

Piezometric error derived from some demand lumped models in water distribution

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Abstract. Allocation of the internal demand in a line to the end nodes of the line may help reduce the size of the mathematical model of a water distribution network (WDN). Such a reduction is desirable as it allows hydraulic simulations at a lower computational cost. Moreover, this reduction is inevitable in the case of WDN models of large cities, due to its size, which require huge amounts of computational resources. However, such simplified models are not at zero cost, since they produce various errors in the calculations. In this contribution we provide a calculation mechanism that allows the engineers responsible for the hydraulic model of a WDN to know the errors in terms of piezometric head produced by allocating the internal demands of a line to the end nodes of the line.

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1. Introduction

Today, with the widespread use of Geographic Information Systems (GIS), models containing up to hundreds of thousands of pipes [3] are built.

Currently, it is possible to build detailed models of a WDN in its entirety. However, even ignoring the aspects related to the uncertainty - something that is not realistic, such models produce massive amounts of data, and require sophisticated computational tools and efficient mechanisms to reasonably interpret the results obtained.

Frequently, some simplifications are performed without further insight. One such simplification consists in the allocation of the internal demand of a line to its end nodes [1, 2], since the consideration of every one of the consumption points would imply the inclusion of an equal number of points in the calculation model. In a large WDN, this would amount to consider hundreds of thousands of calculation points, which may be an insurmountable impediment to build the network model so that calculations may be efficiently performed, and the results obtained reasonably interpreted.

In [2] we have analyzed the errors produced by the most common application of this simplification: the so-called 50% rule, which systematically allocates half of the internal demand of a line to its end nodes. In this contribution we provide a calculation mechanism to estimate the magnitude of the error in terms of pressure head inside the line, which derives from such lumped model.

In view of the results, the analyst will have a criterion for deciding whether the approximate representation is sufficient or, on the contrary, it is necessary to include some intermediate point of the line into the pool of the model calculation points to obtain a more accurate representation.

2. Problem statement

Let's consider a single line associated with some internal consumption under steady state condition. The characteristics of the line are: length: L ; diameter: D ; upstream head (boundary condition at the upstream node): H_0 ; friction factor: f ; and inflow: Q_{in} . Let's consider an arbitrary consumption scenario associated with two characteristics: total demand in the line with regard to line inflow, and specific distribution of the demand along the line. Let's assume that the flow consumed within the line (total in-line demand) represents a percentage of the line inflow through the upstream end 0. If this fraction is represented by F_Q , $0 < F_Q \leq 1$, the gross demand in the line is given by the expression $Q_d = F_Q Q_{in}$. Let's now consider a demand distribution on the line whose accumulated demand is given by a function $Q(x) = Q_d q(x)$, where $q(x)$ is the accumulated demand ratio, a function defined in $[0, L]$ increasing monotonically from 0 to 1. Finally, let F_{Qd} be the factor that allocates a fraction of the line distributed demand, Q_d , to its upstream end. Thus, the demand allocated to this upstream node is $Q_0 = F_{Qd} Q_d$. As a result, $Q_l = Q_{in} - Q_0$ is the flow rate through the line.

In [3] we have shown that if $q(x)$ corresponds to demands d_k at points x_k , with $0 < x_1 < x_2 < \dots < x_{n-1} < x_n < L$, such that $d_1 + \dots + d_n = Q_d$, then to get the same piezometric value at L using both the lumped and the distributed model of demands, the value of F_{Qd} must be

$$F_{Qd}(F_Q, \lambda, \mu) = \frac{1}{F_Q} \left(1 - \sqrt{\sum_{k=0}^n (\lambda_{k+1} - \lambda_k) \left(1 - F_Q \sum_{j=0}^k \mu_j \right)^2} \right), \quad (1)$$

where

$$\mu_i = d_i/Q_d, \mu_0 = 0, \lambda_i = x_i/L, \lambda_0, \lambda_{n+1} = 1, \text{ for } i = 1, \dots, n.$$

Note that μ_i represents the demand ratio withdrawn at x_i , the consumption point in the line that is at relative distance λ_i from 0.

3. Piezometric discrepancy when using the proposed formula

The reduction of the model size using this lumped demand model is at the price of accepting some piezometric head errors at the inner points of the line.

In the (real) case of a discrete demand along the line, the maximum discrepancy occurs at one of the points x_k , since the real HGL

$$H_R(F_Q, x_i) = H_0 - KLQ_{in}^2 \sum_{k=0}^i (\lambda_{k+1} - \lambda_k) \left(1 - F_Q \sum_{j=0}^k \mu_j \right)^2$$

is a decreasing concave upwards polygonal, and the calculated HGL

$$H_C(F_{Qd}, x) = H_0 - KxQ_{in}^2 (1 - F_{Qd}F_Q)^2$$

is a straight line. Then, the problem reduces to identify the first x_{k0} for which the next section of the polygonal has a slope equal to or lower than the slope of the HGL for the lumped model (if equal, all the points between x_{k0} and x_{k0+1} will provide the maximum since the mentioned section of the polygonal and $H_C(F_{Qd}; x)$ runs parallel between both points):

$$\text{Find the first point } x_{k0} \text{ such that } \left(1 - F_Q \left(\sum_{i=1}^{k0} \mu_i \right) \right)^2 \leq (1 - F_{Qd}F_Q)^2.$$

This problem may be rewritten as

$$\text{Find the first point } x_{k0} \text{ such that } \sum_{i=1}^{k0} \mu_i \geq F_{Qd}. \quad (2)$$

These calculations are straightforward and be easily organized, for example in a standard worksheet.

4. Conclusions

This research focuses on the study of the maximum head point discrepancy associated with the concentration of a distributed demand in a line. We use a formula, (1), derived in [2], that performs such a distribution between the end nodes of the line with zero error at L . Note that other allocations produce bigger errors, in general. Calculations are straightforward and involve direct methods easy to apply for example using a very simple worksheet.

If not satisfied with the obtained results from the lumped model for one line, the expert, using these results, may include in the model one additional point of the line, obviously the one where the maximum head discrepancy occurs, given by (2). This divides the line into two new lines to which the same criteria may be applied. Also, including an interior point of one line into the model may be useful in the case that a line is fed by both ends.

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