Space Phasor Theory and Control of Multiphase Machines through their Decoupling into Equivalent Three-Phase Machines

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Abstract. This paper, first, introduces the space phasor theory from a general mathematical viewpoint and, above all, from a physical one as well. The theory is then applied to the particular case of constant airgap multiphase machines (MM´s) fed by arbitrary voltage waveforms. It is mathematically proven, and thereafter checked by simulations too, that these machines can be split into a set of equivalent three phase machines mechanically coupled but electrically independent. This electrical decoupling leads immediately to develop the very fast torque control schemes of the MM´s. All of them boil down to a mere extension of those already used in the homologous three phase machines.

Keywords *Multiphase machines*, *torque control*, *space phasor theory*, *GAFTOC principle*

1. Introduction

The electrical machine user is mainly interested in its external quantities (voltages, currents, speed, torque). On the other hand, understanding the machine behaviour in depth needs the analysis of its main internal quantities (current sheet, induction wave, etc.). In this respect it should be said that the classical theory shows clear differences in its analysis method, depending on which of these two quantity types is under survey.

As for the external quantities a number of precise techniques (geometric constructions, equivalent circuits, etc.) were early developed, although most of them were only valid for particular applications. However, as for the internal quantities, what predominated were rather a graphical description and an intuitive reasoning. In addition, there was no analytical tool which showed in a precise and easy way how the evolution of the machine internal quantities results in changes of its external ones. Thus, the correlation between both quantity types became blurred, and with the increasing mathematical formalism it was gradually set aside in many books on electrical machines which focus, almost exclusively, on their external quantities.

After Kapp´s proposal in [1] the theory of electrical machines in *steady state* makes use of *time phasors*[1] to represent sinusoidal *waves in time*. Representing the induction or the m.m.f. *spatial wave in steady state* through phasors was sometimes used, although mainly as a mere tool for

[1] Time phasors, after Steinmetz´s work [2], have been mathematically characterized in A.C. circuits theory by means of complex quantities

graphical explanations. In spite of that, as early as the 1910´s Dreyfus proposed *to characterize through phasors the m.m.f. spatial waves in transient state* and to take them as essential tools for dynamic studies, as shown in [3]. Yet, his method, restricted to m.m.f. space waves, could hardly compete with Park's one [4,5].

Park introduced his complex quantities in order to simplify the transient equations of three phase machines. Thereafter, Kron indicated that Park's equations could be interpreted as "a transformation of the coordinates axes from the moving terminals to the stationary polar and interpolar space for the purpose of eliminating the troublesome cos θ term". ([5], p. 355).He also indicated in passing that there was a second interpretation for the Park's current complex quantity: it could be considered as a "current linear density wave"([5], p. 354].

The first interpretation is of operational type and general, for it can be applied to all of the variables in Park's equations. Yet, this is not the case with the second one. There is indeed no difficulty in defining and visualizing a current sheet space wave and to correlate it with Park’s current complex quantity. However, in his later highly abstract and mathematical publications, Kron does not seem to have been interested in defining or searching for similar correlations between other (if at all existing) machine space waves and Park’s voltage and flux linkage complex quantities. In any case, when he generalized Park's work, he decidedly insisted on the first interpretation. These facts greatly influenced the Generalized Machine Theory (GMTh) development, in which the concept of coordinates transformation (and, more in general, the concept of matrix transformations) played from the beginning an outstanding role.

In the 1950´s the authors in [6] emphasized that when analyzing the transients in A.C. machines it was important to provide a picture and a physical insight into the phenomena. To this end, the interpretation of Park’s current complex quantity as a symbolical representation of a transient space wave is much more suitable than that given in the GMTh. This interpretation is the one they promoted and widely spread, and leads immediately to analyze the transient state in terms of sinusoidal space waves. And since in that context the main variables are sinusoidal waves (although their amplitude and speed are not constant) it Dreyfuss´s old idea of representing these *space waves* by means of "space vectors”. emerges again. It is worth mentioning that resorting to space vectors (thus, for three-phase machines without space harmonics) an interesting proposal was made in [7] several decades later to relate machine transient condition to the propagation process in space of distributed magnetic fields.

The Space Vector Theory (SVTh) in [6] aroused somehow as an alternative and reaction to the GMTh´s exacerbated mathematical formalism, which was most pronounced in some books, where the machine is viewed completely "from outside", as a “black box” to which different matrix transformations are applied. By contrast, one of the main ideas in [6] was to look at the machine “from the inside”, to start its dynamic analysis on the base of its space waves, and to proceed in this direction as far as possible.

However, the SVTh achieved his goal only in a modest scope. Indeed, their authors, which always assume the hypothesis of *no space harrmonics*, analyzed first in all detail [6, pp 61-67] from a physical perspective the dynamic m.m.f. space wave produced by a *three phase* winding. They characterized it in an original manner by a graphical tool they called “current space vector”, **i.** The expression of **i** has a direct correlation with the phase currents and coincides with the current complex quantity Park had introduced years ago following a different path. Later on, as they could not find *space* quantities related to the phase flux linkage and voltage *time* quantities, they introduced the formulae for the so called voltage (**u**) and flux linkage (**Ψ)** space vectors by a mere mathematical analogy to the **i** space vector [6, p.75]. Obviously, due to their mathematical definition, **u** and **Ψ** *were restricted to three phase machines*. However, although, as just said, there was no physical meaning for them in terms of space waves, the authors in [6] profusely used these **u** and **Ψ** space vectors (together with the **i** vector) in a graphical manner in order to explain the machine equations. This new graphical viewpoint of approaching and illustrating the transients, in contrast to the abstract matrix transformation perspective, provided a much better insight into the phenomena, even in the sophisticated cases when machines are fed through electronic converters. That is why space vectors became in addition very useful for electronic control studies in three phase machines, and as early as the 1970´s they were widely used in Central Europe in numerous books on this subject [8 – 11]. The important influence of the new space vector viewpoint on the development of modern control methods for three phase machines was also pointed out in [12]. It should be added that the approach in [6] was mainly spread in German language and remained practically unnoticed in the English literature until the eighties of the past century. (See, e. g., the prefaces to books [13] and [14]).

Combining the approach in [6], restricted to three phase machines without space harmonics, with the work in [15], Stepina showed that it was also possible to characterize through space phasors (this correct term was introduced by him) the m.m.f. wave produced by a multiphase winding including its space harmonics. This was an important contribution. Relying on it, he tried to develop a general theory encompassing all kind of machines (arbitrary air gap structure and arbitrary number of space harmonics). Unfortunately, his general formulae [16,17] are incorrect (The starting point of his deductions is erroneous for machines with no small air gap, since in this case, contrary to his assumption, the airgap reluctance is different for different m.m.f. harmonics, as already proven in [18, pp 916 and 938] or in [19, p 102]. Moreover, the fundamental **u** and **Ψ** space phasors are fully unknown in his work [16, 17]. Yet, partial aspects of this work are very interesting, especially for design studies of single phase inductions motors (small air gap machines in which, moreover, phasors **u** and **Ψ** are not needed). His very valuable contribution in this field [20] should be underlined.

The approach actually underlying Park´s work, the so called magnetic coupling circuit approach (MCCA), was very much enhanced by [4,5] and became the approach overwhelmingly used all over the world in machine transients analysis. Put it simply: the MCCA regards the machine as a network made up of resistances, and inductances, many of which vary with the rotor position. *Park* applied in [4,5] the MCCA to three phase machines (m = 3) taking only into account the fundamental space wave. The method was extended first to machines with m > 3 in the GMTh and later on to machines with arbitrary number of space harmonics and phases (e. g. [21]).

Interesting investigations on converted fed multiphase machines (MM´s) have been known for several decades (e. g. [22]). But it is nowadays when MM´s in conjunction with power electronics are becoming more usual. This trend, very probably, will increase in the future (See, e. g., the surveys [23, 24] with more than 140 and 220 references, respectively). Yet, MM´s can not be analyzed by means of the SVTh in [6] for, as already indicated, it is restricted to three phase machines without space harmonics. Since, *besides these two limitations*, the **u** and **Ψ** space vectors, even in this simple case of three phase machines, are introduced in the same formal way as in the MCCA (i. e., *lacking physical interpretation*), it could be argued that the advantages and usefulness of trying to extend the new path open by [6] were clearly questionable. And that, in summary, as to the MM´s, it was much better to simply let aside digressions on space waves and to further develop the theory on the powerful base of the MCCA with the help of matrix transformations. On the light of these facts, it can hardly surprise that the great and expansive force of the SVTh in Central and East Europe during the second half of the past century has gradually vanished. Actually it matches quite better the facts evolution to state that, even before the MCCA was applied to MM´s, it already had "absorbed and digested" the SVTh to a great extent within its mathematical formalism. In view of the SVTh limitations and the undeniable power and success of the MCCA, needless to say that in today's publications on MM´s, the old (and only partially successful) efforts in favour of a *space wave approach* have been *fully abandoned worldwide*. They are regarded as a waste of time and considered to be doomed to failure in MM´s analysis and control.

This paper holds just the opposite thesis. It states that for transients analysis and especially for developing and understanding the control schemes of MM´s, it is highly advantageous to make use of the space wave approach. Yet to find out and show the very high potential of this approach, a starting point and procedure quite different from those used in [6] are needed. This results in the space phasor theory (SPhTh) as presented in this paper.

The structure of this paper is as follows: the fundamental idea underlying the SPhTh is presented in chapter 2. Chapter 3 introduces the concepts of a dynamic polyphase system of sequence "g" and its associated dynamic time phasor. Chapter 4 defines the concept of dynamic space phasor, introduces the **u** and **Ψ** phasors and analyzes them in all detail. It shows that each one of the different **u** and **Ψ** phasors existing in the MM is related to one specific and independent group of MM space harmonics. This fact (valid for the **i** phasors too) is extensively discussed, since *it constitutes the hard core of the later MM decoupling into independent machines.* Chapter 5 analyzes in a similar manner, but now much more briefly, the different MM i space phasors. The conclusions and formulae in chapters 4 and 5, of general validity, are applied to the particular case of constant air gap MM´s fed by arbitrary voltages (chapter 6). It is then proven that the MM can be decomposed into a set of mechanically coupled machines, equal to the original one, but each of them fed exclusively by just one voltage space phasor, so that the problem of controlling the MM is tantamount to control a set of equivalent three phase machines mechanically coupled, but electrically independent. This theoretical conclusion, also reinforced by simulations, is then used (chapter 7) to deduce the very fast torque control schemes of the MM´s, which become a mere extension of those applied to three phase machines. In this regard it is also shown that the GAFTOC principle is the truly (although often unknown) driving idea behind all of the schemes. This fact, combined with the space wave approach, enables explaining in a rigorous, and at the same time very physical and didactic manner, how the whole MM control system actually works.

2. FUNDAMENTAL IDEA UNDERLYING THE SPACE PHASOR THEORY

Let it be very long (negligible end effects) salient pole machine with axial conductors at the stator surface. As

$$\vec{B} = \mathrm{rot}(\vec{A}) \qquad (1)$$

the total flux linkages of an arbitrary stator single turn, "ab" (Fig. 1) of any stator phase is given by

$$\begin{aligned}\Psi_{ab} &= \iint_S \vec{B}\cdot\vec{dS} = \iint_S \mathrm{rot}(\vec{A})\cdot\vec{dS} = \oint \vec{A}\cdot\vec{dl} = \\ &= (A_{za} - A_{zb})\cdot l = l \sum_{y=a,b} \pm A_{z,y}\end{aligned} \qquad (2)$$

that is, Ψ_{ab} is the sum of the vector potential values, $\mathbf{A_z}$, at the positions of the two turn conductors, multiplied by the machine length, l. (Notice that **A** and **B** are constant in the axial direction, z, since, as usual, the magnetic field distribution in the machine is considered to be bidimensional).Therefore, the flux linkages of the whole phase are simply the algebraic sum of the **A** values at its conductor positions. The sign ± in (2) corresponds to the direction attached to each conductor (aa´ or a´a in Fig.1) when moving all along the phase. The conductors are assumed to be of negligible cross section. Notice that eq (2) always holds, no matter the rotor shape or whether the magnetic circuit be saturated or not. Thus, if the magnetic vector potential space wave at the stator surface is known, the phase flux linkages *are obtained immediately in the most general case*.

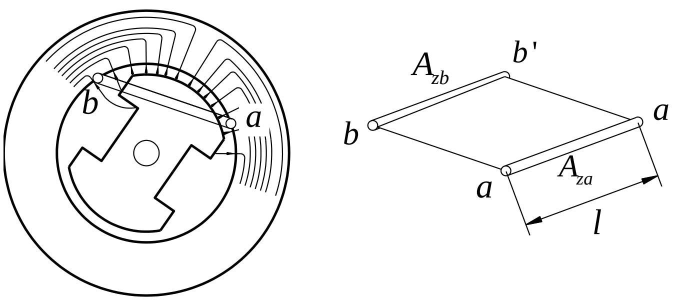

Fig. 1 Flux linking a stator coil of a salient pole machine.

Analogously, if the space distribution (the space wave) of the stator electric potential difference in axial direction, φ_z, is known (potential difference between the two ends of an axial conductor), the voltage of any stator phase K is obtained by simply adding up the electric potential difference values at all of the conductor positions:

$$u_K(t) = \sum_{y=a,b,c,d....} \pm \varphi_{z,y} \qquad (3)$$

Let us now reproduce one of Kron's basic ideas (see introduction to [25]): "The terminology and presentation of many engineers actually assumes that electricity may be transported across a network as if it were a package of merchandise.... Not one of the writers on the theory of such networks ever stops for a minute to ask the key question: In what truly basic respect does an electric network differ from the large variety of non-electric networks? Even a layman feels instinctively that transporting electric current across a network requires a different mechanism from transporting a package of butter across the same network¡ *An electric network differs from all other types of non-electric network in that an electric network is always surrounded by a dynamic electromagnetic field of its own creation*…" (underlined by Kron).

In keeping with this fundamental fact (which, surprisingly, many engineers tend to ignore completely), the SPhTh in this paper considers that a rotating electrical machine can be regarded as an electromechanical device that, when connected to an electric source, produces electromagnetic (field) waves with a restricted propagation capacity, namely they are forced to turn inside the air gap. These space waves, especially the scalar electric potential difference and the vector magnetic potential waves (and the linear current sheet wave) can be characterized very easily by space phasors (the **u**, **Ψ** and **i** phasors, respectively). These space phasors:

- enable to formulate the MM transient equations in a very simple manner, giving at the same time a very powerful and physical insight into the machine transient behaviour.

- make possible the fast decoupling of the MM into a set of equivalent three phase machines (chapter 6). This turns out to be extremely useful for developing the MM torque control schemes (chapter 7).

Thus, the SPhTh in this paper *fully rejects* space phasors to be mere abstract mathematical entities (an idea often found in papers on three phase machines, let alone in MM´s). Its starting point is just the opposite: space phasors are always to be introduced representing well defined machine internal (space) quantities. And what else these space quantities could be, if not Maxwell's field theory quantities? Furthermore, they must be the most significant and powerful ones. These requirements are fully met by **A** and φ which are, by far, the most important quantities in electromagnetics [26, 27].

According to the statement above, it can hardly surprise that with the help of **Ψ** (just the space phasor assigned to **A**), all of the modern control methods for d. c. and three phase machines (e. g. field oriented or direct torque control) can be deduced in a simple and systematic way, proving that all of them are particular variants of a very simple and much more general principle [28].

How to get an intuitive picture of **A** in simple cases is shown in a didactic manner, e.g., in [29]. Nevertheless, since the average electrical engineer is less familiar with flux linkages than with voltage, this last quantity will be used in the next chapter to introduce some previous concepts which are essential in the SPhTh.

3. DYNAMIC POLYPHASE SYSTEMS AND DYNAMIC TIME PHASORS OF SEQUENCE "g"

Since all the essential features of the SPhTh can be illustrated with systems having an odd number of phases, henceforth this number, m, will be assumed odd unless otherwise stated. Likewise, phase 1 axis will be assumed to be always placed on the real axis. Term γ stands for 2π / m.

Let it be a three phase symmetrical winding with arbitrary voltages, $u_1(t)$, $u_2(t)$ y $u_3(t)$, but without homopolar components. It must be possible to obtain these voltages by means of the projection of a "rotating vector"[2] (the voltage phasor $\mathbf{u_A}$) onto the phase axes, that is ($\Re e$ stands for "real part of"):

$$u_1(t) = \Re e\left[\overrightarrow{u_A}\right]; \quad u_2(t) = \Re e\left[\overrightarrow{u_A}\, e^{-j\gamma}\right] \qquad u_3(t) = \Re e\left[\overrightarrow{u_A}\, e^{-j2\gamma}\right] \tag{4}$$

Indeed, since the sum of the phase voltages is zero, there are only two independent equations in (4), which just determine magnitude and angle of $\mathbf{u_A}$. After a very simple calculation it follows

$$\overrightarrow{u_A} = u_A(t)\, e^{j\varepsilon(t)} = \frac{2}{3}\left[u_1(t) + u_2(t)\, e^{j\gamma} + u_3(t)\, e^{j2\gamma}\right] \tag{5}$$

Let it be now a five phase winding with arbitrary voltages, but again without homopolar components. Since there are four independent voltages,

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[2] The term "rotating vector" should be interpreted in this paper from a graphical viewpoint, as often done in the literature in similar cases. From a mathematical viewpoint, these "rotating vectors" (as well as time and space phasors) are complex time varying quantities.

it seems at first sight that the two independent voltage phasors

$$\overrightarrow{u_A} = u_A(t) e^{j\varepsilon_A(t)}; \quad \overrightarrow{u_B} = u_B(t) e^{j\varepsilon_B(t)} \qquad (6)$$

would suffice to determine the voltage of any phase (sum of the two phasor projections onto the phase). Yet, this way all the voltages would actually be derived from the projections of only one effective phasor (sum of $\mathbf{u_A}$ and $\mathbf{u_B}$). Therefore, and for the purpose of specifying the phase voltages, it is necessary to impose the additional condition that $\mathbf{u_A}$ and $\mathbf{u_B}$ also differ from one another as to the way in which their projections over the winding phases take place. This could be done in many ways. The most direct one is to image that the phase positions are exchanged in cyclic manner depending on the phasor considered (e.g. for $\mathbf{u_A}$ the phases are placed in their actual sequence, 1, 2, 3, 4, 5 whereas for $\mathbf{u_B}$ they are assumed[3] to be in the sequence 1, 3, 5, 2, 4. Mathematically we get:

$$\begin{aligned}
u_1(t) &= \Re e\left[\overrightarrow{u_A}\ e^{-j0}\right] + \Re e\left[\overrightarrow{u_B}\ e^{-j0}\right] \\
u_2(t) &= \Re e\left[\overrightarrow{u_A}\ e^{-j\cdot 1\,\gamma}\right] + \Re e\left[\overrightarrow{u_B}\ e^{-j\cdot 1\cdot 2\,\gamma}\right] \\
u_3(t) &= \Re e\left[\overrightarrow{u_A}\ e^{-j\cdot 2\,\gamma}\right] + \Re e\left[\overrightarrow{u_B}\, e^{-j\cdot 2\cdot 2\,\gamma}\right] \qquad (7) \\
u_4(t) &= \Re e\left[\overrightarrow{u_A}\, e^{-j\cdot 3\,\gamma}\right] + \Re e\left[\overrightarrow{u_B}\, e^{-j\,3\cdot 2\,\gamma}\right] \\
u_5(t) &= \Re e\left[\overrightarrow{u_A}\, e^{-j\,4\,\gamma}\right] + \Re e\left[\overrightarrow{u_B}\, e^{-j\,4\cdot 2\,\gamma}\right]
\end{aligned}$$

Solving (7) with the additional condition of no homopolar voltage components, a very simple calculation yields:

$$\begin{aligned}
\overrightarrow{u_A} &= u_A(t) e^{j\varepsilon_A(t)} = \frac{2}{5} \sum_{x=1}^{5} u_x(t) e^{\,j\,(x-1)\,\gamma} \\
\overrightarrow{u_B} &= u_B(t) e^{j\varepsilon_B(t)} = \frac{2}{5} \sum_{x=1}^{5} u_x(t) e^{\,j\,(x-1)\,2\,\gamma}
\end{aligned} \qquad (8)$$

If there is a homopolar component, $u_0(t)$, in the phase voltages, this value must be simply added to the right of the expressions in (7). Of course, $u_0(t)$ can also be obtained by the projections of a third phasor. The condition as to the phase sequence in this case reads obviously that all the phase axes coincide. The expression for this homopolar voltage phasor is:

$$\overrightarrow{u_0} = u_0(t) = \frac{1}{m} \sum_{x=1}^{m} u_x(t) \qquad (9)$$

$\mathbf{u_0}$, $\mathbf{u_A}$ and $\mathbf{u_B}$ are called voltage phasors of sequence 0, 1 and 2 respectively. It is important to keep in mind that, obviously, only phasors of the same sequence can be vectorially combined and added up. In other words trough the phasors of sequence 0, 1 and 2, the original five voltages system has been decomposed into three independent systems (originated by three independent phasors of different sequence).

Analogously, to characterize the arbitrary voltages of a seven phase winding, three independent voltage phasors, $\mathbf{u_A}$, $\mathbf{u_B}$ and $\mathbf{u_C}$ plus one homopolar phasorr, $\mathbf{u_0}$ (given by (9), with m = 7) are needed. The phase sequence for the phasor projections are (1, 2, 3, 4, 5, 6, 7), (1, 3, 5, 7, 2, 4, 6) and (1, 4, 7, 3, 6, 2, 5) for $\mathbf{u_A}$, $\mathbf{u_B}$ and $\mathbf{u_C}$, respec-

[3] Two other cyclic changes seem possible: instead of phase 3, we could place phase 4 or 5 behind phase 1. The first change leads to a phasor $\mathbf{u_B}$ conjugate of the $\mathbf{u_B}$ in (8) The second one simply corresponds to the inverse phase sequence 1, 5, 4, 3, 2, 1. So there is actually only one effective symmetrical and cyclic phases exchange. Analogous conclusions are obtained no matter the phases number.

tively. Extending (7) to the projections of the three phasors $\mathbf{u_A}$, $\mathbf{u_B}$ and $\mathbf{u_C}$ and solving the system, it quickly follows by analogy to (8):

$$\overrightarrow{u_A} = \frac{2}{7} \sum_{x=1}^{7} u_x(t) e^{j(x-1)\gamma} \quad \overrightarrow{u_B} = \frac{2}{7} \sum_{x=1}^{7} u_x(t) e^{j(x-1)2\gamma}$$
$$\overrightarrow{u_C} = \frac{2}{7} \sum_{x=1}^{7} u_x(t) e^{j(x-1)3\gamma} \qquad (10)$$

This process can be easily extended to symmetrical windings with an arbitrary number of phases, m. In that case (m – 1)/2 phasors of sequences "g" = 1, 2, … (m – 1)/2 plus a homopolar voltage phasor given by (9) are needed. The expression for the general phasor of sequence "g" is:

$$\left[\overrightarrow{u}\right]_g = \frac{2}{m} \sum_{x=1}^{m} u_x(t) e^{j(x-1)\, g\gamma} \qquad (11)$$

Next, the concept of a dynamic m–phase system (DmPhS) of sequence "g" will be introduced. By definition, the m quantities (m currents, m voltages, etc) of a m-phase symmetrical winding are said to constitute a DmPhS of sequence "g", if they meet the following equations

$$\begin{aligned} x_1(t) &= x(t)\cos[\varepsilon(t) - g\cdot 0\cdot \frac{2\pi}{m}] \\ x_2(t) &= x(t)\cos\left[\varepsilon(t) - g\cdot 1\cdot \frac{2\pi}{m}\right] \\ &\cdots\cdots\cdots\cdots \\ x_m(t) &= x(t)\cos\left[\varepsilon(t) - g\cdot (m-1)\cdot \frac{2\pi}{m}\right] \end{aligned} \qquad (12)$$

x(t) and ε(t) can be arbitrary time functions. For g = mq+1, mq+2, mq+3, etc., where q is any positive natural number, the values in (12) are the same than for g= 1, 2, 3 etc. respectively. For g = mq or zero the system is called the homopolar DmPhS.

A DmPhS of sequence "g" only has two independent variables, x(t) and ε(t). Thus, it can be fully characterized by a new mathematical tool that will be called in this paper "dynamic time phasor of a DmPhS of sequence g"

$$\left[\overrightarrow{X}\right]_g = \left[x(t)\; e^{j\,\varepsilon(t)}\right]_g \qquad (13)$$

Variables x(t) and ε(t) correspond with the instantaneous amplitude and position of the *dynamic phasor* in the complex plane. Notice that the quantity of any phase with its axis placed at the actual position "y" is obtained by simply projecting the phasor onto the axis that would belong to this phase after having performed with sequence "g" the cyclic exchange of phases above referred to. Mathematically:

$$x_y(t) = \Re e\left[\overrightarrow{X}\; e^{-j\,g\,(y-1)\frac{2\pi}{m}}\right] \qquad (14)$$

The sum of currents, voltages, etc. of several DmPhS´s, S1, S2, etc of the same sequence, "g", produces a DmPhS which also has the sequence "g". The dynamic phasor of the resultant DmPhS equals the vectorial sum of the dynamic phasors associated to S1, S2, etc. (This is by total analogy to the classical polyphase sinusoidal systems, which are a very simple family of DmPhS´s, the phasors of which turn at constant speed with constant amplitude). Notice that although functions x(t) and ε(t) in (12) may be arbitrary and very different for the different systems S1, S2, etc. ., *the statement above on the vectorial sum of dynamic phasors always holds*.

Sequences "g" and (m – g) are called complementary sequences. Any DmPhS of sequence "g" defined by its phasor (13) can be converted into a DmPhS of sequence (m – g) according to the formula (* stands for conjugate complex):

$$\left[\vec{X}\right]_g = \left[\vec{X}^*\right]_{m-g} \qquad (15)$$

Indeed, according to (12) the quantity in the general phase "y" for the DmPhS , X , in (13) , of sequence "g", is

$$x_y(t)\Big|_{X_g} = x(t)\cos\left[\varepsilon(t) - g\,(y-1)\cdot\frac{2\pi}{m}\right] \qquad (16)$$

and the quantity in the same phase "y" for the DmPhS, X^*, of sequence (m – g)

$$x_y(t)\Big|_{X^*_{m-g}} = x(t)\cos\left[-\varepsilon(t) - (m-g)\cdot(y-1)\cdot\frac{2\pi}{m}\right] =$$
$$= x(t)\cos\left[-\varepsilon(t) + g\,(y-1)\cdot\frac{2\pi}{m}\right] = x_y(t)\Big|_{X_g} \qquad (17)$$

These formulae show that DmPhS´s with equal or complementary sequences can be added up, and the result can be characterized by just one dynamic time phasor.

Notice on the other hand that the "voltage phasors of sequence g" previously introduced (see text after (9)) are simply dynamic time phasors of sequence "g".

In summary, in this section it has been shown how to decompose a m–phase system of arbitrary time quantities into its independent DmPhS´s (e. g. eq. (7) for m = 5) and how to calculate their corresponding dynamic time phasors (e. g., equations (8) and (9) for m = 5). *The dynamic time phasors in this section can be regarded as a powerful extension of the steady state time phasors.* Their amplitude and speed may vary following an arbitrary law. They incorporate the phase sequence concept and are very useful for dealing (in analytical and graphical manner) with *time quantities* of arbitrary evolution in multiphase windings. The suitable tools for dealing with machine *space quantities* of arbitrary evolution are introduced in the next section.

4. SPACE PHASOR CONCEPT. PHASORS U AND Ψ AND THEIR ASSOCIATED SPACE WAVES. CORRELATING PHASE QUANTITIES WITH AIRGAP WAVES

Electrical machines studies require operating with certain quantities which are spatially distributed (current sheet, induction, etc). *Dynamic space phasors* are very suitable to this task. By definition a dynamic space phasor is an oriented segment in the complex plane that symbolically represents the spatial sinusoidal distribution of an internal machine quantity. The phasor always points to the positive maximum of the wave (in the case of bipolar waves) and its modulus is equal to the wave's amplitude. Both wave amplitude and speed may vary in an arbitrary manner

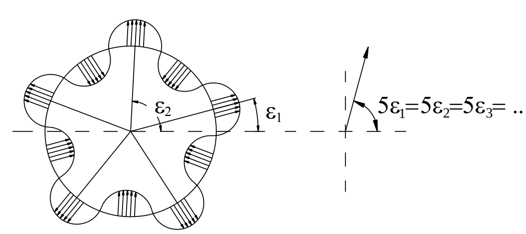

Fig. 2 Space wave of five pole pairs in the machine domain (left) and its corresponding space phasor in the phasorial domain (right).

Usually the internal quantity is not sinusoidal. Then a harmonic space phasor is assigned to each space harmonic of its Fourier expansion. To this end a domain transformation is defined in such a manner that any angle, α, in the machine domain becomes an angle να (ν = absolute harmonic order) in its corresponding phasorial domain. Notice that in this way every multipolar wave is characterized by one only space phasor (the same coordinate in the *phasorial domain* corresponds to all its positive crests in the machine. Fig. 2). Since all the harmonic space waves become bipolar waves in their corresponding phasorial domains, this transformation (which boils down to transform the mechanical angles into electrical ones) turns out to be very useful, not only for the mathematical treatment, but also for the physical and graphical interpretation of later equations.

Let it be a sinusoidal wave with hp pole pairs distributed over the stator cylindrical surface of a salient pole machine (for instance, an induction wave, a wave of electrical potential difference in axial direction, etc.). If $x_{hp}(t)$ is the instantaneous amplitude of the space wave and ε(t) defines any of its instantaneous positive crests in the machine, the wave expression becomes:

$$x_{hp}(\alpha,t) = x_{hp}(t)\cos\left\{hp\left[\varepsilon(t)-\alpha\right]\right\} \qquad (18)$$

On the other hand this wave, as just stated, is fully characterized in its phasorial domain by its space phasor, whose most general expression is:

$$\overrightarrow{X_{hp}} = x_{hp}(t)\,e^{jhp\,\varepsilon(t)} \qquad (19)$$

From (19) and (18) it follows immediately

$$x_{hp}(\alpha,t) = \Re e\left(\overrightarrow{X_{hp}}\; e^{-jph\alpha}\right) \qquad (20)$$

that is, the quantity (e. g. induction, electric potential difference in axial direction, etc.) at any point (or at any axial straight line) of the stator surface specified by the coordinate α in the machine domain is given by the projection of the space phasor onto the straight line defined by hpα in the phasorial domain (Fig. 3)

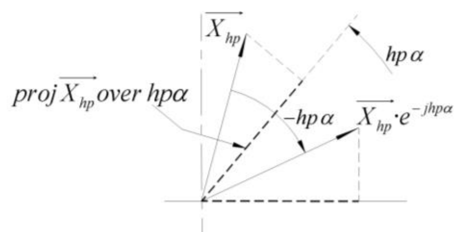

Fig. 3 Geometric Interpretation of eq. (20)

Therefore the voltage of a stator phase k due exclusively to one electric potential wave with 2hp poles is equal to the sum of the projections of the wave space phasor, u_{hp}, onto the Z_k phase conductors translated into the phasorial domain. This sum becomes (see appendix A):

$$\left[u(t)_{phase\ k}\right]_{wave\ hp} = Z_k\,\Re e\left(\overrightarrow{u_{hp}}\;\overline{\xi_{hp,k}}^{\,*}\right) \qquad (21)$$

where complex winding factor, $\overline{\xi_{hp,k}}$, in (21) is a complex constant whose modulus equals the one of the classic winding factor for the harmonic of relative order h. (More details in Appendix A).

In summary, (21) states that the k-phase instantaneous voltage due exclusively to the wave with hp pole pairs *is simply the projection of the wave phasor onto the corresponding phasorial axis of phase* k (multiplied by a constant).

Let it be a 2p poles salient pole machine with a m-phase symmetrical winding on the stator. Let us choose as abscise axis the symmetry axis of first phase, and define $\xi_{h,wndg}$ as the winding factor of relative order *h* of this first phase. Due to winding symmetry, the phase in the generic position "*k*" reproduces the configuration of phase 1 but with an angular displacement in the machine $2\pi(k\text{-}1)/(p\cdot m)$.Therefore its complex winding factor in the phasorial domain becomes:

$$\left(\overline{\xi_{hp}}\right)_{phase\ k} = \left(\overline{\xi_{hp}}\right)_{phase1} e^{j\,h\,p\frac{2\pi(k-1)}{p\cdot m}} = \xi_{h,wndg}\, e^{j\,h(k-1)\frac{2\pi}{m}} \tag{22}$$

By simply applying (21) und (22) it follows that the instantaneous phase voltages of a m–phase stator winding due exclusively to the axial electric potential difference space wave with hp pole pairs characterized by its phasor (19) become:

$$\left[u(t)_{phas1}\right]_{wave\ hp} = Z\xi_{h,wnd}u_{hp}(t)\cos[h\,p\,\varepsilon(t)]$$

$$\left[u(t)_{fase2}\right]_{wave\ hp} = Z\xi_{h,wnd}u_{hp}(t)\cos\left[h\,p\,\varepsilon(t) - h\cdot 1\cdot\frac{2\pi}{m}\right]$$

....

$$\left[u(t)_{fase\,m}\right]_{wave\ hp} = Z\xi_{h,wnd}u_{hp}(t)\cos\left[h\,p\,\varepsilon(t) - h(m-1)\frac{2\pi}{m}\right] \tag{23}$$

For instance, let it be a seven phase (m = 7) winding, and assume the general case of space waves with arbitrary changes in their amplitude and speed. According to (23), each of the waves with p, 8p, 15p, etc. pole pairs (that is, each of the harmonic waves of relative order h = 7q + 1 with q = 0, 1, 2, 3, etc.) produces a voltage DmPhS of sequence 1. Likewise, each of the waves of order h = 7q + 2; h = 7q +3, etc. produces a voltage DmPhS of sequence 2, 3, etc, respectively.

Therefore, for a 7-phase stator winding all of the electric potential waves in axial direction at the stator surface can be classified into seven families. The instantaneous amplitudes and positions of the different waves belonging to the same family are, in the general case, quite different from one another, and so are as well their corresponding space phasors. However, all of them originate DmPhS´s of the same sequence which can be added up to give a resultant DmPhS of this same sequence. Thus*, the combined action of all of the waves of a family on the phase voltages can be characterized by just one equivalent or effective voltage space phasor. Or alternatively: the resultant voltage DmPhS produced by a whole family of waves can be imagined to be produced by one only effective or equivalent space wave, the phasor of which is just the effective phasor of the waves family*.

Moreover, the two voltage DmPhS´s produced by the families with h = 7q +1 and h = 7q +6 are DmPhS´s of complementary sequence. Therefore, they can be added up so that just one space phasor suffices to characterize the combined action of both families. The same statement applies to the families with h = 7q +2 and h = 7q + 5 as well as to the families with h = 7q + 3 and h = 7q + 4. Thus, the combined effect of all of the electrical potential waves on the voltages of the seven phases is fully characterized by just four effective voltage space phasors.

This reasoning can be extended to symmetrical windings with arbitrary phases number. For instance, in order to obtain the voltages of a five phase winding *due to all* the machine space waves three effective voltage space phasors (one of which being the effective homopolar space phasor) suffice in the most general case.

As just seen, the *set of effective voltage space phasors determines the instantaneous voltage of any winding phase in a very simple way: by projecting the phasors onto the phase axis and summing up the projections*. The inverse problem (determining the effective voltage space phasors out of the instantaneous phase voltages) is quite simple too. Notice that each one of these space phasors produces one of the independent DmPhS´s into which the voltage system of the polyphase winding can be decomposed (simply compare equations (23) and (12)). *Therefore, from a merely mathematical viewpoint, obtaining the effective space phasors out of the phase voltages boils down to get the dynamic time phasors which define the polyphase voltage system*. This problem has already been solved in section 3 (see especially its final remarks). Anyway, it is useful an additional comment on the physical meaning of the solution obtained this way by means of an example.

Let it be again a salient pole machine with, for instance, a five phase stator winding without homopolar components. In this case, two effective stator voltage space phasors, $\mathbf{u_A}$ and $\mathbf{u_B}$ must be computed. Their expressions are given in (8). Phasors $\mathbf{u_A}$ and $\mathbf{u_B}$ symbolize (at a given scale) two effective stator electrical potential difference space waves, with different pole pair number, p and 3p, respectively. (More precisely: these two effective phasors characterize the total action on the phase voltages of two different sets of actual stator electric potential difference space waves. The first phasor refers to the families with waves of relative order h = 5q +1 and h = 5q+4. The second one to the families with h = 5q+3 and h=5q+2). Amplitude and speed of both effective space waves vary in due manner to give the phase voltages.

In summary, for an m-phase stator symmetrical winding, the numerous and actual stator electric potential difference space waves which appear at the stator cylindrical surface can be classified, in the most general case, into m families. For m odd, (m – 1)/2 of these families have a complementary family. Yet, the combined action on the stator phases voltage of all the waves of two complementary families can be accounted for by just one effective stator voltage space phasor. Thus *(m – 1)/2 effective independent space phasors (plus one homopolar phasor) suffice to characterize the action of all of the stator electric po-*

tential waves. These phasors fully define the instantaneous phase voltages and, conversely, according to (11), they are determined out of the stator phase voltages as follows:

$$\overrightarrow{u_{familes\ g,\ (m-g)}} = \frac{2}{m} \sum_{x=1}^{m} u_x(t) e^{j(x-1)\ g\ \frac{2\pi}{m}} \quad (24)$$

The statement above as well as the corresponding equation (24) hold no matter the rotor shape or whether the magnetic circuit be saturated or not.

Analogous statements apply to the magnetic vector potential waves over the cylindrical stator surface. Again, the combined action on the stator phase flux linkages of all of the magnetic potential space waves of two complementary families can be accounted for by just one effective stator magnetic vector potential space phasor, Ψ_{families}, Its expression can be obtained out of the flux linkages of the stator phases, no matter the rotor shape or whether the machine be saturated or not. Operating as done with the electric potential waves, it follows, by analogy with (24),

$$\overrightarrow{\Psi_{families,\ g,\ m-g}} = \frac{2}{m} \sum_{x=1}^{m} \Psi_x(t) e^{j(x-1) g \frac{2\pi}{m}} \qquad (25)$$

It is convenient to insist on the physical content of (23). It states that, given a m-phase winding, any electric (or vector magnetic) potential space wave with hp pole pairs *always produces in the winding a voltage (or flux linkage) DmPhS of a sequence "g", no matter the changes in the wave amplitude and speed.* Sequence "*g*" only depends on the wave poles number. Waves with different poles number but belonging to the same family produce DmPhS´s of the same sequence.

It is worth mentioning that even in the simple case of three phase machines, and even in good books (e.g. [30], p. 286 or [31], p. 21]) it has been explicitly stated that phasors **u** and **Ψ** lack a physical interpretation. Yet assigning a physical meaning to the **u** phasor was already done in [32], where the **u** phasor definition was carried out from a field perspective (axial electric potential difference) and a circuit perspective (average value of conductors voltages).The second definition was later used to also introduce the **Ψ** phasor and, unfortunately, it was the one almost exclusively used by this author in his later publications on the SPhTh (e. g. [33,34]). The procedure and definitions in those publications are in itself legitimate and correct, but they are useful for transients analysis only if the space harmonics are neglected, otherwise, the expressions for **u** and **Ψ** become too complex, as explicitly acknowledged in [33,34]. Therefore, those space phasor definitions and their corresponding developments in previous author's publications which do not coincide with the ones given here or in [35] are to be replaced in due manner. In other words, the "field perspective" in [32] is the truly suitable one and the one to be chosen for introducing the space phasors definitions and formulae, as done and explained in detail in this paper.

Formulae (24) and (25) coincide with the so called *instantaneous symmetrical components (ISC)* of voltages and flux linkages. Although these ISC have been profusely used all over the world for more than half a century, it has been impossible so far to attach a clear physical interpretation to them. They keep on being introduced directly as a mere (and fortunate) transformation of variables that simplify the solution of the system equations. Yet, according to previous paragraphs, they have a deep and double physical meaning: effective voltage and flux linkage dynamic space phasors associated to the different (and independent) space waves families (first interpretation. Machine viewpoint). Dynamic (and decoupled) time phasors which fully describe (by simple projection) the time evolution, in the most general case, of phase voltages and flux linkages of a m-phase symmetrical winding (second interpretation. Circuits viewpoint. See eq. (11)).

5. CURRENT SPACE PHASORS AND THEIR ASSOCIATED SPACE WAVES

Any phase A placed in the cylindrical stator (D = inner diameter) of an electrical machine produces a current sheet space wave that can be split into space harmonics. If $i_A(t)$ is the phase current, the space phasor, **a**, that characterizes the harmonic of relative order *h* of the current sheet wave produced by the phase A is [35]

$$\overrightarrow{a_{h\cdot p,A}} = \frac{2Z_A}{\pi D} i_A(t)\, \overline{\xi_{h\cdot p,A}} \quad (26)$$

The current sheet space phasor of a polyphase winding is the sum of the space phasors of all of its phases Thus the space harmonic of relative order h of the current sheet wave produced by a m-phase symmetric winding fed by arbitrary currents can be represented by its space phasor, whose expression, deduced immediately from (26) and (22), is:

$$\overrightarrow{a_{hp,wnd}} = \sum_{phases} \overrightarrow{a_{hp}} = \frac{2Z}{\pi D} \overline{\xi_{hp,wnd}} \sum_{k=1}^{m} i_k(t)\, e^{jh(k-1)\frac{2\pi}{m}} \quad (27)$$

Instead of the current sheet space phasor it is often advantageous to use the so called current space phasor, defined as:

$$\overrightarrow{i_{hp,wndg}} = \frac{\overline{\xi_{hp,wndg}}}{\left|\xi_{hp,wndg}\right|} \frac{2}{m} \sum_{k=1}^{m} i_k(t)\, e^{jh(k-1)\frac{2\pi}{m}} \quad (28)$$

Notice that both phasors only differ by a constant and therefore they represent (at different scales) the same space quantity. For h = 1 (fundamental wave) and m=3 the current space phasor in (28) results in the Park´s current vector.

As readily expected, like the **u** phasor (electric potential) and the **Ψ** phasor (magnetic vector potential), also the **i** phasor has a direct relationship with one of the Maxwell´s field theory quantities. The current sheet (**i** phasor) at any point of the stator surface is equal to the tangential component of the magnetic field intensity, H_t, at this point, assuming the classical hypothesis of μ_{Fe} to be infinite (e. g., [30, p. 342]) Furthermore, the vectorial product of the space phasors of voltage (related with the axial electric field) and current (related with the tangential magnetic field) is directly related with the well known Poynting vector, **S** = **E** x **H**. Starting from **S** the different electrical machines and their main formulae can also be deduced, as done in a systematic and beautiful manner in [36]. This allows a deep physical insight into the machine behaviour from a perspective quite different from the usual one.

In view of the relationships between DmPhS´s and waves families for phasors **u** and **Ψ**, it is only logical to expect that a current DmPhS applied to a symmetrical m–phase winding will produce current sheet waves belonging to specific families. And this is just what happens: if the sequence of the current DmPhS is "g", all the waves are of order h = qm ± g, (See appendix B). For instance, a current DmPhS of g = 2 applied to a seven phase winding *only* produces space waves of relative order h = 2, 5, 9, 12 etc.

Finally, notice that, like equations (24) and (25), equation (28) always holds too, no matter the rotor shape or whether the magnetic circuit be saturated or not.

6. DECOUPLING A CONSTANT AIR GAP MULTIPHASE MACHINE INTO INDEPENDENT MACHINES. THEORETICAL PROOF AND VALIDATION THROUGH SIMULATIONS

This section refers to the following constant airgap MM´s: induction machines (IM), doubly fed asynchronous machines (DFAM) and permanent magnet synchronous machines (PMSM) with a stator winding constituted by m phases symmetrically distributed.

The hard core of the problem of establishing the machine equations actually lies on developing the relationships between current sheet and flux linkage space phasors (in other words, between currents and fluxes).

Contrary to equations (24), (25) and (27), of general validity, the relationship between current sheet and magnetic vector potential space waves (or flux linkage waves, Ψ) changes for every machine type and, for some machines, it turns out to be very complex, even if the phase leakage flux is considered (as will be done here too) proportional to the phase current. However, the solution is simple for those machines in which the two classic hypotheses of ideal magnetic circuit and constant air gap of negligible width can be assumed. In such a case, *any sinusoidal current sheet wave results in just one flux linkage wave (or in just one airgap induction wave) with the same pole number* (there are no saturation or reluctance induction waves). The relationship between these current sheet and induction waves is very simple: their amplitudes are proportional and the induction wave always lies 90 degrees apart from its exciting current sheet wave in the phasorial domain. These space waves relationships are very simple to write down by means of space phasors, no matter the machine harmonics and phases number, as has been shown in detail in [35].

Yet, it is not at all necessary to develop the machine equations in order *to prove in a precise and rigorous way that the MM can be decoupled into a set of independent machines. This will be done in the next paragraphs*.

Let it be an ideal IM with an arbitrary phase number, m, in stator and rotor, and let us apply to the stator an arbitrary current DmPhS, S_i, of sequence g. Assume first that the rotor circuit is open. S_i produces a current sheet waves family with h_{est} = qm ± g (see Appendix B), which in turn, as above commented, produces a homologous family (the same h values) of stator and of air gap vector potential space waves. This last waves family generates a stator common flux linkages DmPhS of sequence g. Therefore, the electromotive forces (EMF´s) induced in the stator phases constitute at any moment a DmPhS of sequence g too. And the m-phase winding voltages system, S_V (sum of the EMF´s and the phase impedance voltage drops systems) also constitutes a DmPhS of sequence g. *In summary, a current DmPhS of sequence g can only produce flux linkages and voltages DmPhS´s of the same sequence.* (Of course, if instead S_i the input is the voltage system S_V, the output will be then the current system S_i).

Assume the rotor winding be closed now (It should be underlined that *in the much more simple case of a PMSM this second step of the analysis is unnecessary*). At the first moment, the air gap flux linkage space waves family (due to the stator currents) induces in the rotor a EMF DmPhS of sequence g, which in turn, as there are no external rotor voltages[4], produces a rotor current DmPhS of sequence g too. This happens no matter the changes in the amplitude and speed of **Ψ** with respect to the rotor (see comments after eq (25)). The rotor current DmPhS produces (see again appendix B) a family of rotor current sheet waves with h_{rot} = qm ± g (Notice that the general rotor family term h_{rot} coincides with the one of the stator family, h_{str}). Each pair of stator and rotor current sheet waves of the same order (or their phasors expressed in a common reference frame) combine to give at any moment a machine resultant wave of the same order, so that we get a machine resultant waves family with h_{mach} = qm ± g. This resultant current sheet waves family originates a homologous $\mathbf{\Psi}_{airgap}$ waves family which induces a EMF DmPhS of sequence g in stator and another in the rotor. This modifies the stator and rotor currents in the next moment, but, obviously, they keep on being two DmPhS´s of sequence g. Thus, they produce two homologous families of current sheet waves with h = qm ± g, which combine together, and the process repeats again. In summary, *in constant airgap symmetrical MM´s with equal stator and rotor phases number, two arbitrary stator and rotor voltage DmPhS´s of same sequence, g, produce at any moment only stator and rotor current DmPhS´s of sequence g*

On the other hand it is well known that only the interaction of stator and rotor space waves with the same pole number can produce torque. This

[4] Of course, the reasoning in the text also applies to a doubly fed asyncronous machine provided that the external rotor voltages system be an arbitary *DmPhS*, $S_{V,rot}$ of the same sequence, *g*.

means that only stator and rotor voltage DmPhS´s of the same sequence can produce torque (h_{est} = h_{rot} only in families associated to DmPhS´s of the same sequence).

Therefore, each one of the two arbitrary stator and rotor voltage systems of the MM analyzed above has to be expanded into (m + 1)/2 independent DmPhS´s (m odd), one of which being the homopolar system (g = 0). *The MM can be regarded as a set of (m + 1)/2 mechanically coupled but electrically independent machines with the same windings*. Stator and rotor of machines 0, 1, 2 etc. are supplied by the (stator and rotor) voltage DmPhS´s of sequence g = 0, 1, 2, etc respectively. The MM torque (or current) equal the sum of the torques (or currents) produced by all of the independent machines. Notice that this statement applies to those IM´s or DFAM´s in which m_{str} = m_{rot}, as well as to the more simple case of the PMSM.

The above conclusions have been confirmed by numerous simulations carried out with the space phasor model developed in detail in [35]. The model applies to machines with any stator and rotor phase number, fed by arbitrary voltage waveforms and taking into account the space harmonics. The author would like to emphasize that very much attention was paid to a reliable model validation by comparing in [35] torque simulations with direct torque measurements, that is, with precise measurements of mechanical (not electrical) quantities performed in a squirrel cage motor. In this sense, the great difficulties of measuring with accuracy pulsating electromagnetic torques of several hundreds of hertz were brought into light, the possible measurement techniques were critically reviewed and the solution chosen was extensively discussed. As far as the author knows, it is the first time in the literature that *transient torques of hundreds of hertz* due to winding space harmonics have been directly measured and compared with model simulations.

Figures 4 to 7 show, by way of example, the simulations results of a direct on line no load starting-up of an induction motor, “a”, with seven phases in stator and rotor. The simulations were performed applying the model in [35] first to motor “a” fed with the stator voltage system S1 defined as

$$u_{ax}(t) = 300\cos\left[\omega t - (x-1)2\pi/7\right] + 200\cos\left[3\omega t - 3(x-1)2\pi/7\right]$$
$$+100\cos\left[5\omega t - 5(x-1)2\pi/7\right] = u_{bx}(t) + u_{cx}(t) + u_{dx}(t) \qquad (29)$$

where $u_{ax}(t)$ stands for voltage at phase “x” (x = 1 to 7) of machine “a”. Thereafter the model was applied to three machines (b, c and d) mechanically coupled, all of them equal to machine “a”, and fed with the three voltage DmPhS´s $u_{bx}(t)$, $u_{cx}(t)$, and $u_{dx}(t)$ obtained from S1 in (29).

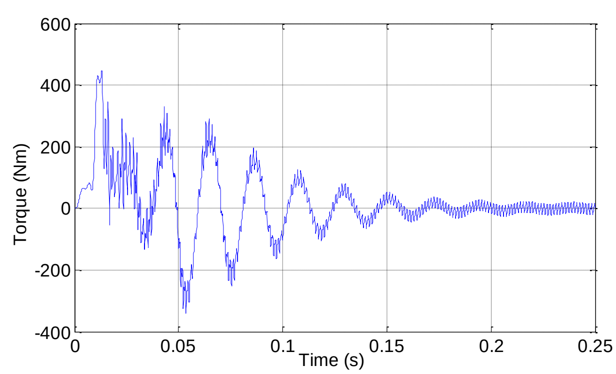

Fig. 4. Machine “a” no load starting-up torque. Main machine data R_{str}=0,41 Ω; L_{str}= 100 mH; $L_{\sigma\ str}$= 2,5 mH; R_{rot}= 0,28mΩ; L_{rot}= 14 μH; $L_{\sigma\ rot}$= 1,12 μH; J = 0.03 Kgm^2; h_{max}= 25. Full pitch stator and rotor coils placed in 56 and 28 slots

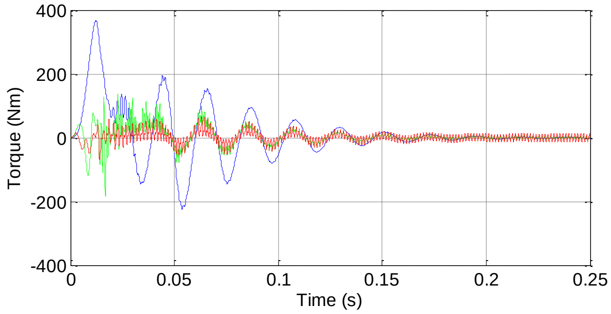

Fig 5 Torques of machines “b” (blue), “c” (green) and “d” (red). Remaining data as in Fig. 4.

The actual data in figures 4 and 5 confirm that T_a is exactly equal to T_b+ T_c +T_d at any moment. Analogous statement holds for the currents in figures 6 and 7. These results were observed in all simulations, no matter voltage input, load value, space harmonics number or machine parameters, as predicted by the theory.

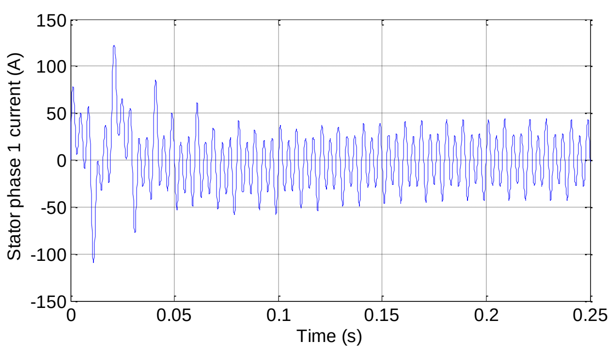

Fig. 6. Machine “a” stator phase 1 current during no load starting-up. Remaining data as in Fig. 4.

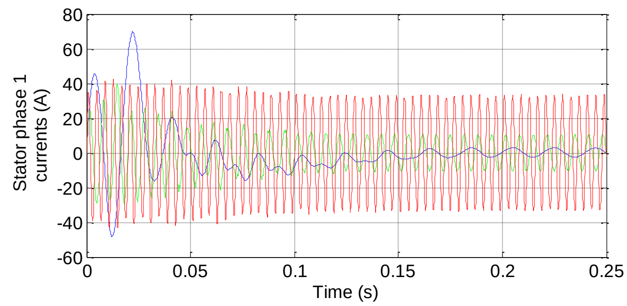

Fig. 7 Stator phase 1 currents in machines “b” (blue), “c” (green) and “d” (red). Remaining data as in Fig. 4.

It should be added that the authors in [37] have dealt with the particular case of PMSM following a completely different approach. Resorting, to different analysis tools (endomorphism, orthonormal base of eigenvectors, etc) they arrive at decoupling laws for PMSM fully equivalent to the results in this paper.

If m_{str} and m_{rot} are not equal in the IM or in the DFAM, then the exact decoupling of the MM in the most general case is not possible, as also confirmed, e. g., by direct on line starting up simulations (Physical explanation: the field harmonics belonging to one only stator waves family have their rotor counterparts distributed among several rotor families; and conversely).Yet, even in these machines, if they are fed through a converter, the decoupling above referred to keep on being valid with small or negligible error, as shown in the next section.

7. FAST TORQUE CONTROL OF MULTI-PHASE MACHINES

The general strategy (GAFTOC principle) to get a very fast machine torque control is based on keeping the pulsational EMF´s of all of its phases null during transients and achieving the torque changes by exclusively enhancing the rotational EMF´s [28]. Three phase stator or rotor windings without homopolar currents produce only one set of flux linkages space waves of general order h = 3q±1 (that is, *only one* $\boldsymbol{\Psi_{str}}$ *and one* $\boldsymbol{\Psi_{rot}}$*. phasors*). Therefore, keeping the pulsational EMF´s null in *all* stator or rotor phases is achieved in a three phase machine by simply keeping constant the magnitude of its $\mathbf{\Psi_{str}}$ or $\mathbf{\Psi_{rot}}$ phasor (See Fig 2 in [28]). Field orientation control (FOC) and direct torque control are simply two particular methods of implementing the GAFTOC principle.

It has also been shown in the previous section that a constant airgap MM with m stator and rotor phases can be split up into a machine with homopolar voltages (always to be avoided for well known reasons) plus (m-1)/2 independent machines. *Each of these independent machines has only one* $\boldsymbol{\Psi_{str}}$ *and one* $\boldsymbol{\Psi_{rot}}$*, space phasor which fully define the flux linkages of any phase,* just as happens in three phase machines without homopolar components. Thus, the MM (more concretely: PMSM, IM and DFAM) has been decoupled into a set of equivalent and electrically independent three phase machines which are mechanically coupled.

Moreover, in converter-fed machines it is not necessary to take into account all the members of each waves family. Indeed, it has been theoretically justified as well as confirmed by numerous simulations in [35] that the field harmonics effect on the transient torque of three phase induction motors is always negligible *at low slips*. Obviously this conclusion can be extended to the different and independent waves families of converter fed MM´s (small slips, even in transients). This means that in these MM´s only the space harmonics which are head members of their groups (that is, the harmonics with the lowest pole number) need to be considered as to the torque, since they alone are able to provide a non negligible torque contribution [35].

Certainly, if $m_{str} \neq m_{rot}$ in the IM or the DFAM, the exact decoupling of the MM is not possible. However, as to the torque control, this fact is scarcely relevant, since only the head members of the families should be considered. In fact, although often ignored, this simplified assumption underlies all the control schemes in the literature on converter-fed three phase induction machines, where only the fundamental stator and rotor waves are considered as to the torque production. (Notice that the three phase stator winding produces just one family of, e. g., Ψ waves. However, the multiphase squirrel cage produces several families. Yet, only the head members of the stator family and of the rotor main family are taken into account in the torque equation). In other words, if only the head members of each waves family are to be considered (converter fed MM´s) then the effective decoupling of the MM still holds, no matter m_{str} and m_{rot} be equal or not.

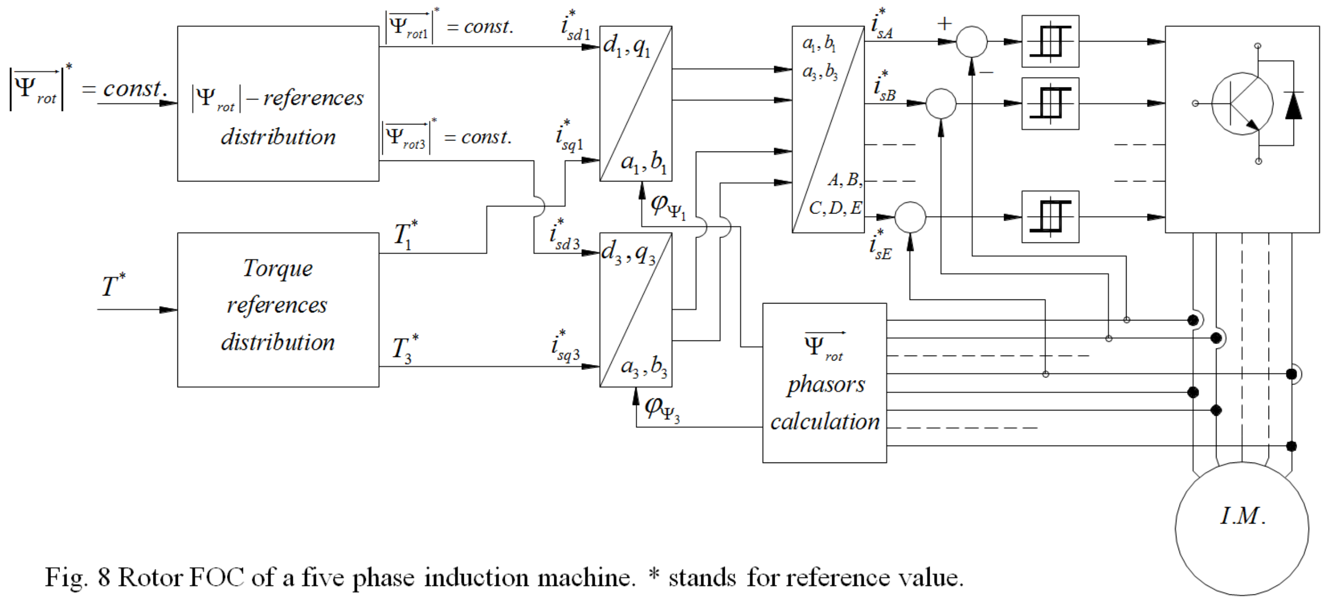

Fig. 8 Rotor FOC of a five phase induction machine. * stands for reference value.

Thus, establishing the scheme for a very fast torque control of the MM´s of type IM, DFAM and PMSM boils down to applying the GAFTOC principle *to each one of* the *independent machines* into which the MM can be split up. In other words, any of the schemes in [28] for three phase machines can be extended directly to their homologous MM´s. By way of example, Fig. 8 (a mere extension of Fig. 3 in [28]) shows the rotor FOC of a five phase induction motor.

The scheme in Fig.8 is associated to two fictitious independent machines each of them with one only $\mathbf{\Psi}_{\mathbf{rot}}$ phasor. Stator current and voltages of both machines are known (ISC of the actual five phase machine), as well as their electric parameters and rotor positions (equal to the ones of the actual machine). On the other hand, obtaining in a three phase machine its $\mathbf{\Psi}_{\mathbf{rot}}$ phasor (actually only its angle φ_Ψ with respect to the stator is needed) from rotor position, stator currents and machine parameters is a classical problem solved a long time ago. In this sense, the block in Fig. 8 “$\mathbf{\Psi}_{\mathbf{rot}}$ phasors calculation” represents the set of operations to get the $\mathbf{\Psi}_{\mathbf{rot}}$ phasor of each one of the two fictitious machines (its counterpart in Fig. 3 of [28] must calculate only one $\mathbf{\Psi}_{\mathbf{rot}}$ phasor, of course). The block “Torque references distribution” simply specifies the constant percentages of the total torque which must be provided by the fundamental and the third space harmonics of the actual machine (These harmonics are the head members of the two space waves groups). An analogous statement holds for the block “Flux linkages references distribution”.

It is useful to make some additional comments on how the system in Fig. 8 actually works. The machine space waves are characterized by space phasors in cartesian or polar coordinates (e.g., **i** phasors represent current sheet waves).The main waves, by far, are the $\mathbf{\Psi}$ waves, which are forced by the control structure to keep their amplitudes constant (*this way, only rotational EMF´s are induced in the windings. GAFTOC principle*).This task is particularly simple if operating with the phasors in reference frames tied to the **Ψ** waves. Naturally, the changes in the *space* waves result in changes in their associated phase *time* waves (phase quantities). The values of these phase quantities u(t), Ψ(t) etc. are obtained by simply projecting (in their phasorial domains) the set of **u**, **Ψ** etc space phasors onto the phase axis and summing up the projections, as proven in chapter 4. This is just what is done by the operations in block (a_1 b_1 a_3 b_3) → (A B C D E) which are presented in the literature as abstract matrix transformations. Actually they simply represent the way in which the machine internal quantities are correlated with its external ones. The drawback in the MCCA of lacking tools to perform this correlation in a precise and simple way was underlined at the beginning of the introduction section. This correlation is no problem in the SPhTh (sum of phasors projections) so that one can go from space to time quantities and conversely at any time.

Finally, notice that, as already said, Fig. 8 (an extension of the first control scheme in [28]) *is just one example. All the other three phase control schemes in [28] for PMSM, IM and DFAM can be extended in a similar manner to MM´s.*

8. Conclusions

Kapp´s time phasors refer to sinusoidal *time waves* and are only valid for *steady* states. For *transient* states analysis Dreyfus already proposed in [3] to use m.m.f. *space waves.* Yet, his method could hardly compete with Park´s one.

The later SVTh by Kovacs and Racz [6], profusely used in Central Europe in the second half of the past century, can be regarded as a second and more vigorous attempt to place the machine space waves at the central point of the study. Unfortunately, the SVTh in [6] is restricted to three phase machines without field harmonics and has no physical interpretation for the essential **u**, and **Ψ** phasors.

This paper is fully pervaded by the following driving idea: for analysing transients in electrical machines and developing their control schemes it is highly advantageous to connect again with the old *"wave approach"*; that is, *to connect with the way of thinking and the mental stream of analysis* that flowing through the SVTh goes back to Dreyfuss´s work (and even to Kapp´s phasor concept); a mental stream (developed specially in Central Europe) that, as to transient studies, has been almost completely blocked in the last 30 years by the undeniable power and success of the MCCA. Yet, to overcome this block, a starting point and procedure clearly different from the ones in [6] are necessary, namely one has to go resolutely and directly to the physical roots of the machine behaviour. And, as in any electric system, these roots, ultimately, always are Maxwell´s field theory quantities (see the, in author's opinion, excellent book [26]). To put it concretely: by contrast to the MCCA (machine as a network), the SPhTh in this paper states that a machine can also be regarded as an electromechanical device which produces electromagnetic waves inside its air gap. These space waves, especially the fundamental *u*, *Ψ* and *i* waves, can be characterized very easily by space phasors which are to be taken as essential tools of the theory. Moreover, they also can be correlated easily with the phase time quantities.

With regard to this second point, since a space wave, no matter the changes in its speed and amplitude, always produces a DmPhS of a fixed sequence, it follows that also DmPhSs should become key elements of the SPhTh. To operate with them a new tool called “dynamic time phasor of a DmPhS of sequence *g*” has been introduced in this paper. *It can be regarded as a powerful extension of Kapp´s phasor*. Dynamic time and space phasors, just as Kapp´s phasors, can be dealt with mathematically (complex time varying quantities) and graphically (vectorial addition) and also enable drawing dynamic time and space diagrams. This way the SPhTh extends to transients problems a method of analysis and graphical representation very familiar to any electrical engineer. In this sense, space phasors are much more intuitive than abstract matrix transformations. And, conceptually, more suitable. For instance, the statements often found that time quantities like currents, flux linkages, etc. are subjected to transformations of the coordinates axes can hardly be accepted from a correct physical viewpoint (only space quantities can be subjected to space transformations). It is dubious that such statements can lead to a clear insight into the machine dynamics and control. Yet, in view of the increasing use of converter-fed MM´s, an accurate, physical and graphical description of these processes becomes a must.

Each one of the different **u** and **Ψ** dynamic space phasors existing in the MM is related to one specific and independent group of space harmonics. Likewise, a current DmPhS of sequence g (characterized by its dynamic time phasor) produces only current sheet space waves of relative order h = qm ± g. These facts have been extensively discussed in the paper, since they constitute the *hard core of the MM decoupling*.

Thereafter, making use of these two analysis tools (*dynamic space phasor in the hp-phasorial domain; dynamic time phasor of a a DmPhS of sequence “g”*) it is theoretically proven that constant airgap MM´s (either PMSM or IM and DFAM with $m_{str} = m_{rot}$) can always be decoupled (m odd) into one machine with homopolar components plus a set of (m-1)/2 independent and equivalent three phase machines without homopolar components (This MM decoupling had been previously proven only for the particular case of PMSM). The theoretical conclusions above have been also confirmed by numerous simulations with the model in [35] (Notice that whereas the **u** and **Ψ** formulae and physical interpretation for the general case are given in this paper, the whole set of machine equations for the *particular* case of constant airgap machines has been deduced in detail in [35]). For converter fed MM´s the decoupling holds too, with small or negligible error, even if $m_{str} \neq m_{rot}$. This enables establishing directly the schemes for a very fast torque control of the MM´s, since they boil down to a mere extension of those already known for their homologous three phase counterparts. Moreover, *it is shown that the GAFTOC principle is the truly driving idea behind all of these schemes.* This fact, combined with the space wave approach, allows explaining in a rigorous, and at the same time very physical and didactic manner, how the whole MM control system actually works.

Finally, this paper gives for the first time *the interpretation of the* ***u*** *and* ***Ψ*** *space phasors from a space wave perspective*. It also gives the double and deep physical meaning of *the u and Ψ instantaneous symmetrical components* in polyphase machines and windings.

Appendix

A. Deduction of eq (21)

Let it be a phase A with Z_{aA} conductors placed at (the straight line defined by) α_a, Z_{bA} conductors placed at α_b, etc. According to eq (20) the sum of the projections of phasor u_{hp} onto all of the series-connected conductors of phase A translated into the hp–phasorial domain becomes:

$$\left[u(t)_{phas\ A}\right]_{wav\ hp} = \Re e\left[\overrightarrow{u_{hp}}\left(\pm Z_{aA}e^{-jph\alpha_a} \pm Z_{bA}e^{-jph\alpha_b} \pm \ldots\right)\right] =$$
$$= Z_A \,\mathrm{Re}\,al\left(\overrightarrow{u_{hp}}\ \overline{\xi_{hp,A}}^{*}\right)$$

where $\overline{\xi_{hp,A}} = (1/Z) \square \sum_{y=a,b,c\ldots} \pm Z_y\, e^{+jhp\alpha_{yA}}$ is a complex constant (the complex winding factor of relative order h). More details in [35].

B. Space waves produced by a current DmPhS.

Let it be a current DmPhS of sequence g as defined in (12). The current sheet space waves produced by this system are obtained by replacing in (28) the values of (12). Making use of Euler´s formula ($2\cos\alpha = e^{j\alpha} + e^{-j\alpha}$) it follows, after simple mathematical manipulations:

$$\vec{i}_h = \frac{1}{m}\frac{\overline{\xi_h}}{\left|\overline{\xi_h}\right|}\, i(t)\left[e^{j\varepsilon(t)}\sum_{y=1}^{y=m} e^{j(y-1)(h-g)\frac{2\pi}{m}} + e^{-j\varepsilon(t)}\sum_{y=1}^{y=m} e^{j(y-1)(h+g)\frac{2\pi}{m}}\right]$$

Inside the bracket in the above equation there is a first sum of m unitary vectors which are symmetrically distributed with an angle (h – g) * 2π/m between two consecutive vectors, and another analogous sum, but with an angle which is equal to (h + g)* 2π/m. Each one of these sums is different from zero only if the angle between two consecutive vectors is zero or a 2π multiple. Therefore a current DmPhS of sequence g can only produce current sheet space waves of relative order h = qm ± g

C. Comparison between SPhTh and MCCA

MCCA and SPhTh are two *very different* approaches (electric network with variable inductances; electromagnetic device with airgap field waves) which nevertheless describe *the same* process. Yet, when two approaches with same simplifying hypotheses are used for analyzing a process, they produce two equation sets which must be either equal or formally equivalent. The MCCA has been successfully used all over the world for the last fifty years. Thus, it would be sheer madness to present the SPhTh in this paper as a new method that, relying on the same hypotheses as the ones of the MCCA, brings new and improved equations which no one could have been able to obtain previously or for which there are no equivalent ones in the MCCA. In other words, in the end, the equations of both theories must be mathematically equivalent. But this does not mean that both theories are equally suitable. In the author's opinion, the following aspects are to be considered in favour of the SPhTh:

Use of analytical tools with a clear physical content and meaning

Space phasors are much more intuitive than matrix transformations. They characterize the dynamic space distribution of a well defined internal or space quantity.

Relating space waves to external phas quantities.

Space phasors show in a precise and easy way how the evolution of the machine internal quantities results in changes of its external ones. (Projections of the space phasors into the phase axis in their corresponding phasorial domains).

Simplicity of the model and of the methodological procedure.

The MCCA regards the machine as a system of magnetically coupled coils. A chain of abstract mathematical transformations (almost always lacking any underlying physical explanation) is applied in order to decouple the variables and to simplify the machine structure. In the SPhTh no abstract phase reduction or transformation matrices are required, and the dynamic equations are formulated in an easy way.

Excellent suitability for electronic control studies

First: space phasors provide a very good physical insight into the machine behaviour. This way, they enable to "untangle" (with little mathematics) the machine structure, and to decouple the MM (and to understand its underlying physical base!) into a set of equivalent and independent three phase machines (machines with one only $\mathbf{\Psi}$ space phasor in stator and rotor).

Second: electromechanical energy conversion is concerned with motional emfs. Thus, pulsational emfs should be considered a waste of resources (increasing in machine and converter sizes) and time (slower dynamic answer) and are always to be avoided. This simple idea constitutes the GAFTOC principle [28], from which all the other control methods can be deduced. Yet, implementing the GAFTOC becomes very simple in the SPhTh: it boils down to keep constant the magnitude of every **Ψ** space phasor and to change only its rotational speed.

Graphical representation of transients.
A sinusoidal space wave can always be graphically represented by a space phasor, no matter the changes in its amplitude and speed. Dynamic diagrams provide en excellent insight into transient processes.

Torque formula very simple and intuitive
The torque formula is quite simple and has a direct and very physical interpretation: tendency to the alignment of two magnets.

Removing the abrupt gap between steady and transient state studies.
Space phasors make use of equations and diagrams formally analogous to the ones in steady state. Thus, they enable extending to transient problems a mathematical and graphical analysis technique very familiar to any electrical engineer from his steady state studies.

D. Torque expression with space phasors.

For the analytical description of the machine with space harmonics, the MCCA [21] and the SPhTh assume the hypothesis of $\mu_{Fe} = \infty$. In linear systems the torque can be obtained from the stored magnetic energy trough the general expression (e.g. [38], chapter 2)

$$\mathrm{T} = \left.\frac{\partial \mathrm{W_{mag}}}{\partial \lambda}\right|_{\mathrm{i1,i2...=cte}} \tag{1.1}$$

In the case of IM´s and DFAM´s with three phase windings on stator and rotor (PMSM is a more simple case) the torque due to the interaction of the main waves produced by stator and rotor (fundamental machine torque) can also be expressed by means of space phasors as

$$T(t) = \frac{9}{4}\, p\, L_{1,mut.} \left(\overrightarrow{i_{1p,rot}}\, e^{j\,p\lambda(t)} \times \overrightarrow{i_{1p,\,str}} \right) \tag{1.2}$$

That is, the torque is proportional (in a formal language) to the vectorial product of the two fundamental current space phasors of stator and rotor. Thus, the torque can be understood as the tendency to alignment of two magnets or of their associated current sheet waves. (For deduction of (1.2), see, e. g. [14]). Only space waves with the same pole pair number, hp, produce torque. Therefore, in the general case of a constant airgap MM with space harmonics, applying exactly the same procedure as in [14] to each pair (same hp) of stator and rotor harmonic current space phasors, given by eq. (28), and summing up all of the harmonic torques, if follows (λ: rotor-stator angle. $L_{h,mut}$ = stator–rotor phases maximal mutual inductance for the induction wave or order h. More details in [35]):

$$T(t) = \frac{m_{est} m_{rot}}{4} \sum_{h=1}^{h_{max}} h\, p\, L_{h,mut.} \left(\overrightarrow{i_{hp,rot}}\, e^{j\,h\,p\lambda(t)} \times \overrightarrow{i_{hp,est}} \right)$$

(1.3)

(1.3)coincides with (1.1) for constant airgap MM

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