Summary

The Ph.D. thesis "Weighted Banach Spaces of harmonic functions" presented here, treats several topics of functional analysis such as weights, composition operators, Fréchet and Gâteaux differentiability of the norm and isomorphism classes. The work is divided into four chapters that are preceded by one in which we introduce the notation and the well-known properties that we use in the proofs in the rest of the chapters.

In the first chapter we study Banach spaces of harmonic functions on open sets of $\mathbb{R}^d$ endowed with weighted supremun norms. We define the harmonic associated weight, we explain its properties, we compare it with the holomorphic associated weight introduced by Bierstedt, Bonet and Taskinen, and we find differences and conditions under which they are exactly the same and conditions under which they are equivalent.

The second chapter is devoted to the analysis of composition operators with holomorphic symbol between weighted Banach spaces of pluriharmonic functions. We characterize the continuity, the compactness and the essential norm of composition operators among these spaces in terms of their weights, thus extending the results of Bonet, Taskinen, Lindström, Wolf, Contreras, Montes and others for composition operators between spaces of holomorphic functions. We prove that for each value of the interval $[0, 1]$ there is a composition operator between weighted spaces of harmonic functions such that its essential norm attains this value.

The third chapter is related with the study of Gâteaux and Fréchet differentiability of the norm. The Šmulyan criterion states that the norm of a real Banach space X is Gâteaux differentiable at $x \in X$ if and only if there exists x^* in the unit ball of the dual of X weak* exposed by x and the norm is Fréchet differentiable at x if and only if x^* is weak* strongly exposed in the unit ball of the dual of X by x. We show that in this criterion the unit ball of the dual of X can be replaced by a smaller convenient set, and we apply this extended criterion to characterize the points of Gâteaux and Fréchet differentiability of the norm of some spaces of harmonic functions and continuous functions with vector values. Starting from these results we get an easy proof of the theorem about the Gâteaux differentiability of the norm for spaces of compact linear operators announced by Heinrich and

published without proof. Moreover, these results allow us to obtain applications to classical Banach spaces as the space H^∞ of bounded holomorphic functions in the disc and the algebra $A(\overline{\mathbb{D}})$ of continuous functions on $\overline{\mathbb{D}}$ which are holomorphic on $\mathbb{D}$. The content of this chapter has been included by E. Jordá and the author in [46].

Finally, in the forth chapter we show that for any open set U of $\mathbb{R}^d$ and weight v on U, the space $h_{v_0}(U)$ of harmonic functions such that multiplied by the weight vanishes at the boundary on U is almost isometric to a closed subspace of c_0, extending a theorem due to Bonet and Wolf for the spaces of holomorphic functions $H_v(U)$ on open sets U of $\mathbb{C}^d$. Likewise, we also study the geometry of these weighted spaces inspired by a work of Boyd and Rueda, examining topics such as the v-boundary and v-peak points and we give the conditions that provide examples where $h_{v_0}(U)$ cannot be isometric to c_0. For a balanced open set U of $\mathbb{R}^d$, some geometrical conditions in U and convexity in the weight v ensure that $h_{v_0}(U)$ is not rotund. These results have been published by E. Jordá and the author in [45].