Summary

The scope of this thesis is the abstract finite group theory. All the groups we will consider will be finite. hence, the word “group” will be understood as a synonimous of “finite group”. We say that a subgroup H of a group G is *solitary* when no other subgroup of G is isomorphic to H. A normal subgroup H of a group G is said to be *normal solitary* when no other normal subgroup of G is isomorphic to H. A normal subgroup N of a group G is said to be *quotient solitary* when no other normal subgroup K of G gives a quotient isomorphic to G/N. Solitary subgroups, normal solitary subgroups, and quotient solitary subgroups have been recently studied by authors like Thévenaz [Thé93], who named the solitary subgroups as *strongly characteristic subgroups*, Kaplan and Levy [KL09, Lev14], Tărnăuceanu [Tăr12b, Tăr12a], and Atanasov and Foguel [AF12].

The aim of this PhD thesis project is to deepen into the analysis of these subgroup embedding properties, by refining the knowledge of their lattice properties, by obtaining general properties related to classes of groups, and by analysing groups in which the members of some distinguished families of subgroups satisfy these embedding properties.

The basic results of group theory that will be used in the memoir appear in Chapter 1. Among them, we comment on some results about soluble groups, supersoluble groups, nilpotent groups, classes of groups, and p-soluble and p-nilpotent groups for a prime p. In Chapter 2, we present the basic concepts about these embedding properties, as well as some basic results satisfied by them.

Chapter 3 is devoted to the study of lattice properties of these types of subgroups. In this chapter we deepen into the study of the lattices of solitary subgroups and quotient solitary subgroups developed by Kaplan and Levy [KL09] and by Tărnăuceanu [Tăr12b] and we check

that, even though these lattices consist of normal subgroups, they are not sublattices of the lattice of normal subgroups. We also check that the set of all normal solitary subgroups does not constitute a lattice, which motivates the introduction of the concept of subnormal solitary subgroup as a more suitable tool to deal with lattice properties.

In Chapter 4, we study in depth the relations between these embedding properties and classes of groups. We observe that the subnormal solitary subgroups behave well with respect to radicals for Fitting classes and that the residuals for formations are quotient solitary subgroups. We also study conditions under which the radicals with respect to Fitting classes are quotient solitary subgroups and the residuals with respect to formations are solitary subgroups. To finish, we state the natural question of whether the solitary or subnormal solitary subgroups can be regarded as radicals for suitable Fitting classes or whether the quotient solitary subgroups are residuals for suitable Fitting classes. We give a negative answer to this question

Chapter 5 is devoted to the study of groups whose minimal subgroups are solitary, that is, groups with a unique subgroup of order p for each prime p dividing its order. We give a complete classification of these groups and we make some remarks about related problems.

Our contributions to this research line appear in the paper [ERLssb], accepted to be published in *Communications in Algebra*, whose results appear mainly in Chapters 3 and 4, and in [ERLssa], accepted to be published in *Journal of Algebra and its Applications*, whose results appear mainly in Chapter 5. They have been also presented in the *IX Encuentro en Teoría de Grupos* (Spanish Meeting in Group Theory) [ERL12], in the *Predoc Seminar* of the Universitat de València [LC13] and in the *X Congreso Internacional de Investigación Científica* (X International Conference of Scientific Research) organised by the Universidad Autónoma de Santo Domingo [ERL14].