Multivariate Nakagami-*m* Distribution with Constant Correlation Model

Juan Reig

Abstract **In this paper, a multivariate Nakagami-*m* distribution is derived using Royen's gamma distributions of one-factorial accompanying matrices for a constant correlation model. The cumulative distribution function (CDF), the probability density function (PDF) and the covariances are obtained in infinite series form. From these results, we derive outage probabilities of selection combiners (SC) in both interference-limited and noise-limited scenarios with a constant correlation model over Nakagami-*m* fading assuming arbitrary average powers at each input of the combiner.**

Keywords **Correlated fading, diversity, selection combiners, Nakagami-*m* fading channels.**

1. Introduction

The effect of correlated fading has been extensively analyzed on the performance metrics of wireless communication systems. The bivariate Nakagami-*m* probability density function (PDF) [1] was expanded in infinite series form by Tan and Beaulieu in [2]. In [3], Simon and Alouini presented an integral expression of cumulative distribution function (CDF) with Marcum functions of the bivariate Nakagami-*m* distribution.

The classical Khrishnamoorthy and Parthasarathy [4] multivariate gamma distribution has been used in [5] for evaluating performances of selection combiners (SC) over Nakagami-*m* correlated channels. Nevertheless, the joint probability density function of [4] involves infinite series of Laguerre polynomials. Several correlation models have been proposed and used in the literature for evaluating performances of diversity systems. The constant correlation model corresponds to a scenario with closely placed diversity antennas and circular symmetric antenna arrays. It was used by Aalo [6] for analyzing the performance metrics of maximal-ratio diversity combiners.

In recent works [7] and [8], the joint PDF and CDF of the multivariate Nakagami-*m* and Rayleigh distributions, respectively, with exponential correlation model have been derived.

In this paper, we derive the Nakagami-*m* joint probability density function using Royen's one-factorial multivariate gamma distributions for a constant correlation model. This joint PDF obtained is infinite series mixture of products of Nakagami-*m* probability density functions. These results are applied to the derivation of outage probabilities of selection combiners over correlated Nakagami-*m* fading.

Received September 30, 2007.

Dr.- Ing. J. Reig, Departamento de Comunicaciones, Universidad Politécnica de Valencia, Valencia, Spain.

This paper is organized as follows: in section 2, the derivation of multivariate gamma PDF, CDF and covariances with constant correlation model is addressed. Section 3 includes the analysis of the outage probabilities of multiple-branch SC over correlated Nakagami-*m* fading with constant correlation model in noise-limited as well interference-limited scenarios. Finally, conclusions are discussed in section 4.

2. Multivariate Nakagami-*m* Distribution

2.1 Derivation of Multivariate CDF and PDF

Let $\mathbf{R} = [r_{ij}]\,(i,j = 1,\ldots,n)$ be the "accompanying" Gaussian correlation matrix of the *n*-variate gamma distribution. $\mathbf{R}$ matrix is said to be "one factorial" if there are any numbers $a_1,\ldots,a_n$ with

$$r_{ij} = a_i a_j (i \neq j), \quad |a_k| \leq 1,\ i,j,k = 1,\ldots,n, \tag{1}$$

or

$$r_{ij} = -a_i a_j (i \neq j), \quad \mathbf{R} \text{ positive semidefinite.} \tag{2}$$

The multivariate gamma CDF for $\mathbf{R}$ is given by [9, (3.12)]

$$\begin{aligned} F(s_1,\ldots,s_n) &= \frac{1}{\Gamma(m)}\int_0^{\infty} \exp(-y) y^{m-1} \\ &\prod_{j=1}^{n} \exp\left(\frac{\mp a_j^2 y}{1 \mp a_j^2}\right) \sum_{k=0}^{\infty} \mathcal{G}_{m+k}\left(\frac{s_j}{1 \mp a_j^2}\right) \\ &\left(\frac{\mp a_j^2 y}{1 \mp a_j^2}\right)^k / k!\, dy \end{aligned} \tag{3}$$

with marginal gamma PDF given by

$$p(x_j) = \mathcal{F}_m(x_j) = \frac{1}{\Gamma(m)} x_j^{m-1} \exp(-x_j) \tag{4}$$

where the upper signs of (3) hold for (1) and the lower ones for (2), $\mathcal{G}_m(s_j)$ denotes the cumulative distribution of the standard gamma distribution whose PDF is given by (4)

$$\mathcal{G}_m(s_j) = \frac{1}{\Gamma(m)} \gamma(m, s_j) \tag{5}$$

and $\gamma(\cdot,\cdot)$ is the incomplete gamma function [10, (6.5.2)].

In particular for $a_{ij} = \sqrt{r}(i \neq j), r_{ij} = r(i \neq j)$ and regrouping terms of (3), one can obtain

$$F(x_1, ..., x_n) = \frac{1}{\Gamma(m)} \sum_{k_1=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} \mathcal{G}_{m+k_1}\left(\frac{x_1}{1-r}\right) \cdot \ldots \cdot \mathcal{G}_{m+k_n}\left(\frac{x_n}{1-r}\right) \frac{1}{k_1! \cdot \ldots \cdot k_n!}\left(\frac{r}{1-r}\right)^{k_1+\ldots+k_n} \int_0^{\infty} \exp\left(-\frac{nry}{1-r} - y\right) y^{k_1+\ldots+k_n+m-1}\, dy \quad (6)$$

Integrating (6), it yields

$$F(x_1, \ldots, x_n) = \frac{(1-r)^m}{(1+(n-1)r)^m \Gamma(m)} \sum_{k_1=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} \mathcal{G}_{m+k_1}\left(\frac{x_1}{1-r}\right) \cdot \ldots \cdot \mathcal{G}_{m+k_n}\left(\frac{x_n}{1-r}\right) \frac{\Gamma(m+k_1+\ldots+k_n)}{k_1! \cdot \ldots \cdot k_n!} r^{k_1+\ldots+k_n} \left(\frac{1}{1+(n-1)r}\right)^{k_1+\ldots+k_n} \quad (7)$$

If $\mathbf{P} = [\rho_{ij}]$ is the correlation matrix of a *n*-variate gamma distribution and $\mathbf{R} = [r_{ij}]$ is that of the "accompanying" Gaussian distribution, then [11, Lemma1] $\rho_{ij} = \left[r_{ij}^2\right]$ $(i, j = 1, \ldots, n)$. Therefore, the multivariate gamma PDF can be obtained as

$$p(x_1, \ldots, x_n) = \frac{\partial F(x_1, \ldots, x_n)}{\partial x_1 \ldots \partial x_n} = \frac{(1-\sqrt{\rho})^m}{\Gamma(m)} \sum_{k_1=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} \mathcal{F}_{m+k_1}\left(\frac{x_1}{1-\sqrt{\rho}}\right) \cdot \ldots \cdot \mathcal{F}_{m+k_n}\left(\frac{x_n}{1-\sqrt{\rho}}\right) \frac{\Gamma(m+k_1+\ldots+k_n)}{k_1! \cdot \ldots \cdot k_n!} \rho^{\frac{k_1+\ldots+k_n}{2}} \left(\frac{1}{1+(n-1)\sqrt{\rho}}\right)^{m+k_1+\ldots+k_n} \quad (8)$$

where $\mathcal{F}_m(\cdot)$ is the univariate gamma PDF given by (4). Using the transformation

$$r_1 = \sqrt{\frac{\Omega_1 x_1}{m}}, \ldots, r_n = \sqrt{\frac{\Omega_n x_n}{m}} \quad (9)$$

in (8), the Nakagami-*m* joint PDF with constant correlation model is derived as

$$p(r_1, \ldots, r_n) = \frac{(1-\sqrt{\rho})^m}{\Gamma(m)} \sum_{k_1=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} \mathcal{N}_{m+k_1}(A_1 r_1) \cdot \ldots \cdot \mathcal{N}_{m+k_n}(A_n r_n) \frac{\Gamma(m+k_1+\ldots+k_n)}{k_1! \cdot \ldots \cdot k_n!} \rho^{\frac{k_1+\ldots+k_n}{2}} \left(\frac{1}{1+(n-1)\sqrt{\rho}}\right)^{m+k_1+\ldots+k_n} \quad (10)$$

where

$$A_j = \frac{m}{\Omega_j\left(1-\sqrt{\rho}\right)}, \; j = 1, \ldots, n \quad (11)$$

The marginal distributions of (10) are the univariate Nakagami-m distributions whose PDF are given by [1, (3)]

$$p(r_j) = \frac{2}{\Gamma(m)}\left(\frac{m}{\Omega_j}\right)^m r_j^{2m-1} \exp\left(-\frac{m r_j^2}{\Omega_j}\right) \quad (12)$$

where Ω_j is the average power of the distribution, m is the fading parameter and $\mathcal{N}_a(br)$ is a function defined as

$$\mathcal{N}_a(br) = \frac{2}{\Gamma(a)} b^a r^{2a-1} \exp\left(-br^2\right) \quad (13)$$

Note that the joint PDF is the *n*th-order infinite summation over k_j of the product of n Nakagami-*m* PDFs with average powers $\Omega_{eq} = \Omega_j \cdot (m + k_j) / \left(m\left(1-\sqrt{\rho}\right)\right)$ and fading parameters $m_{eq} = m + k_j$. The convergence of (8) and (10) is subjected to $0 \leq \rho < 1$. Substituting $n = 2$ into (10), the bivariate Nakagami-*m* PDF can be obtained as

$$p(r_1, r_2) = \frac{4}{(1-\rho)^m \Gamma(m)}\left(\frac{m}{\Omega_1}\right)^m \left(\frac{m}{\Omega_2}\right)^m \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\Gamma(m+k_1+k_2)}{\Gamma(m+k_1)(m+k_2)\, k_1! k_2!} \rho^{\frac{k_1+k_2}{2}} \left(\frac{1}{1-\rho}\right)^{k_1+k_2} A_1^{k_1} A_2^{k_2}\, r_1^{2(m+k_1)-1} r_2^{2(m+k_2)-1} \exp\left(-A_1 r_1^2\right) \exp\left(-A_2 r_2^2\right) \quad (14)$$

Eqn. 14 agrees with [1, (136)]. Both series provide different representations of the same distribution. They can be derived from different forms of the inverse Laplace-transform of the bivariate Nakagami-m characteristic function [1, (129)].

2.2 Covariances

The covariances of multivariate Nakagami-*m* distribution with constant correlation model can be calculated as

$$E\left[{r_1}^{l_1} \cdot \ldots \cdot {r_n}^{l_n}\right] = \int_0^{\infty} \cdots \int_0^{\infty} {r_1}^{l_1} \cdot \ldots \cdot {r_n}^{l_n} \times p(r_1, \ldots, r_n)\, dr_1 \ldots dr_n \quad (15)$$

Substituting (10) and (13) into (15) and using [12, (3.381/4)], we can obtain

$$E\left[{r_1}^{l_1} \cdot \ldots \cdot {r_n}^{l_n}\right] = \left(\frac{1-\sqrt{\rho}}{1+(n-1)\sqrt{\rho}}\right)^m \left(\frac{1}{A_1}\right)^{\frac{l_1}{2}} \cdot \ldots \cdot \left(\frac{1}{A_n}\right)^{\frac{l_n}{2}} \frac{\Gamma\left(m+\frac{l_1}{2}\right) \cdot \ldots \cdot \left(m+\frac{l_n}{2}\right)}{\Gamma^n(m)} F_A\Bigg(m; m+\frac{l_1}{2}, \ldots, m+\frac{l_n}{2}; m, \ldots, m; \frac{\sqrt{\rho}}{1+(n-1)\sqrt{\rho}}, \ldots, \frac{\sqrt{\rho}}{1+(n-1)\sqrt{\rho}}\Bigg) \quad (16)$$

where $F_A(\ldots)$ denotes the denotes the Appell hypergeometric function defined as [12, (9.19)]

$$F_A\left(\alpha;\beta_1,\ldots,\beta_n;\gamma_1,\ldots,\gamma_n;z_1,\ldots,z_n\right)= \sum_{k_1=0}^{\infty}\cdots\sum_{k_n=0}^{\infty}\frac{(\alpha)_{k_1+\ldots+k_n}(\beta_1)_{k_1}\cdot\ldots\cdot(\beta_n)_{k_n}}{(\gamma_1)_{k_1}\cdot\ldots\cdot(\gamma_n)_{k_n}k_1!\cdot\ldots\cdot k_n!} \cdot {z_1}^{k_1}\cdot\ldots\cdot {z_n}^{k_n} \quad (17)$$

From (16), the covariances for the bivariate Nakagami-*m* distribution can be calculated as

$$E\left[{r_1}^{l_1}\cdot{r_2}^{l_2}\right]=\left(\frac{1-\sqrt{\rho}}{1+\sqrt{\rho}}\right)^m\left(\frac{1}{A_1}\right)^{\frac{l_1}{2}} \left(\frac{1}{A_2}\right)^{\frac{l_2}{2}}\frac{\Gamma\left(m+\frac{l_1}{2}\right)\left(m+\frac{l_2}{2}\right)}{\Gamma^2(m)} F_A\left(m;m+\frac{l_1}{2},m+\frac{l_2}{2};m,m;\frac{\sqrt{\rho}}{1+\sqrt{\rho}},\frac{\sqrt{\rho}}{1+\sqrt{\rho}}\right) \quad (18)$$

where $F_2(\ldots)$ is given by [12, (9.180/2)] as

$$F_A\left(\alpha;\beta_1,\beta_2;\gamma_1,\gamma_2;z_1,z_2\right)= \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(\alpha)_{k_1+k_2}(\beta_1)_{k_1}(\beta_2)_{k_2}}{(\gamma_1)_{k_1}(\gamma_2)_{k_2}k_1!k_2!}{z_1}^{k_1}{z_2}^{k_2} \quad (19)$$

Using [12, (9.182/3)] and [12, (9.131)], (18) can be reduced to

$$E\left[{r_1}^{l_1}\cdot{r_2}^{l_2}\right]=\left(\frac{\Omega_1}{m}\right)^{\frac{l_1}{2}}\left(\frac{\Omega_2}{m}\right)^{\frac{l_2}{2}} \frac{\Gamma\left(m+\frac{l_1}{2}\right)\Gamma\left(m+\frac{l_2}{2}\right)}{\Gamma^2(m)} {}_2F_1\left(-\frac{l_1}{2},-\frac{l_2}{2};m;\frac{\sqrt{\rho}}{1+\sqrt{\rho}},\frac{\sqrt{\rho}}{1+\sqrt{\rho}}\right) \quad (20)$$

in agreement with [1, (137)].

3. Outage Probability of Selection Combiners

3.1 Noise-limited Scenario

In a noise-limited situation, the outage probability is to be calculated only for thermal-noise, where the cochannel interference is negligible. From (10) and using the transformation $\epsilon_j = {r_j}^2, \quad (j=1,\ldots,n)$, the multivariate PDF of the short-term power of signals at each input of the combiner is given by

$$p\left(\epsilon_1,\ldots,\epsilon_n\right)=\frac{\left(1-\sqrt{\rho}\right)^m}{\Gamma(m)} \sum_{k_1=0}^{\infty}\cdots\sum_{k_n=0}^{\infty}\frac{\Gamma\left(m+k_1+\ldots+k_n\right)}{\Gamma\left(m+k_1\right)\cdot\ldots\cdot\Gamma\left(m+k_n\right)} \left(\frac{1}{1+(n-1)\sqrt{\rho}}\right)^{m+k_1+\ldots+k_n} \frac{1}{k_1!\cdot\ldots\cdot k_n!}\rho^{\frac{(k_1+\ldots+k_n)}{2}}A_1^{m+k_1}\cdot\ldots\cdot A_n^{m+k_n} {\epsilon_1}^{m+k_1-1}\cdot\ldots\cdot{\epsilon_n}^{m+k_n-1} \exp\left(-A_1\epsilon_1\right)\cdot\ldots\cdot\exp\left(-A_n\epsilon_n\right) \quad (21)$$

Let us define $s_j=\epsilon_j E_S/N_0, \quad (j=1,\ldots,n)$ as the instantaneous signal-to-noise ratio (SNR) per symbol at each input of the combiner, where E_S/N_0 is the symbol energy-to-Gaussian noise spectral density ratio. The outage probability at the output of the SC, defined as the probability of SNR is less than a protection ratio q, can be calculated as

$$\begin{aligned} P_{out}(q) &= \text{Prob}\left(s_1<q,\ldots,s_n<q\right) \\ &= \int_0^{\infty}\cdots\int_0^{\infty}p\left(s_1,\ldots,s_n\right)ds_1\ldots ds_n \end{aligned} \quad (22)$$

where q is related to the required threshold probability of error as follows

$$q=\begin{cases}\frac{1}{a}\ln\left(\frac{1}{2p_{th}}\right) & \begin{cases}a=\frac{1}{2} & \text{for NCFSK}\\ a=1 & \text{for DCPSK}\end{cases}\\ \frac{1}{2}\left(Q^{-1}\left(p_{th}\right)\right)^2 & \begin{cases}a=\frac{1}{2} & \text{for CFSK}\\ a=1 & \text{for CPSK}\end{cases}\end{cases} \quad (23)$$

where NCFSK and DCPSK apply to non-coherent frequency shift keying and differentially coherent phase shift keying, respectively, CFSK and CPSK represent coherent frequency shift keying and coherent phase shift keying, respectively, Q^{-1} is the inverse of the Gaussian probability integral defined as

$$Q(x)=\int_x^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left(-t^2/2\right)\,dt \quad (24)$$

and p_{th} is the required threshold probability of error. Note that (24) is applied to optimum detection with optimum matched filter received [13].

The integral (22) can be solved as

$$
\begin{aligned}
P_{out} = & \frac{\left(1-\sqrt{\rho}\right)}{\Gamma(m)} \\
& \sum_{k_1=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} \frac{\Gamma\left(m+k_1+\ldots+k_n\right)}{\Gamma\left(m+k_1\right)\cdot\ldots\cdot\Gamma\left(m+k_n\right)} \\
& \left(\frac{1}{1+(n-1)\sqrt{\rho}}\right)^{m+k_1+\ldots+k_n} \\
& \frac{1}{k_1!\cdot\ldots\cdot k_n!}\rho^{\frac{(k_1+\ldots+k_n)}{2}} \\
& \gamma\left(m+k_1, \frac{mq}{\overline{s}_1\left(1-\sqrt{\rho}\right)}\right)\cdot\ldots\cdot \\
& \gamma\left(m+k_n, \frac{mq}{\overline{s}_n\left(1-\sqrt{\rho}\right)}\right)
\end{aligned} \tag{25}
$$

where $\overline{s}_j, (j=1,\ldots,n)$ are the average SNR at each input of the combiner. Substituting $\rho=0$ into (25) only the $k_1=\ldots=k_n=0$ term of summation is non-zero, thus, the outage probability for independent signals is reduced to

$$
\begin{aligned}
P_{out}(q) = & \frac{1}{\Gamma^n(m)} \\
& \times\gamma\left(m, \frac{mq}{\overline{s}_1}\right)\cdot\ldots\cdot\gamma\left(m, \frac{mq}{\overline{s}_n}\right)
\end{aligned} \tag{26}
$$

Fig. 1 shows outage probabilities versus average SNR at each branch of the SC combiner assuming $\overline{s}_1=\ldots=\overline{s}_n$ for NCFSK and $p_{th}=10^{-4}$. The fading parameters of signals at each input of the combiner are $m=1.2$. Curves are plotted for non-diversity environment with solid lines. Outage probabilities for $n=2,3$ and 4 branches are drawn with dashed lines for $\rho=0, 0.3$ and 0.7. The differences in outage probabilities between low and high correlation coefficients increase as n grows.

3.2 Interference-limited Scenario

In interference-limited systems, thermal noise power is negligible compared to the cochannel interference power contribution. Assuming total independence between interferences received on any pair of inputs of the combiner, the joint PDF of the signal-to-interference ratios (SIR) at each input of the combiner can be written as

$$
\begin{aligned}
& p\left(\gamma_1,\ldots,\gamma_n\right) = \\
& \times\int_0^{\infty}\ldots\int_0^{\infty} i_1\cdot\ldots\cdot i_n\, p\left(i_1\right)\cdot\ldots\cdot p\left(i_n\right) \\
& \times p\left(i_1\gamma_1,\ldots,i_n\gamma_n\right)\, di_1\ldots di_n
\end{aligned} \tag{27}
$$

where $p\left(i_1\right),\ldots,p\left(i_n\right)$ are the PDFs of the total interference power received at each branch of the combiner,

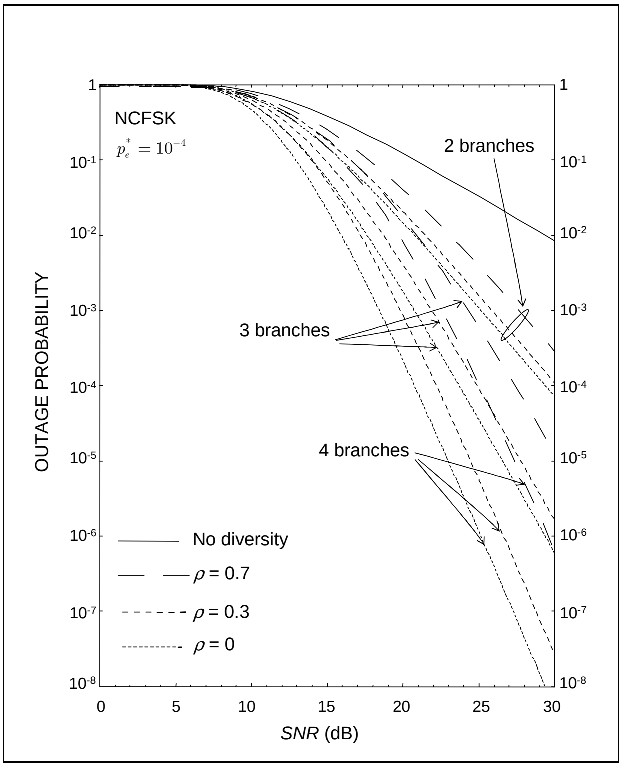

Fig. 1. Outage probability versus average signal-to-noise ratio in a selection combiner for NCFSK over correlated Nakagam-m channel with $m=1.2$ and $p_{th}=10^{-4}$.

$i_1,\ldots,i_n$, respectively, given by

$$
\begin{aligned}
p\left(i_j\right) = & \frac{1}{\Gamma\left(m_{i_j}\right)}\left(\frac{m_{i_j}}{\Omega_{i_j}}\right)^{m_{i_j}} \\
& \times\, i_j^{m_{i_j}}\exp\left(-\frac{m_{i_j}}{\Omega_{i_j}}i_j\right), \; j=1,\ldots,n
\end{aligned} \tag{28}
$$

and Ω_{i_j} and m_{i_j} are the average short-term power and fading parameter, respectively, of the interference received on *j*-th input of the combiner. Substituting (21) and (28) into (27), the joint PDF of the SIR is obtained as

$$
\begin{aligned}
& p\left(\gamma_1,\ldots,\gamma_n\right) = \frac{\left(1-\sqrt{\rho}\right)^m}{\Gamma(m)\Gamma(m_{i_1})\cdot\ldots\cdot\Gamma(m_{i_n})} \\
& \sum_{k_1=0}^{\infty}\cdots\sum_{k_n=0}^{\infty}\frac{\Gamma\left(m+m_{i_1}+k_1\right)\cdot\ldots\cdot\Gamma\left(m+m_{i_n}+k_n\right)}{\Gamma(m+k_1)\cdot\ldots\cdot\Gamma(m+k_n)} \\
& \frac{\Gamma(m+k_1+\ldots+k_n)}{k_1!\cdot\ldots\cdot k_n!}\left(\frac{1}{1+(n-1)\sqrt{\rho}}\right)^{m+k_1+\ldots+k_n} \\
& \frac{1}{k_1!\cdot\ldots\cdot k_n!}\rho^{\frac{(k_1+\ldots+k_n)}{2}}\cdot\left(\frac{m_{i_1}}{\Omega_{i_1}}\right)^{m_{i_1}}\cdot\ldots\cdot\left(\frac{m_{i_n}}{\Omega_{i_n}}\right)^{m_{i_n}} \\
& A_1^{m+k_1}\cdot\ldots\cdot A_n^{m+k_n}\,\gamma_1^{m+k_1-1}\cdot\ldots\cdot\gamma_n^{m+k_n-1} \\
& \left(\frac{m_{i_1}}{\Omega_{i_1}}+A_1\gamma_1\right)^{-m-m_{i_1}-k_1}\cdot\ldots\cdot \\
& \left(\frac{m_{i_n}}{\Omega_{i_n}}+A_n\gamma_n\right)^{-m-m_{i_n}-k_n}, \; \gamma_1,\ldots,\gamma_n\leq 0
\end{aligned} \tag{29}
$$

The probability of outage can be derived as

$$P_{out}(q) = \int_0^\infty \ldots \int_0^\infty p(\gamma_1, \ldots, \gamma_n)\, d\gamma_1 \ldots d\gamma_n \quad (30)$$

From (29) and (30), the outage probability is given by

$$P_{out}(q)(\gamma_1, \ldots, \gamma_n) = \frac{\left(1-\sqrt{\rho}\right)^m}{\Gamma(m)\Gamma(m_{i_1})\cdot\ldots\cdot\Gamma(m_{i_n})}$$
$$\sum_{k_1=0}^\infty \cdots \sum_{k_n=0}^\infty \frac{\Gamma\left(m+m_{i_1}+k_1\right)\cdot\ldots\cdot\Gamma\left(m+m_{i_n}+k_n\right)}{\Gamma(m+k_1)\cdot\ldots\cdot\Gamma(m+k_n)}$$
$$\frac{\Gamma(m+k_1+\ldots+k_n)}{k_1!\cdot\ldots\cdot k_n!}\left(\frac{1}{1+(n-1)\sqrt{\rho}}\right)^{m+k_1+\ldots+k_n}$$
$$\frac{1}{k_1!\cdot\ldots\cdot k_n!}\rho^{\frac{(k_1+\ldots+k_n)}{2}} \cdot q^{nm+k_1+\ldots+k_n}$$
$$\left(\frac{\Omega_{i_1}}{m_{i_1}}\right)^{m+k_1}\cdot\ldots\cdot\left(\frac{\Omega_{i_n}}{m_{i_n}}\right)^{m+k_n} \quad (31)$$
$$A_1^{m+k_1}\cdot\ldots\cdot A_n^{m+k_n}\frac{1}{(m+k_1)\cdot\ldots\cdot(m+k_n)}$$
$${}_2F_1\left(m+k_1, m+m_{i_1}+k_1; m+k_1+1; -\frac{A_1\Omega_{i_1}}{m_{i_1}}q\right)\cdot\ldots\cdot$$
$${}_2F_1\left(m+k_n, m+m_{i_n}+k_n; m+k_n+1; -\frac{A_n\Omega_{i_n}}{m_{i_n}}q\right)$$

where ${}_2F_1(\cdot,\cdot;\cdot;\cdot)$ is the Gauss hypergeometric function [10, (15.1.1)]. For independent signals at each input of the combiner ($\rho = 0$), (32) agrees with [14, (13),(25)]. Let the average signal-to-interference ratio be defined as

$$SIR_{av} = \frac{\Omega}{\Omega_e} = \frac{\left(\frac{\Omega_1+\ldots+\Omega_n}{n}\right)}{\left(\frac{\Omega_{i_1}+\ldots+\Omega_{i_n}}{n}\right)} \quad (32)$$

In Fig. 2, the outage probability of SC versus SIR_{av}/q, where q is the protection ratio, is plotted for $n = 1$, 2, 3 and 4 branches assuming $\Omega_1 = \ldots = \Omega_n, m = 1.2$ and $m_{i_1} = \ldots = m_{i_n} = 1.5$. The outage probability in a non-diversity scenario is drawn with solid line. Curves with dashed lines correspond to SC for correlation coefficients between desired signals $\rho = 0$, 0.3 and 0.7. Again, the outage probability behavior improves as the diversity order (number of branches) increases.

4. Conclusions

A new form of the probability density function, the cumulative density function and the covariances of the multivariate Nakagami-m distribution with constant correlation model has been obtained in infinite series expansion. Previous functions have been contrasted with results of literature for the bivariate distribution. The distribution derived is applied to outage probabilities calculation for both noise-limited and interference-limited scenarios.

Acknowledgement

I would like to thank Dr. T. Royen of University of Bingen for his fruitful comments and remarks on the multivariate gamma distribution.

References

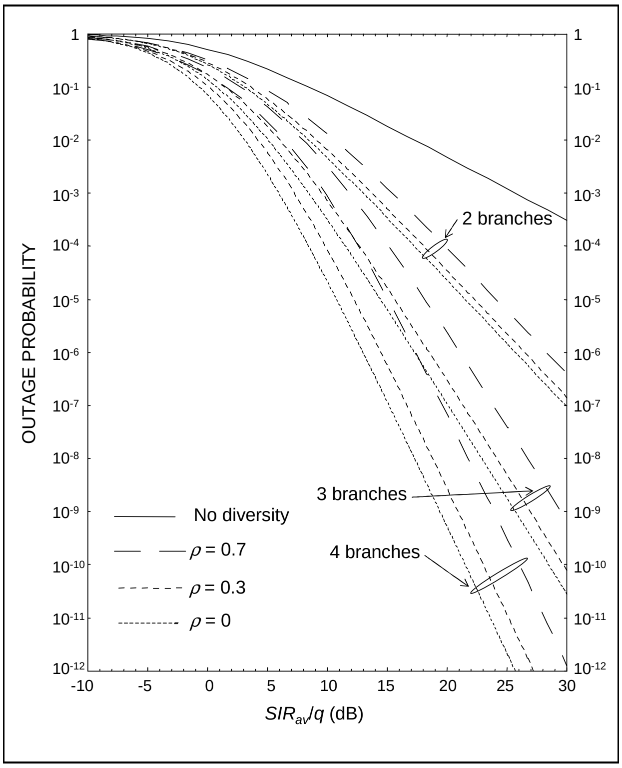

Fig. 2. Outage probability versus average signal-to-interference ratio normalized with protection ratio in a selection combiner over correlated Nakagam-*m* channel with $m = 1.2$ and $m_{i_1} = \cdots = m_{i_n} = 1.5$.

[1] Nakagami, M.: The m-distribution-a general formula of intensity distribution of rapid fading. statistical methods of radio wave propagation. Pergamon, England: W. G. Hoffman, Ed. Oxford, 1964.

[2] Tan, C. C.; Beaulieu, N. C.: Infinite series representation of the bivariate rayleigh and nakagami-*m* distributions. IEEE Trans. Commun. COM– **45** (1997), 1159–1161.

[3] Simon, M. K.; Alouini, M. S.: A simple integral representation of the bivariate rayleigh distribution. IEEE Commun. Lett. **2** (1998), 128–130.

[4] Krishnamoorthy, A. S.; Parthasarathy, M.: A multivariate gamma-type distribution. Annals of Math. Statistics **22** (1951), 549–557.

[5] Ugweje, O. C.; Aalo, V. A.: Performance of selection diversity system in correlated nakagami fading. Proceedings of IEEE Vehicular Technology Conference Spring. Phoenix. AZ, May 1997. 1488–1492.

[6] Aalo, V. A.: Performance of maximal-ratio diversity systems in a correlated nakagami fading environment. IEEE Trans. Commun. COM– **43** (1995), 2360–2369.

[7] Karagiannidis, G. K.; Zogas, D. A.; A., K. S.: On the multivariate nakagami-*m* distribution with exponential correlation. IEEE Trans. Commun. COM– **51** (2003), 1240–1244.

[8] Mallik, R. K.: On multivariate rayleigh and exponential distributions. IEEE Trans. Inform. Theory **49** (2003), 1499–1515.

[9] Royen, T.: Multivariate gamma distributions with one-factorial accompanying correlation matrices and applications to the distribution of the multivariate range. Metrika **38** (1991), 299–315.

[10] Abramowitz, M.; Stegun, I. A.: Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. New York: Dover, 9th.ed., 1972.

[11] Krishnaiah, P. R.; Rao, M. M.: Remarks on the multivariate gamma distribution. Amer. Math. Monthly **68** (1961), 342–346.

[12] Gradsthteyn, I. S.; Ryzhik, I. M.: Table of Integrals, Series and Products. San Diego: Academic, 5th.ed., 1994.

[13] Schwartz, M.; Bennett, W. R.; Stein, S.: Communication Systems and Techniques. New York: McGraw-Hill, 1966.

[14] Yao, Y.; Sheikh, A. U. H.: Investigations into cochannel interference in microcellular mobile radio systems. IEEE Trans. Veh. Technol. VT– **41** (1992), 114–123.

Juan Reig received the MS and PhD degrees in Telecommunications Engineering from the Technical University of Valencia, Spain, in 1993 and 2000, respectively. He has been a faculty member in the Department of Communications at the Technical University of Valencia, Spain since 1994, where he is now Associate Professor of Telecommunication Engineering. He is a member of the Radio and Wireless Communications Group (RWCG) of the Telecommunications and Multimedia Applications Research Institute (iTEAM). His areas of interest include fading theory, diversity, ultra-wide band systems and radio resource management in 3G systems.