Abstract

In the stock markets, the process of estimating a fair price for a stock, option or commodity in the next few months or a year is consider the corner stone for this trade. There are several attempts to obtain a suitable mathematical model in order to enhance the estimation process for evaluating the options for short or long periods. The Black-Scholes equation (1973) is considered a breakthrough in the mathematical modeling for the stock markets. It presented a practical mathematical model to estimate a fair value for a given option at that time. Based on Black-Scholes assumptions, they obtained a linear partial differential equation and it is solved analytically.

Since that time the stock trade has tremendously grown and several factors have been incorporated which lead to new complex financial products to appear. Black-Scholes assumptions as constant volatility and that the stock follows standard Brownian motion cannot keep up with these developments in the financial market. Consequently, these constraints need to be changed. There have been numerous efforts to develop alternative asset models that are capable of capturing the leptokurtic features found in financial market data, and subsequently to use these models to develop option prices that accurately reflect the volatility smiles and skews found in market traded options. Two strategies have been done to capture these behaviors; the first modification is to add jumps into the price process for the underlying asset, as originally was proposed by Merton; the second is to allow the volatility to evolve stochastically, introduced by Heston. The first modification leads to the so-called jump diffusion and Lévy models which are described by a partial integro-differential equation (PIDE) with two independent variables the underlying asset and time. Following the second approach, it leads to a partial differential equation (PDE) with two spatial variables; the underlying asset and the volatility apart from the time.

Here in this work, we solve numerically PIDEs for a wide class of Lévy processes using finite difference schemes for European options and also, the associated linear complementarity problem (LCP) for American option. Moreover, the models for options under stochastic volatility incorporated with jump-diffusion are considered. Numerical analysis for the proposed schemes is studied since it

is the efficient and practical way to guarantee the convergence and accuracy of numerical solutions. In fact, without numerical analysis, careless computations may waste good mathematical models.

This thesis consists of four chapters; the first chapter is an introduction containing historically review for stochastic processes, Black-Scholes equation and preliminaries on numerical analysis. Chapter two is devoted to solve the PIDE for European option under CGMY process. The PIDE for this model is solved numerically using two distinct discretization approximations; the first approximation guarantees unconditionally consistency while the second approximation provides unconditional positivity and stability. In the first approximation, the differential part is approximated using the explicit scheme and the integral part is approximated using the trapezoidal rule. In the second approximation, the differential part is approximated using the Patankar-scheme and the integral part is approximated using the four-point open type formula. After constructing the finite difference scheme for each case, the positivity, stability and consistency are studied. Also several examples and simulations are provided.

Chapter three provides a unified treatment for European and American options under a wide class of Lévy processes as CGMY, Meixner and Generalized Hyperbolic. First, the reaction and convection terms of the differential part of the PIDE are removed using appropriate mathematical transformation. After that the differential part for European case is discretized using the explicit scheme, while the integral part is approximated using Laguerre-Gauss quadrature formula. Numerical properties such as positivity, stability and consistency for this scheme are studied. For the American case, the differential part of the LCP is discretized using a three-time level approximation while the Laguerre-Gauss quadrature has been used to approximate the integral term. Next, the Projected successive over relaxation and multigrid techniques have been implemented to obtain the numerical solution. Several numerical examples are given including discussion of the errors and computational cost.

Finally in Chapter four, the PIDE for European option under Bates model is considered. Bates model combines both stochastic volatility and jump diffusion approaches resulting in a PIDE with a mixed derivative term. Since the presence of cross derivative terms involves the existence of negative coefficient terms in the numerical scheme deteriorating the quality of the numerical solution, the mixed derivative is eliminated using suitable mathematical transformation. The new PIDE is solved numerically and the numerical analysis is provided. Moreover, the LCP for American option under Bates model is studied.