + \mathrm{j}\,\check{Y}_s \quad ; \quad p_k = \hat{p}_k + \mathrm{j}\,\check{p}_k \quad ; \quad u_k = \hat{u}_k + \mathrm{j}\,\check{u}_k \tag{5}$$

Application of the minimization conditions

$$\frac{\partial\Lambda_r^u}{\partial\hat{u}_s} = \frac{\partial\Lambda_r^u}{\partial\hat{Y}_s} = \frac{\partial\Lambda_r^u}{\partial\check{Y}_s} = 0 \;\; ; \;\; \frac{\partial\Lambda_i^u}{\partial\check{u}_s} = \frac{\partial\Lambda_i^u}{\partial\hat{Y}_s} = \frac{\partial\Lambda_i^u}{\partial\check{Y}_s} = 0 \tag{6}$$

gives the source magnitudes as the solution of the following linear systems:

$$\begin{bmatrix} N & -B_1 & B_2 \\ B_1 & -C_1 & D \\ B_2 & -D & C_2) \end{bmatrix} \begin{bmatrix} \hat{u}_s \\ \hat{Y}_s \\ \check{Y}_s \end{bmatrix} = \begin{bmatrix} A_1 \\ E_1 \\ F_1 \end{bmatrix} \;\; ; \;\; \begin{bmatrix} N & -B_2 & -B_1 \\ B_1 & -D & -C_1 \\ B_2 & -C_2 & -D) \end{bmatrix} \begin{bmatrix} \check{u}_s \\ \hat{Y}_s \\ \check{Y}_s \end{bmatrix} = \begin{bmatrix} A_2 \\ F_2 \\ E_2 \end{bmatrix} \tag{7}$$

The different quantities appearing in equation (7) are given by:

$$\begin{gathered} A_1 = \sum_{k=1}^{N} \hat{u}_k \; ; \; A_2 = \sum_{k=1}^{N} \check{u}_k \; ; \; B_1 = \sum_{k=1}^{N} \hat{p}_k \; ; \; B_2 = \sum_{k=1}^{N} \check{p}_k \\ C_1 = \sum_{k=1}^{N} \hat{p}_k^2 \; ; \; C_2 = \sum_{k=1}^{N} \check{p}_k^2 \; ; \; D = \sum_{k=1}^{N} \hat{p}_k\check{p}_k \\ E_1 = \sum_{k=1}^{N} \hat{u}_k\hat{p}_k \; ; \; E_2 = \sum_{k=1}^{N} \check{u}_k\check{p}_k \; ; \; F_1 = \sum_{k=1}^{N} \hat{u}_k\check{p}_k \; ; \; F_2 = \sum_{k=1}^{N} \check{u}_k\hat{p}_k \end{gathered} \tag{8}$$

From equation (7) it is apparent that, while a single solution for $u_s$ is obtained, two different solutions are available for $Y_s$ (and thus for $Z_s$). Considering now the identification in terms of the modulus, the source magnitudes are obtained as the values of $u_s$ and $Y_s$ that minimize the quantity

$$\Lambda_m^u(u_s, Y_s) = \sum_{k=1}^{N} |u_s - (Y_s p_k + u_k)|^2 \tag{9}$$

Therefore, one has to express $\Lambda_m^u$ in terms of the real and imaginary parts of $u_s$ and $Y_s$, and then apply the minimization conditions. Using again the notation in equation (5), equation (9) may be written as:

$$\Lambda_m^u(u_s, Y_s) = N(\hat{u}_s^2 + \check{u}_s^2) + \sum_{k=1}^{N} \Lambda_{m,k}^u(u_s, Y_s, p_k, u_k) \tag{10}$$

where:

$$\begin{aligned} \Lambda_{m,k}^u(u_s, Y_s, p_k, u_k) \;\; = & \;\; \hat{u}_k^2 + \check{u}_k^2 + (\hat{Y}_s\hat{p}_k - \check{Y}_s\check{p}_k)^2 + (\check{Y}_s\hat{p}_k - \hat{Y}_s\check{p}_k)^2 \\ - & \;\; 2(\hat{u}_s\hat{u}_k - \check{u}_s\check{u}_k) - 2\hat{p}_k(\hat{Y}_s\hat{u}_s + \check{Y}_s\check{u}_s) - 2\check{p}_k(\hat{Y}_s\check{u}_s - \check{Y}_s\hat{u}_s) \\ + & \;\; 2\hat{p}_k(\hat{Y}_s\hat{u}_k + \check{Y}_s\check{u}_k) + 2\check{p}_k(\hat{Y}_s\check{u}_k - \check{Y}_s\hat{u}_k) \end{aligned} \tag{11}$$

Now, the minimization conditions to be applied are:

$$\frac{\partial\Lambda_m^u}{\partial\hat{u}_s} = \frac{\partial\Lambda_m^u}{\partial\check{u}_s} = \frac{\partial\Lambda_m^u}{\partial\hat{Y}_s} = \frac{\partial\Lambda_m^u}{\partial\check{Y}_s} = 0 \tag{12}$$

which after some algebra give:

$$\begin{bmatrix} N & 0 & -B_1 & B_2 \\ 0 & N & -B_2 & -B_1 \\ B_1 & B_2 & -(C_1+C_2) & 0 \\ B_2 & -B_1 & 0 & (C_1+C_2) \end{bmatrix} \begin{bmatrix} \hat{u}_s \\ \check{u}_s \\ \hat{Y}_s \\ \check{Y}_s \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ E_1+E_2 \\ F_1-F_2 \end{bmatrix} \tag{13}$$

where again the different matrix elements are given by equations (8). Therefore, additional values for $u_s$ and $Z_s$ are obtained. In summary, one has two different solutions for $u_s$ and three different solutions for $Z_s$. Incidentally, it can be observed that both from equations (7) and (13) the same relationship between $u_s$ and $Y_s$ is obtained, namely

$$N\begin{bmatrix} \hat{u}_s \\ \check{u}_s \end{bmatrix} = \begin{bmatrix} B_1 & -B_2 \\ B_2 & B_1 \end{bmatrix} \begin{bmatrix} \hat{Y}_s \\ \check{Y}_s \end{bmatrix} + \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \tag{14}$$

which could be used to reduce the order of the systems in equations (7) and (13), thus providing a full analytical solution for $Y_s$, from which $U_s$ would be readily obtained making use of equation (14). Instead, equations (7) and (13) will here be solved numerically. In order to ensure that the system matrix is well-conditioned, it is convenient to have similar magnitudes for pressure and velocity, which can be achieved by multiplying the velocity by the characteristic impedance $\rho a_0$, where $\rho$ and $a_0$ are the density and the speed of sound at the source section, respectively. In this way, any robust solution algorithm may be used; in this case, LU decomposition with implicit pivoting and subsequent back-substitution [3].

The result comes thus as the source magnitude in velocity units and a non-dimensional source admittance, $\tilde{Y}_s$. These can be easily transformed into the desired source variables (volume velocity or mass velocity), by simply taking into account that, for the volume velocity $q$,

$$q_s = S u_s \;\; ; \;\; Z_s = \frac{\rho a_o}{S}\frac{1}{\tilde{Y}} \tag{15}$$

and, for the mass velocity $v$,

$$v_s = \rho S u_s \;\; ; \;\; Z_s = \frac{a_o}{S}\frac{1}{\tilde{Y}} \tag{16}$$

Here, $S$ is the cross-sectional area at the source section.

In principle, if a source velocity representation has been achieved, then it can be readily transformed into a pressure source: the impedance is the same, and the source magnitude would be

$$p_s = Z_s u_s \tag{17}$$

However, this is strictly true only when one deals with a linear time-invariant system. Considering the application of this formalism to an engine exhaust, one may only expect that the system is approximately linear and time-invariant. Therefore, if a pressure source is to be identified, it is preferable to search directly for such a representation. Additionally, since the source impedance should ideally be the same for both the pressure and the velocity sources, this provides a simple criterion for the evaluation of the suitability of the linear representation and of the particular set of loads chosen [3].

When trying to identify a pressure source, one starts from equation (2), from which one has

$$p_s = p_k + Z_s U_k \tag{18}$$

for each of the loads considered. Again, one may consider applying the least squares procedure either to the real and imaginary parts or to the modulus. In the first case, the quantities to be minimized are

$$\Lambda_r^p(p_s, Y_s) = \sum_{k=1}^{N}\left(\hat{p}_k - \hat{p}_s + \hat{Z}_s\hat{u}_k - \check{Z}_s\check{u}_k\right)^2 \;\; ; \;\; \Lambda_i^p(p_s, Y_s) = \sum_{k=1}^{N}\left(\check{p}_k - \check{p}_s + \check{Z}_s\hat{u}_k + \hat{Z}_s\check{u}_k\right)^2 \tag{19}$$

whereas in the second case the source magnitudes will now be the values of $p_s$ and $Z_s$ that minimize the quantity

$$\Lambda_m^p(p_s, Y_s) = \sum_{k=1}^{N}|p_s - (p_k + Z_s u_k)|^2 \tag{20}$$

Writing $p_s = \hat{p}_s + \mathrm{j}\,\check{p}_s$, and following the same procedure as that used for the velocity source, one finds now that the matrix equations to be solved are, in the case of the real and imaginary parts,

$$\begin{bmatrix} N & -A_1 & A_2 \\ A_1 & -G_1 & H \\ A_2 & -H & G_2 \end{bmatrix} \begin{bmatrix} \hat{p}_s \\ \hat{Z}_s \\ \check{Z}_s \end{bmatrix} = \begin{bmatrix} B_1 \\ E_1 \\ F_2 \end{bmatrix} \; ; \; \begin{bmatrix} N & -A_2 & -A_1 \\ A_2 & -G_2 & -H \\ A_1 & -H & -G_1 \end{bmatrix} \begin{bmatrix} \check{p}_s \\ \hat{Z}_s \\ \check{Z}_s \end{bmatrix} = \begin{bmatrix} B_2 \\ E_2 \\ F_1 \end{bmatrix} \tag{21}$$

and, in the case of the modulus,

$$\begin{bmatrix} N & 0 & -A_1 & A_2 \\ 0 & N & -A_2 & -A_1 \\ A_1 & A_2 & -(G_1+G_2) & 0 \\ A_2 & -A_1 & 0 & (G_1+G_2) \end{bmatrix} \begin{bmatrix} \hat{p}_s \\ \check{p}_s \\ \hat{Z}_s \\ \check{Z}_s \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ E_1+E_2 \\ F_2-F_1 \end{bmatrix} \tag{22}$$

Here, $A_1$, $A_2$, $B_1$, $B_2$, $E_1$, $E_2$, $F_1$ and $F_2$ are again given by equation (8), while the new parameters are given by

$$G_1 = \sum_{k=1}^{N} \hat{u}_k^2 \; ; \; G_2 = \sum_{k=1}^{N} \check{u}_k^2 \; ; \; H = \sum_{k=1}^{N} \hat{u}_k \check{u}_k \tag{23}$$

Therefore, one has two different solutions for $p_s$ and three different solutions for $Z_s$, which gives a total of six determinations for the source impedance, to which two more can be added, if equation (17) is applied to the two pairs $(p_s, u_s)$ obtained. Again as in equation (14), from both equations (21) and (22) a single relationship between $p_s$ and $Z_s$ is obtained, namely

$$N \begin{bmatrix} \hat{p}_s \\ \check{p}_s \end{bmatrix} = \begin{bmatrix} -A_1 & A_2 \\ -A_2 & -A_1 \end{bmatrix} \begin{bmatrix} \hat{Z}_s \\ \check{Z}_s \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{24}$$

If again the velocity is multiplied by the characteristic impedance $\rho a_0$, one obtains the source magnitude in pressure units and a non-dimensional source impedance, $\tilde{Z}_s$, which can be transformed to be consistent with a volume velocity or a mass velocity representation, as in equations (15) and (16).

In view of the previous developments, independently of the source representation chosen, the data required by the identification algorithm are, for each engine operation point considered, instantaneous pressure and velocity traces (these must be transformed into the frequency domain) at the source location, for each of the acoustic loads considered, and cycle-averaged values of the temperature-related magnitudes (density, speed of sound) at the source location, which should be roughly independent on the acoustic load used, and the cross-sectional area.

## 3. Results and discussion

Two issues to be considered before any of the previous procedures is applied are: the choice of the location of the source section, and the choice of the set of loads to be used. Apart from the obvious requirement that everything upstream of that part of the exhaust system responsible for silencing must be included, it is convenient, in order to avoid the introduction of spurious lengths (corresponding to duct portions), that the source section is located at (or at least close to) some physical discontinuity (manifold junction, catalyst outlet, etc.), as illustrated in Figure 2. Regarding the loads, apart from the requirement that all the loads have the same or very similar backpressure, which is an essential minimum requirement if it is to be assumed that the source is not affected by the change in the load, two important and somehow concurrent constraints must be considered: First, the behaviour of the loads in terms of backpressure should be similar to that of a realistic exhaust design, so that the results obtained may be regarded as sufficiently representative of the actual behaviour of the engine in real operation; secondly, the loads should be sufficiently different so as to provide an essentially non-singular system of equations and thus allow for the identification of the source parameters $u_s$ and $Z_s$.

Different approaches have been proposed in the literature for the proper choice of the loads. The most detailed analysis of the problem is probably that given in [21]; however, the loads considered were uniform ducts of different lengths, which may be applicable to acoustic sources which may not be expected to be affected by backpressure

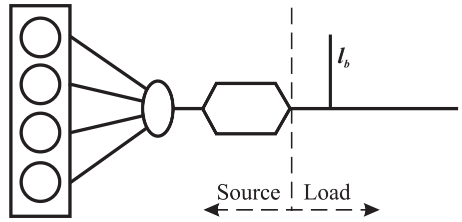


Figure 2: Scheme of the exhaust considered and the loads chosen for source identification

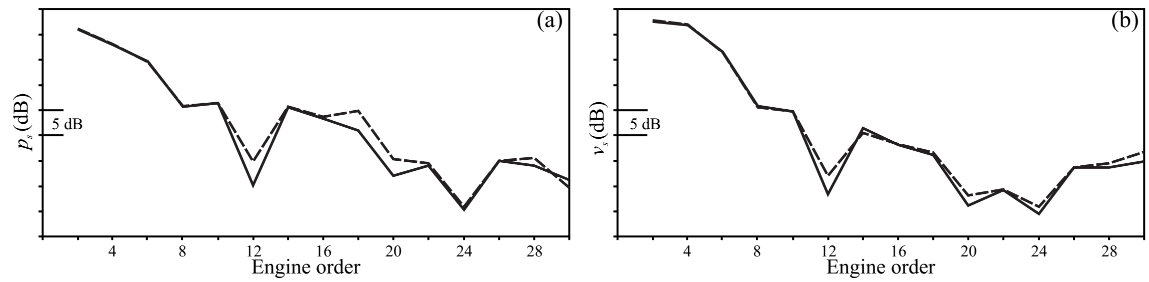


Figure 3: Source magnitude at 2000 rpm: (a) Pressure source; (b) velocity source. Solid line: from modulus (equations (20) and (13)); dashed line: from real and imaginary parts (equations (19) and (7), respectively).

effects, which is not the case when an internal combustion engine is considered. In any case, an important conclusion of the work presented in [21] is that it is not guaranteed that the source obtained is fully independent of the precise set of loads used.

A suitable alternative is that proposed in [16], where quarter-wave resonators of different lengths were used. Actually, this solution provides a sufficiently different acoustic behaviour without any noticeable effect of the backpressure. Moreover, such resonators may be used in conjunction with other elements so that the required backpressure level may be attained. Following those ideas, the arrangement shown in Figure 2 was thus considered. The case with $l_b = 0$, i.e. without any resonator, and five resonators (with lengths of 350, 750, 1350, 1750 and 2350 mm, respectively) were used, giving a total of six loads which, according to [16], should provide suitable results in the frequency range of interest. It can be observed that any possible simple numerical relation between the lengths was avoided.

Gasdynamic simulations with Ricardo WAVE were run on the previous configurations for different engine speeds, and it was first checked that the choice of the loads provided backpressure results with differences below 1% between the different cases, and similar deviations for all the other relevant average magnitudes at the source section (density and speed of sound). Then, instantaneous pressure and velocity traces at the source section were transformed into the frequency domain in order to compute the different magnitudes defined in equations (8) and (23). Results for the different source determinations are commented in the following.

First, results at 2000 rpm are shown in Figures 3 to 5 for the corresponding engine orders. In Figure 3, the two determinations available for the source magnitude (from application of the least-squares fit to the modulus or to the real and imaginary parts) are compared for both the pressure and the velocity source representations. It can be observed that, for the first five engine orders, differences are negligible in both cases, whereas for higher orders some differences appear which are more apparent in the case of the pressure source, with deviations of about 5 dB in some cases.

In Figure 4, six representations of the source impedance are compared. In all the cases, it can be seen that the results for the lower engine orders are quite consistent, whereas for higher orders, and most notably close to the main characteristic frequencies (which are easily identified by the change in sign of the imaginary part), important differences appear. The situation is essentially the same as for the source magnitude, as could be expected in view of the relations expressed by equations (14) and (24) between the source magnitude and the source impedance. Additionally, it is interesting to notice that, whereas in the case of the velocity source the result obtained from equation (13) resembles an average between those obtained from the two equations (7), in the case of the pressure source the results

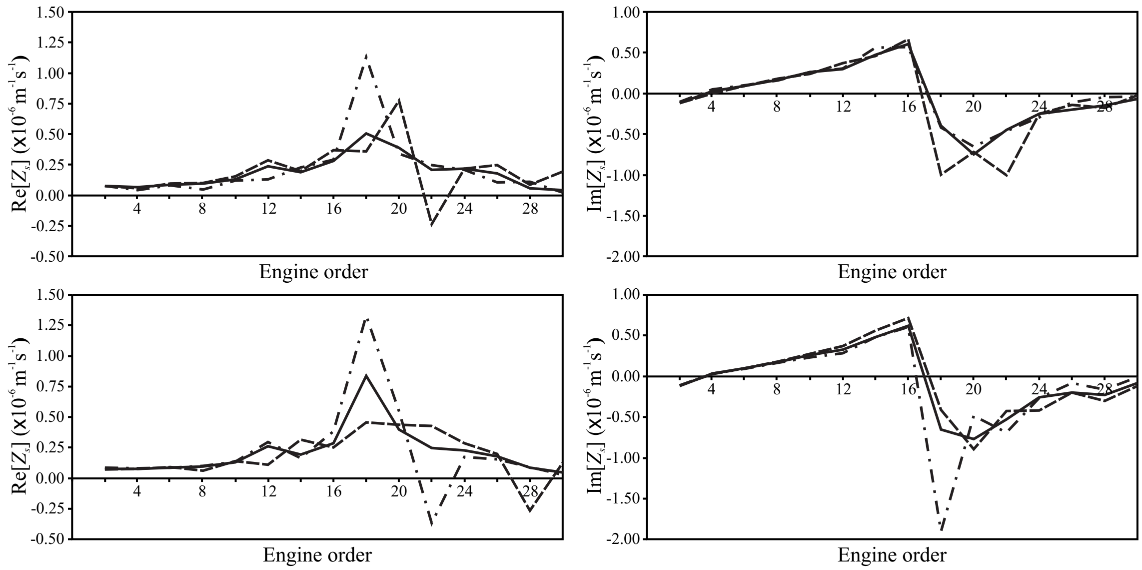


Figure 4: Source impedance at 2000 rpm. Top: Pressure source; bottom: velocity source. Solid line: from modulus (equations (20) and (13), respectively); dashed line: from real part (left system in equations (19) and (7), respectively); dash-dot line: from imaginary part (right system in equations (19) and (7), respectively).

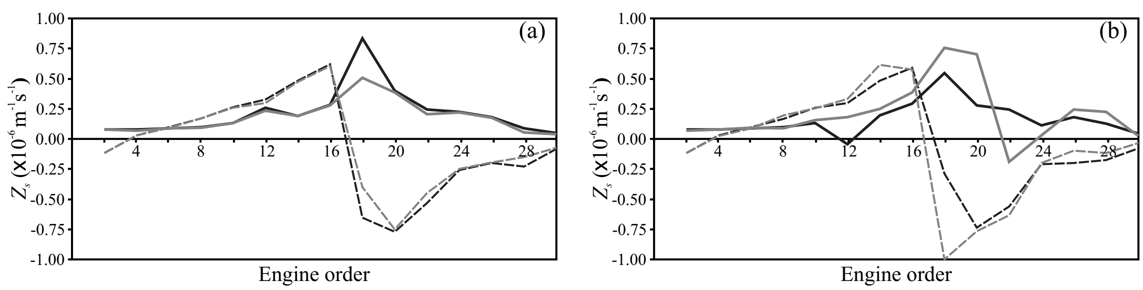


Figure 5: Source impedance at 2000 rpm. (a) Comparison of real (solid lines) and imaginary (dashed lines) parts of source impedance from pressure source (grey lines) and velocity source (black line) identification. (b) Comparison of real (solid lines) and imaginary (dashed lines) parts of source impedance from equation (17) applied to pressure and velocity source magnitudes: identification based on modulus (black lines) and on real and imaginary parts (grey lines).

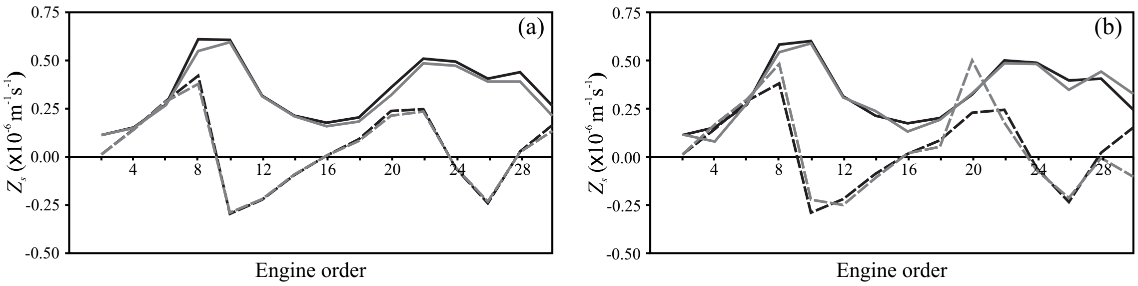


Figure 6: Source impedance at 4000 rpm. Legend as in Figure 5.

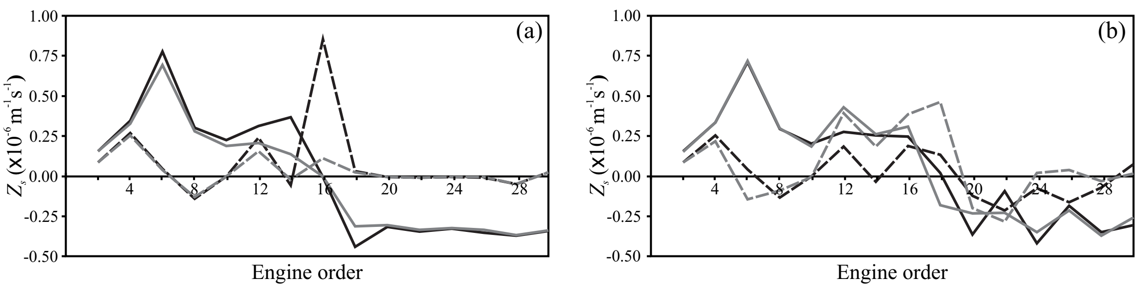


Figure 7: Source impedance at 6000 rpm. Legend as in Figure 5.

obtained from the minimization of the real part differences deviate noticeably from those obtained from the imaginary part and the modulus, which are fairly similar. Also, for relatively high frequencies the resistive impedance becomes sometimes negative; this is not unusual and need not be determinant for the quality of the noise prediction [4], but in any case casts some doubts on the applicability of the source model to these conditions.

Finally, two additional comparisons are presented in Figure 5. In Figure 5(a), source impedances obtained from the pressure and the velocity sources are compared, as suggested in [22] as a possible consistency criterion. It can be checked that both results are essentially coincident for most of the engine orders, which indicates that a criterion based on the comparison between impedances coming from a single source representation would be more exigent in terms of internal consistency. In Figure 5(b), two additional determinations of the source impedance are shown, which are obtained from the direct application of equation (17) to the source magnitude values obtained from equations (13) and (20), and from equations (7) and (19), respectively. Here, it may be noticesd that, even if the amplitudes of the source magnitudes appeared to be rather similar regardless of the identification used, quite important differences appear when such magnitudes are used in order to compute the source impedance. Therefore, these additional determinations provide a more exigent criterion based on the cross-comparison of pressure and velocity sources than that illustrated in Figure 5(a).

The above comments remain essentially valid when other engine speeds are considered, but with distinctive features associated with the engine speed considered. As an example, in Figures 6 and 7 the same representation as in Figure 5 is given at 4000 rpm and 6000 rpm, respectively. It is apparent from Figure 6 that results at 4000 rpm are much more consistent than at 2000 rpm. Impedances obtained from the pressure and velocity sources are almost coincident for most of the engine orders shown; even the results from applying equation (17) provide, except at very precise frequencies, essentially the same result regardless of the identification considered. Moreover, the four impedances shown in Figure 6 are very similar, and all the real parts are positive, i.e. they are effectively "resistances". All these facts indicate that, at this precise engine speed, either the behaviour of the engine actually resembles that of a linear time-invariant source, or that the set of loads chosen is particularly well suited for this case.

On the contrary, the results at 6000 rpm, shown in Figure 7, seem to be acceptable only for the first four engine orders, where the four impedance determinations shown are quite similar. For the fifth and subsequent engine orders, results become rather erratic, with large differences associated both to the source representation and to the identification procedure used. Also, for engine orders above the 16-th one all the real parts, regardless of the impedance considered, become negative. This, together with the substantially flat behaviour observed in Figure 7(a), could suggest that there is some problem in the frequency content of the results of the gas-dynamic computations. In fact, it is not likely that this behaviour can be attributed to the loads chosen, as from the results at 4000 rpm one would expect a reasonable behaviour up to at least the 20-th order.

## 4. Summary and conclusions

With the purpose to check the internal consistency of the representation of an engine exhaust as a linear time-invariant acoustic source, a least-squares extension of the two-load method for source identification has been developed. As velocity fluctuations are considered, together with pressure fluctuations, as a primary input, it may be considered that the method as formulated here is better suited for its use on the results of numerical gas-dynamic simulations. However, if measurement of velocity fluctuations (or reliable estimates from multiple pressure measurements) are available, it could also be applied to the experimental determination of source characteristics.

The formulation of the method allows for the use of different minimization criteria, based either on the modulus or on the real and imaginary parts separately. In this way, two independent source magnitude representations and three independent source impedance determinations have been obtained for a given source representation. The application of the formalism to both pressure and velocity sources provides then six independent determinations for the source impedance; these, together with the two additional determinations coming from crossing information coming from pressure and velocity sources, give a total of eight independent impedance determinations. As, in the ideal case that the source were actually linear and time invariant, and all the loads used were fully independent, all these representations should be essentially the same, any deviations observed should be attributed either to a lack of applicability of the linear representation, to an unfortunate choice of the loads or to some experimental or computational error affecting the input data. However, consideration of different engine speeds may provide additional criteria in order to separate these effects.

The more exigent criterion found seems to be the comparison between source magnitudes obtained from a single source representation but with different least-squares minimization criteria. Such a criterion could be used to assess the expectable accuracy of the resulting noise prediction, or even to optimize the choice of the loads considered.

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