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Additional Information

Evolutionary and variable step size strategies for multichannel filtered-x affine projection algorithms

Alberto Gonzalez^a, Felix Albu^b, Miguel Ferrer^a, Maria de Diego^a

^a Institute of Telecommunications and Multimedia Applications (iTEAM)

Universitat Politècnica de València, Spain. e-mail: {agonzal,mferrer,mdediego}@dcom.upv.es

^b Department of Electronics & Telecommunications

Valahia University of Targoviste, Romania. e-mail: felix.albu@valahia.ro *

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Abstract: This paper is focused on the necessity to improve the performance of the affine projection (AP) algorithm for active noise control (ANC) applications. The proposed algorithms are evaluated regarding their steady-state behavior, their convergence speed and their computational complexity. To this end, different strategies recently applied to the AP for channel identification are proposed for multichannel ANC. These strategies are based either on a variable step size, an evolving projection order, or the combination of both strategies. The developed efficient versions of the AP algorithm use the modified filtered-x structure, which exhibits faster convergence than other filtering schemes. Simulation results show that the proposed approaches exhibit better performance than the conventional AP algorithm and represent a meaningful choice for practical multichannel ANC applications.

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I. Introduction

The Affine Projection (AP) algorithm [1] is a versatile adaptive strategy that improves the speed of convergence of the well know Least Mean Squares (LMS) algorithm, while maintaining its good properties of robustness and stability. The speed of convergence of the AP algorithm increases when an integer parameter called projection order (N) is also increased. However its computational cost and its final residual error (final misadjustment) get worse at same time. Variable step-size affine projection algorithms have already been proposed [2] [3] [4] [5] to overcome this duality and get better performance at steady state without penalizing the convergence speed of the algorithm. Although these strategies achieve better final error in steady state, their computational cost remains invariant through the algorithm execution since it mainly depends on its projection order. Moreover, since the final steady-state error depends also on the projection order [6], it could be improved even more. Therefore, an AP algorithm with a projection order that evolves (evolving projection order AP algorithm) [7],[8] has been proposed in order to adjust the computational burden to the algorithm convergence stage and achieve both a low final error and computational cost at steady state. Other type of AP that dynamically adjusts the update interval for the adaptive filter coefficients has been recently proposed [9]. This kind of algorithms try to improve the AP algorithm performance also reducing its computational cost. In this paper, we introduce some of those variants of the AP algorithm for multichannel active noise control (ANC) systems and analyze their performances in terms of convergence properties, computational cost and final residual error.

The application of the AP algorithm to ANC requires the inclusion of a suitable filtering scheme to compensate the secondary path between the adaptive filter output and the error sensor. To this end, the modified filtered-x structure [10] has been used as adaptive filtering scheme. In Section II, a brief description of the multichannel modified filtered-x AP algorithm is presented. The previously commented efficient strategies for the AP algorithm are introduced for ANC applications in Section III. In Section IV, the computational complexity

of the algorithms in terms of the number of multiplications required at each iteration is reported. Simulations results obtained in a multichannel ANC system are presented in Section V allowing the performance comparison among the different AP approaches. Finally, conclusions are summarized in Section VI.

II. The affine projection algorithm for multichannel active noise control

In order to apply the AP algorithm to ANC, there are some fundamental issues to consider. First, the error signal $e(n)$ is obtained as the acoustical combination of the desired signal $d(n)$ (which represents the undesired signal or disturbance signal to be cancelled) and the adaptive filter output filtered through the secondary acoustic path between the adaptive filter output and the error sensor, instead of the subtraction of an electrical signal. This point causes a sign change in the coefficient update equation and also implies to filter the reference signal through an estimate of the secondary path. Second, due to the unavailability of the disturbance signal $d(n)$ that is needed to calculate the error signal vector, $\mathbf{e}(n)$, in the coefficient update equation, an adaptive filter requires to use a filtering structure to compensate for the secondary path. The AP algorithms are mainly implemented using the modified filtered-x scheme [11], [12]. This scheme allows the recovery of an estimate of $d(n)$. Finally, a multichannel ANC system is comprised of several secondary sources, several error sensors, and even several reference sensors, which involves a set of coefficient update equations. The multichannel transducer configuration requires a considerable computational burden that constrains the kind of algorithms to be implemented on the controller.

Fig. 1 shows a block diagram of a single channel AP algorithm based on the modified filtered-x structure. Block \mathbf{h} represents the secondary path meanwhile $\hat{\mathbf{h}}$ is an FIR filter that models \mathbf{h} . $x(n)$ is the reference signal, $y(n)$ is the adaptive filter output and $d(n)$ is the disturbance signal (unavailable in practice). $e(n)$ results from the acoustic sum of $d(n)$ and $y(n)$ filtered through the secondary path. Moreover, $\hat{v}(n)$, $\hat{d}(n)$ and $\hat{e}(n)$ are internal signals that represent respectively the reference signal filtered through $\hat{\mathbf{h}}$, an estimate of $d(n)$ and a new error signal that is used in the algorithm to update the filter coefficients. A generic multichannel ANC

system has I reference signals, $x_i(n)$, K error sensors that produce K error signals, $e_k(n)$, from the disturbance signals $d_k(n)$, and J secondary sources that generate the $y_j(n)$ signals. Thus IJ adaptive filters, $w_{ij}(n)$, have to be updated at each iteration. Furthermore, the following signals are needed: IJK filtered reference signals, $\hat{v}_{ijk}(n)$, obtained from the reference signal $x_i(n)$ filtered through the estimate of the secondary path $\hat{\mathbf{h}}_{jk}$ that links the j th actuator with the k th error sensor, and the K estimates of the error signals and the disturbance signals ($\hat{e}_k(n)$ and $\hat{d}_k(n)$, respectively).

The multichannel AP algorithm for ANC with the modified filtered-x scheme embedded is described by **Algorithm 1** according to the notation shown in Table 1.

III. Efficient strategies for the modified filtered-x AP algorithm

III.1 Variable step size affine projection algorithms

A high step size speeds up the convergence but worsens the final error, thus a time variable step size that decreases its value when the algorithm is closer to the steady state is advisable. An AP algorithm with variable step size was presented in [2]. We propose an AP algorithm for multichannel ANC systems based on the strategy introduced in [2], which will be named throughout this paper as VSSAP.

The considered strategy computes an estimation of the mean squared value of the auxiliary error vector, $\epsilon_{ij}(n)$, to adjust the convergence step size. The proposed variation rule for the variable step-size parameters is given by

$$\mu_{ij}(n) = \mu_{\max} \frac{\|p_{ij}(n)\|^2}{\|p_{ij}(n)\|^2 + C}, \quad (1)$$

where $p_{ij}(n)$ is an estimation of the mean value of $\epsilon_{ij}(n)$, which is obtained from an exponential weighting of its instantaneous value as $p_{ij}(n) = \alpha p_{ij}(n-1) + (1-\alpha)\epsilon_{ij}(n)$ (with $0 < \alpha < 1$), and C is a positive parameter that depends on the algorithm projection order. It should be noted that this parameter is approximated by $\frac{N}{\text{SNR}}$ in [2] (SNR is the signal to noise ratio), thus it has to be adjusted following the projection order values.

The maximum step-size parameter μ_{\max} in (1) is chosen to guarantee both fast convergence speed and filter stability and ideally should be less than 1 [4]. Furthermore, the adaptive filter coefficients are updated by the following equation

$$\mathbf{w}_{ij}(n+1) = \mathbf{w}_{ij}(n) - \mu_{ij}(n)\epsilon_{ij}(n). \quad (2)$$

Note that this is the expression used instead of the step 7 in **Algorithm 1**. It can be shown that this variation rule guarantees that the mean square deviation of the filter weights undergoes the largest decrease between algorithm iterations.

III.2 Evolving projection order affine projection algorithms

The variable order affine projection algorithms follow the step 7 equation in **Algorithm 1**, but their projection order can dynamically change between iterations. This projection order varies in order to speed up the convergence speed and minimize the computational cost and residual error depending on certain conditions that can differ slightly between different variable order AP approaches. A first version of these algorithms was proposed in [7] where the number of input data vectors to update the filter coefficients were selected within each algorithm iteration. Other examples of this kind of algorithms are given by [8] and [13],[14]. Recently, a similar algorithm is proposed in [15] that changes the projection order in accordance with the adaptation stage and states a linear dependence of the projection order on the logarithm of the filter output error variance. All these strategies guarantee a good behavior at both steady and transient states, but mainly they try to optimize the computational cost when the algorithm stage does not require high projection orders. If we apply a variable order strategy to a multichannel ANC system, different available approaches arise. Generally speaking, each of the IJ adaptive filters may use a different projection order. Nevertheless we proposed a simplified algorithm with the same projection order for all the filters. Therefore, this multichannel algorithm uses the instantaneous residual error power at each error sensor to update the single projection order and keep a single step-size

parameter. Thus,

$$N(n) = \begin{cases} \min \{N(n-1) + 1, N_{\max}\}, & \sum_{k=1}^K e_k^2(n) > \eta(n) \\ N(n-1), & \theta(n) < \sum_{k=1}^K e_k^2(n) \leq \eta(n) \\ \max \{N(n-1) - 1, 1\}, & \sum_{k=1}^K e_k^2(n) \leq \theta(n) \end{cases} \quad (3)$$

The thresholds $\theta(n)$ and $\eta(n)$ are derived from [8] and adjusted to a multichannel system by

$$\eta(n) = K\sigma_v^2 \frac{\mu N(n-1) + 2}{2 - \mu}, \quad (4)$$

and

$$\theta(n) = K\sigma_v^2 \frac{\mu [N(n-1) - 1] + 2}{2 - \mu}, \quad (5)$$

where σ_v^2 is the power of an additive noise uncorrelated with the disturbance signal in the ANC system, and N_{\max} is the maximum projection order. We refer to the AP with the time-varying projection order given by (3) as the evolving order AP algorithm (Evolving AP).

III.3 Variable projection order and step size affine projection algorithms

The AP approaches described in the previous subsections aim to optimize either the final residual error, or the computational complexity or the convergence speed. Thus, the different algorithms adapt the projection order or the step-size parameter. However, since the residual error at steady state and the convergence speed at transient state depend on both the step size and the projection order, algorithms that simultaneously self-adjust both parameters can get profit from both strategies [16][17].

Different strategies can be performed to change both parameters, and some of them are summarized as follows:

- The first approach is a mixture of the two previous strategies, thereby allowing the change in both step size and projection order. In this case, a single evolving projection order for all the adaptive filters is used

and a variable step-size parameter for each filter is needed. The thresholds to adjust the projection order proposed in (4) and (5) should be modified by considering the mean value of the whole set of step-size parameter. Thus, the μ parameter in (4) and (5) is replaced by $\frac{1}{IJ} \sum_{i=1}^I \sum_{j=1}^J \mu_{ij}(n)$.

- A second approach considers the two previous strategies, but a different projection order is used at each adaptive filter. In this case, the thresholds are given by:

$$\eta_{ij}(n) = K\sigma_v^2 \frac{\mu_{ij}(n)N_{ij}(n-1) + 2}{2 - \mu_{ij}(n)}, \quad (6)$$

$$\theta_{ij}(n) = K\sigma_v^2 \frac{\mu_{ij}(n)[N_{ij}(n-1) - 1] + 2}{2 - \mu_{ij}(n)}, \quad (7)$$

what leads to the following projection orders at each iteration

$$N_{ij}(n) = \begin{cases} \min \{N_{ij}(n-1) + 1, N_{\max}\}, & \sum_{k=1}^K e_k^2(n) > \eta_{ij}(n), \\ N_{ij}(n-1), & \theta_{ij}(n) < \sum_{k=1}^K e_k^2(n) \leq \eta_{ij}(n), \\ \max \{N_{ij}(n-1) - 1, 1\}, & \sum_{k=1}^K e_k^2(n) \leq \theta_{ij}(n). \end{cases} \quad (8)$$

- Finally, by using the strategy described in [17], the projection order is modified depending on the variable step size. In this case the step size $\mu_{ij}(n)$ in (1) is compared with appropriate thresholds to adjust the projection order. This algorithm will be called throughout this paper as Evolving VSSAP. The main difficulty of this method is the threshold tuning.

IV. Computational complexity of the algorithms

The computational burden of the AP algorithms for ANC depends on, among other factors: the number of taps of both the adaptive filters (L) and the FIR secondary path models (M), the dimensions of the ANC system (I, J and K) and strongly on the projection order (N). The number of multiplications needed at each iteration of the AP algorithm presented in section II is $(2IJL + JKM + IJKM + IJKLN + IJK(LN + N^2 + O(N^3)))$.

It should be noted that algorithms that change their projection order add a few computational complexity to check the conditions of change. However they save an appreciable computational burden if the projection order decreases. For instance, the VSSAP requires $2IJ$ multiplications more than the AP described in Section II at each iteration. Nevertheless, the projection order remains constant, which means the computational cost does not decrease when the steady-state is reached. Regarding the evolving AP and the Evolving VSSAP, $5IJ$ and $6IJ$ additional multiplications at each iteration, respectively, are needed. These extra multiplications allow to determine if the projection order should be decreased and thus the computational cost will be eventually reduced.

Table 2 shows the multiplications required for AP order $N = 10$, VSSAP, Evolving AP and Evolving VSSAP during the first 10,000 iterations of the experiments for different ANC configurations. The configuration parameters have been chosen in order to guarantee the convergence of the algorithms. Adaptive filters of 150 coefficients and FIR filters of order 199 modelling the secondary paths have been used.

It can be observed that the variable order algorithms exhibit lower computational cost than the algorithms that do not change the order. Moreover, the Evolving AP is computationally less costly in comparison with the Evolving VSSAP. But the last algorithm achieves a better final residual error at steady state as it can be observed in the next section.

V. Simulation Results

In this section, we carry out a series of experiments in an ANC setup to illustrate the performance of the different proposed algorithms. In order to evaluate the new approaches for practical applications, three performance parameters are considered: computational complexity, convergence speed and final residual error.

V.1. Convergence performance

Convergence is defined herein as the ratio between the sum of the instantaneous estimated power at each

error sensor and its mean value without active noise control. An average of 3,000 independent runs was done to reduce the variance of the different curves. The reference signal was Gaussian noise of zero mean and unit variance.

The first experiment analyzes a single-channel ANC system. To evaluate the efficiency of the algorithms within a non-stationary environment, the coefficients of the plant were changed after 500,000 iterations. Thus the adaptive system had to readjust its working parameters from its input signals. The primary and secondary paths have 250 and 232 coefficients, respectively, whereas the adaptive filter has 150 taps. Figure 2 shows the corresponding curves for the AP algorithm when $N = 1$ and $N = 10$, for the VSSAP when $N = 10$, for the Evolving AP and for the Evolving VSSAP with an initial projection order $N = 10$. The step-size parameter was set to $\mu = 0.1$. For the variable step size algorithms it was only the starting μ value. Overall, the proposed Evolving VSSAP outperforms the algorithms analyzed in terms of final residual error and convergence rate. Regarding the residual error at steady state, it achieves the lower values of the AP with $N = 1$. However, it shows a slightly slower convergence behavior than faster algorithms, the AP and the VSSAP with $N = 10$. Furthermore, it is clearly seen that the VSSAP performs similarly to the AP with $N = 10$. The only algorithm with different behavior through the simulation is the Evolving AP. This algorithm behaves similarly to both the AP and the VSSAP with $N = 10$ in the first part of the simulation. However, during the last iterations when the primary path changes, its evolving order is not able to achieve higher values in order to speed up the convergence, so it performs worst than the other algorithms analyzed. This drawback could be improved simply by better adjusting the threshold levels of the projection order changes for automatically switching to the highest projection order when a fast transient is detected.

The second experiment has been carried out in a multichannel system (1:2:2 ANC system). The length of the different FIR filters was the same as in previous experiment but with a stationary environment. The different attenuation curves are shown in Figure 3. Equivalent conclusions as the first experiment can be given.

It is noteworthy that the experiments carried out serve to illustrate how effective can be the process of switching both the step size and the projection order, or only one of these, in the different algorithms proposed.

V.2. Computational performance

The computational requirements of the different algorithms in terms of the number of flops has been also investigated. Fig. 4 shows the computational cost evolution for one realization. The configuration parameter setting is $I = 1$, $J = 1$, $K = 1$, $L = 150$ and $M = 232$. It can be appreciated that the AP ($N = 10$) and the VSSAP ($N = 10$) have a high and constant computational cost due to their time-invariant projection order. The Evolving VSSAP alleviates the computational load dramatically since it decreases the projection order as soon as it reaches the steady-state where high projection orders are not required. Moreover, Fig. 4 shows that the computational cost of the Evolving AP also switches as a result of changes in the projection order. However, the computational cost of the Evolving AP irregularly changes due to the dependency between the projection order and the instantaneous error signal.

In summary, the main features of the analyzed algorithms analyzed are summarized in Table 3.

VI. Conclusions

In this paper, several efficient strategies for the AP algorithm have been proposed and developed for single channel and multichannel ANC systems. Some of these approaches self adjust the step-size parameter and/or the projection order, in order to improve simultaneously the convergence speed, the final residual error and even the computational cost, with respect the approaches that set constant these parameters. The main aim of this contribution has been the development of efficient AP strategies for multichannel ANC systems based on the modified filtered-x scheme. It has been shown by simulations that the strategies that simultaneously adjust the step size and projection order are a recommended choice for ANC multichannel systems. They exhibit the robustness and good convergence properties of AP algorithm together with low complexity at every

convergence stage and low mean squared error at steady state.

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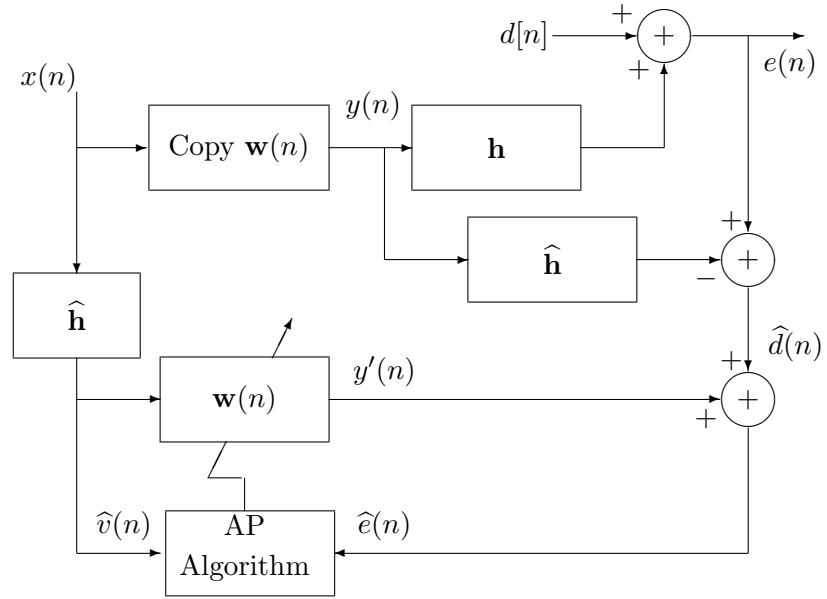


Figure 1: Block diagram of an ANC system using the AP algorithm with the modified filtered-x scheme.

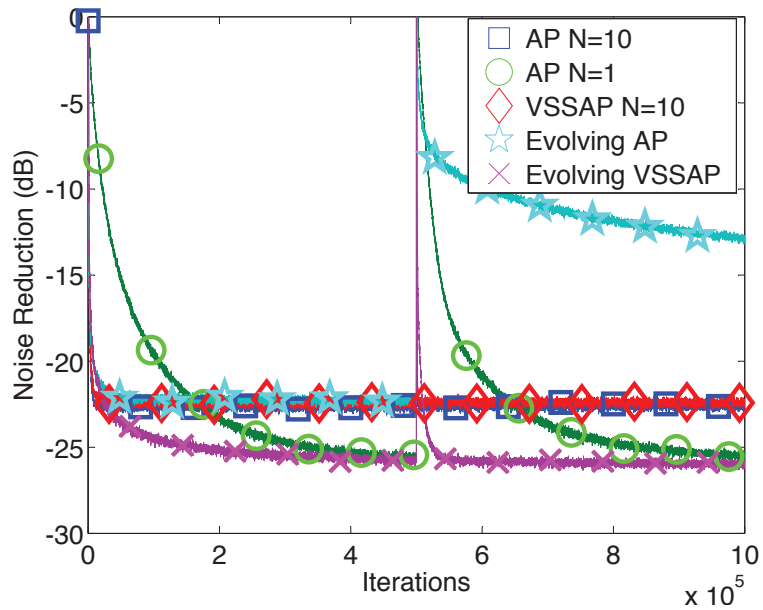


Figure 2: Convergence curves for different AP strategies in a variant single-channel ANC system.

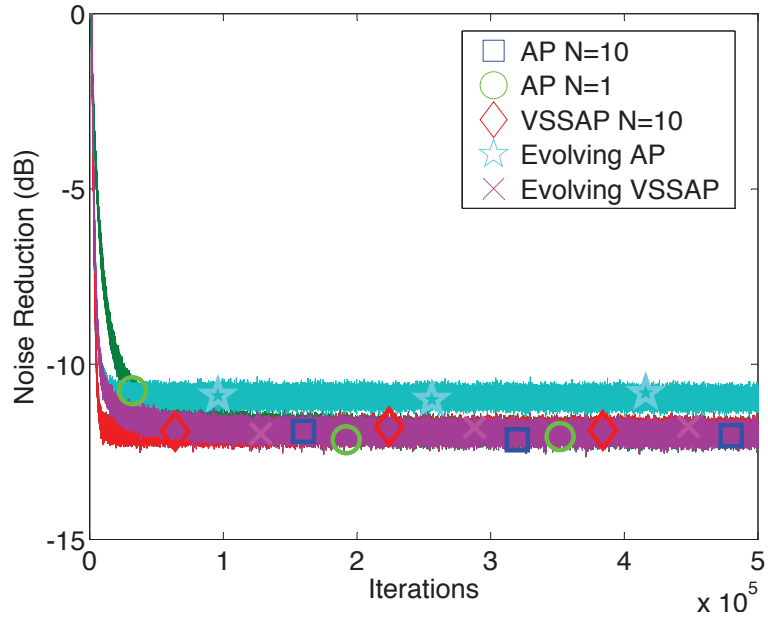


Figure 3: Convergence curves for different AP strategies in a stationary ANC multichannel system.

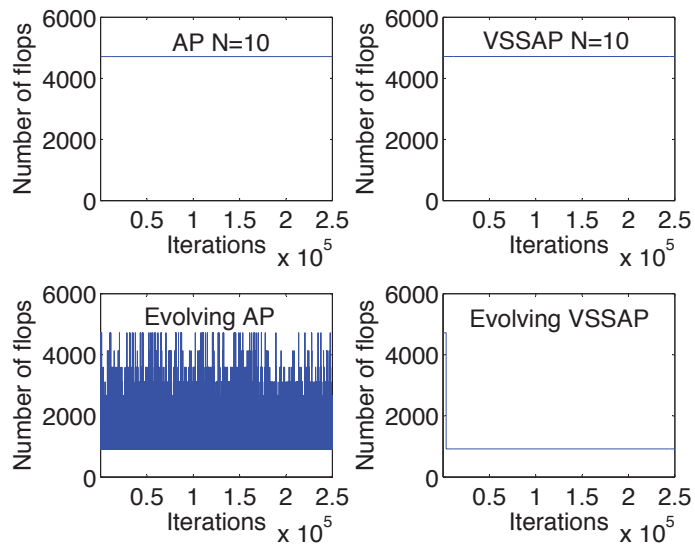


Figure 4: Evolution of computational cost for one realization of the algorithms.

Table 1: Notation of the multichannel modified filtered-x affine projection algorithm.

I	Number of reference sensors
J	Number of actuators
K	Number of error sensors
L	Length of the adaptive filters
N	Projection order
$x_i(n)$	i th reference signal at time n
$y_j(n)$	j th actuator signal at time n
$e_k(n)$	k th error sensor signal at time n
$\hat{\mathbf{h}}_{j,k}$	Estimated FIR filter modelling the acoustic plant $\mathbf{h}_{j,k}$ that links the k th error sensor and the j th actuator
$\epsilon_{ij}(n)$	auxiliary error vector
$\mathbf{w}_{ij}(n)$	$[w_{ij1}(n)w_{ij2}(n) \dots w_{ijL}(n)]^T$
$\mathbf{x}_{Li}(n)$	$[x_i(n)x_i(n-1) \dots x_i(n-L+1)]^T$
$\mathbf{x}_{Mi}(n)$	$[x_i(n)x_i(n-1) \dots x_i(n-M+1)]^T$
$\mathbf{y}_j(n)$	$[y_j(n)y_j(n-1) \dots y_j(n-M+1)]^T$
$\hat{\mathbf{v}}_{ijk}(n)$	$[\hat{v}_{ijk}(n)\hat{v}_{ijk}(n-1) \dots \hat{v}_{ijk}(n-L+1)]^T$
$\hat{\mathbf{d}}_k(n)$	$[\hat{d}_k(n)\hat{d}_k(n-1) \dots \hat{d}_k(n-N+1)]^T$
$\hat{\mathbf{V}}_{ijk}(n)$	$[\hat{\mathbf{v}}_{ijk}(n)\hat{\mathbf{v}}_{ijk}(n-1) \dots \hat{\mathbf{v}}_{ijk}(n-N+1)]$

Algorithm	$I = 1, J = 1, K = 1$	$I = 1, J = 2, K = 2$	$I = 1, J = 2, K = 4$	$I = 1, J = 4, K = 4$
AP (N=10)	46.50	181.50	361.50	721.50
VSSAP	46.52	181.54	361.54	721.58
Evolving AP	8.82	30.63	59.71	117.87
Evolving VSSAP	8.83	30.70	59.78	118.06

Table 2: Comparison of the total number of multiplications (in millions of operations) required for the different algorithms in a single channel ANC system and three multichannel ANC configurations.

Algorithm	Transient	Steady-State	Computational cost	Tuning parameters
AP (high N)	very good	bad	very expensive	simple
AP (low N)	bad	very good	low	simple
VSSAP (high N)	very good	good	very expensive	difficult
VSSAP (low N)	bad	excellent	low	difficult
Evolving AP	good	good	medium	difficult
Evolving VSSAP	very good	excellent	low-medium	very difficult

Table 3: Qualitative performance comparison of different algorithms in terms of steady-state and transient-state behavior, together with the computational complexity and tuning parameters.

Algorithm 1 Multichannel modified filtered-x AP algorithm.

Require: Reference signals $x_i(n)$ and error signals $e_k(n)$

Ensure: Output of the adaptive filter $y_j(n)$

1: Update the vectors $\mathbf{x}_{Li}(n)$ and $\mathbf{x}_{Mi}(n)$

2: $y_j(n) = \sum_{i=1}^I \mathbf{w}_{ij}^T(n) \mathbf{x}_{Li}(n)$ $J \times LI$ multiplications

3: $\hat{\mathbf{d}}_k(n) = e_k(n) - \sum_{j=1}^J \mathbf{y}_j^T(n) \hat{\mathbf{h}}_{jk}$ $K \times MJ$ multiplications

4: $\hat{v}_{ijk}(n) = \mathbf{x}_{Mi}^T(n) \hat{\mathbf{h}}_{jk}$ $IJK \times M$ multiplications

5: $\hat{\mathbf{e}}_k(n) = \hat{\mathbf{d}}_k(n) + \sum_{i=1}^I \sum_{j=1}^J \hat{\mathbf{v}}_{ijk}^T(n) \mathbf{w}_{ij}(n)$ $K \times IJNL$ multiplications

6: $\epsilon_{ij}(n) = \sum_{k=1}^K \hat{\mathbf{v}}_{ijk}(n) [\hat{\mathbf{V}}_{ijk}^T(n) \hat{\mathbf{v}}_{ijk}(n)]^{-1} \hat{\mathbf{e}}_k(n)$ $IJ \times K(LN + N^2 + O(N^3))$ multiplications

7: $\mathbf{w}_{ij}(n+1) = \mathbf{w}_{ij}(n) - \mu \epsilon_{ij}(n)$ $IJ \times L$ multiplications
