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Corrigendum: Some class size conditions implying solvability of finite groups

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The authors have realized that the proof of Step 1 in [1, Theorem A] is incomplete. Precisely, in the proof of Step 1 we suppose that there exists a π-element x of index m where π is the set of primes dividing m and we have the following situation: $C_G(x)$ is a direct product of a p-subgroup and a p'-subgroup and the class sizes of such p'-subgroup are at most two numbers: 1 or n. However, the case in which all of them are exactly equal to 1, that is, when the p'-subgroup is abelian, is omitted and this is the case we complete here by showing that it cannot happen.

Proof of Step 1 *of Theorem* A. Let π be the set of primes dividing m and suppose that there exists a π-element x of index m. By using the primary decomposition of x we may assume that x is a p-element for some prime $p \in \pi$. Also, if y is a p'-element of $C_G(x)$, then $C_G(xy) = C_G(x) \cap C_G(y) \subseteq C_G(x)$ and thus y has index 1 or n in $C_G(x)$, which is a p'-number. By Lemma 1, $C_G(x)$ can be written as a direct product of a p-subgroup and a p'-subgroup; so $C_G(x) = C_G(x)_p \times C_G(x)_{p'}$. Moreover, the class sizes of $C_G(x)_{p'}$ are 1 or n. If there are elements in this p'-subgroup having both class sizes, then by applying [1, Theorems 4 and 7] we deduce that G is solvable and the theorem is proved. We will prove that the other case leads to a contradiction. We assume then that $C_G(x)_{p'}$ is abelian, whence we may write $C_G(x) = S_x \times T_x$, where S_x is a π-subgroup and T_x is an abelian π-complement of G. By a theorem of Wielandt (see for instance [2, Theorem 9.1.10]) all π-complements of G are conjugate and every π'-subgroup of G lies in some π-complement of G. Also, we notice that every noncentral π'-element of G has index m and that its centralizer is of the same type as of x. In fact, if w is a noncentral π'-element of G, then $w \in T_x^g$ for some $g \in G$. Thus $C_G(x^g) = S_x^g \times T_x^g = C_G(w)$.

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Now, we claim that $n = |G|_{\pi'}/|\mathbf{Z}(G)|_{\pi'}$. Let z be an element of index mn and let us consider its decomposition $z = z_\pi z_{\pi'}$. If $w = z_{\pi'}$ is noncentral, by the above paragraph we have $C_G(w) = S_w \times T_w$, with T_w an abelian π-complement of G. Then, $T_w \subseteq C_G(z)$, which is a contradiction. Hence, we can suppose z to be a π-element. We prove that $C_G(z) = H_z \times K_z$, with H_z a π-subgroup and K_z an abelian π'-subgroup. To see this, take any π'-element $t \in C_G(z)$ and notice that $C_G(zt) = C_G(z) \cap C_G(t) \subseteq C_G(z)$. The maximality of the index of z implies that $C_G(t) \cap C_G(z) = C_G(z)$, that is, $t \in \mathbf{Z}(C_G(z))$, so $C_G(z)$ factorizes as wanted. Now, suppose that there exists a noncentral element $t \in K_z$. Then

$$C_G(z) \subseteq C_G(t) = S_t \times T_t,$$

where T_t is a π-complement of G, and thus, $z \in S_t$ and $T_t \subseteq C_G(z)$, which is a contradiction. Therefore, K_z is central, and so the claim is proved.

Now, we take α an element of index n. By the equality obtained in the paragraph above, we can write $C_G(\alpha) = H_\alpha \times \mathbf{Z}(G)_{\pi'}$ where H_α is some Hall π-subgroup of G. Then, we can assume that α is a π-element, lying in $\mathbf{Z}(H_\alpha)$. On the other hand, H_α contains some Sylow p-subgroup P of G, so in particular, for the p-element x fixed at the beginning of the proof, we have $x^g \in P \subseteq H_\alpha$ for some $g \in G$. It follows that $\alpha \in C_G(x^g) = S_x^g \times T_x^g$, and consequently $\alpha \in S_x^g$, so $T_x^g \subseteq C_G(\alpha)$ and this is the final contradiction. □

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