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A parametric study of optimum road frame bridges by threshold acceptance

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This paper examines the economic optimization of reinforced concrete road frame bridges by threshold acceptance. The formulation of the problem includes 50 discrete variables: three geometrical ones, three types of concrete and 44 reinforcement bars and bar lengths. Design loads are in accordance with the national codes for road bridges. An internal matrix method program computes the stress resultants and envelopes of the frame bridges. The evaluation module includes the ultimate limit state of fatigue plus other commonly specified limit states of service and ultimate flexure, shear and deflections. Solutions are evaluated following the Spanish code for structural concrete. This study reviews the main factors affecting the design of frame bridges. The study then presents a parametric study of commonly used road frame bridges from 8 to 16 m in horizontal span for different fills and earth covers conditions. The evolution of the total, concrete and steel cost is examined with regard to the key parameters, resulting in practical rules of thumb for optimum frames. Finally, it is shown that the steel-to-concrete cost has a fair influence on the characteristics of the optimum road frame bridges.

Keywords: Economic optimization, Heuristics, Concrete structures, Structural design

The present design of economical concrete structures is much conditioned by the experience of structural engineers. Most traditional procedures adopt cross-section dimensions and material grades arising from sanctioned common practice. Once the structure is defined, it follows the analysis of stress resultants and the computation of passive and active reinforcement that satisfy the limit states prescribed by concrete codes. If the dimensions or material grades are insufficient, the structure is redefined on a trial-and-error basis. Such process leads to safe designs, but the economy of the concrete structures is, therefore, very much linked to the experience of the structural designer.

An alternative approach to design can be found in structural optimization procedures. In this sense, artificial intelligence has dealt since its appearance in the mid 1950s with a variety of fields that include the solution of constrained problems[1,2], such as the design of structures which is a problem of selection of design variables subject to structural constraints. The optimization approach defines the structure on the basis of the design variables, automatically calculates and validates the structure and then redefines it by means of an optimization algorithm that controls the flow of a large number of iterations in the search for the optimum structure. This approach includes full verification of a code of practice and, hence, it goes beyond an optimization procedure and becomes an explicit way of designing the structure. However, it is worth mentioning that experience is crucial for the development of computer design models since design involves more than a simple application of codes of practice.

Structural optimization methods are clear alternatives to experience-based designs. They may be categorized as either exact methods or heuristic methods. The exact methods are the traditional approach. They are based on the calculation of optimal solutions following iterative techniques of linear programming[3,4]. These methods are very efficient for a few design variables, but computing time becomes prohibitive for large numbers of variables. The heuristic methods are linked to the evolution of artificial intelligence procedures. This group includes numerous search algorithms[5-9], such as genetic algorithms, simulated annealing, tabu search, neural networks and ant colonies among others. These methods have been applied successfully in areas other than structural engineering, for example, transport engineering[10]. A thorough review of structural optimization methods can be found in the study by Cohn and Dinovitzer[11], who reported on the gap between theoretical studies and the practical application of optimization methods in civil and aeronautical engineering. Additionally, they affirmed

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that most studies focused on steel structures while only a small fraction dealt with reinforced concrete (RC) structures. A review of non-heuristic structural concrete optimization studies is reported by Sarma and Adeli[12].

Among the first studies of heuristic optimization applied to steel structures, the contributions of Jenkins[13] and of Rajeev and Krishnamoorthy[14] in the early 1990s are to be mentioned. As regards concrete structures, pioneering applications include the 1997 work by Coello *et al*.[15], who applied GA to sections of simply supported RC beams, together with the study of prestressed concrete beams by Leite and Topping[16]. Various studies[17-25] describe examples of beams, columns, building frames, flat slab buildings, T-beam bridge decks and water tanks. Recently, the authors' research group has presented studies of earth retaining walls, building frames, road vaults and rectangular section hollow bridge piers[26-31].

The present study concentrates on optimum frame bridges, an example of which is given in Fig. 1. Box frame bridges are used to solve the intersection of transverse traffic or hydraulic courses with the main upper road. Spans range from 3 to 20 m. Box frames are preferred when there is a low bearing strength terrain. The depth of the top and bottom slabs is typically designed between 1/10 and 1/15 of the horizontal free span. The depth of the walls is typically designed between 1/12 of the vertical free span and the depth of the slabs. Bridge box frames must satisfy all the limit states required for an RC structure to sustain the traffic and earth loads prescribed by the codes.

Proposed Optimization Model for Frame Bridges

The problem of optimization established in the present study consists of an economic optimization of the structural design of frame bridges. It deals with the minimization of the objective function F of Eq. (1), satisfying also the constraints of Eq. (2).

$$F(x_1, x_2, ...x_n) = \sum_{i=1,r} p_i * m_i(x_1, x_2, ..., x_n) \qquad \text{... (1)}$$

$$g_j(x_1, x_2, ...x_n) \le 0 \qquad \text{... (2)}$$

Note that the objective function in Eq. (1) is an economic function expressed as the sum of unit prices multiplied by the measurements of construction units (concrete, steel, formwork, etc). And that the constraints in Eq. (2) are all the service and ultimate limit states that the structure has to satisfy, as well as the geometrical and constructability constraints of the

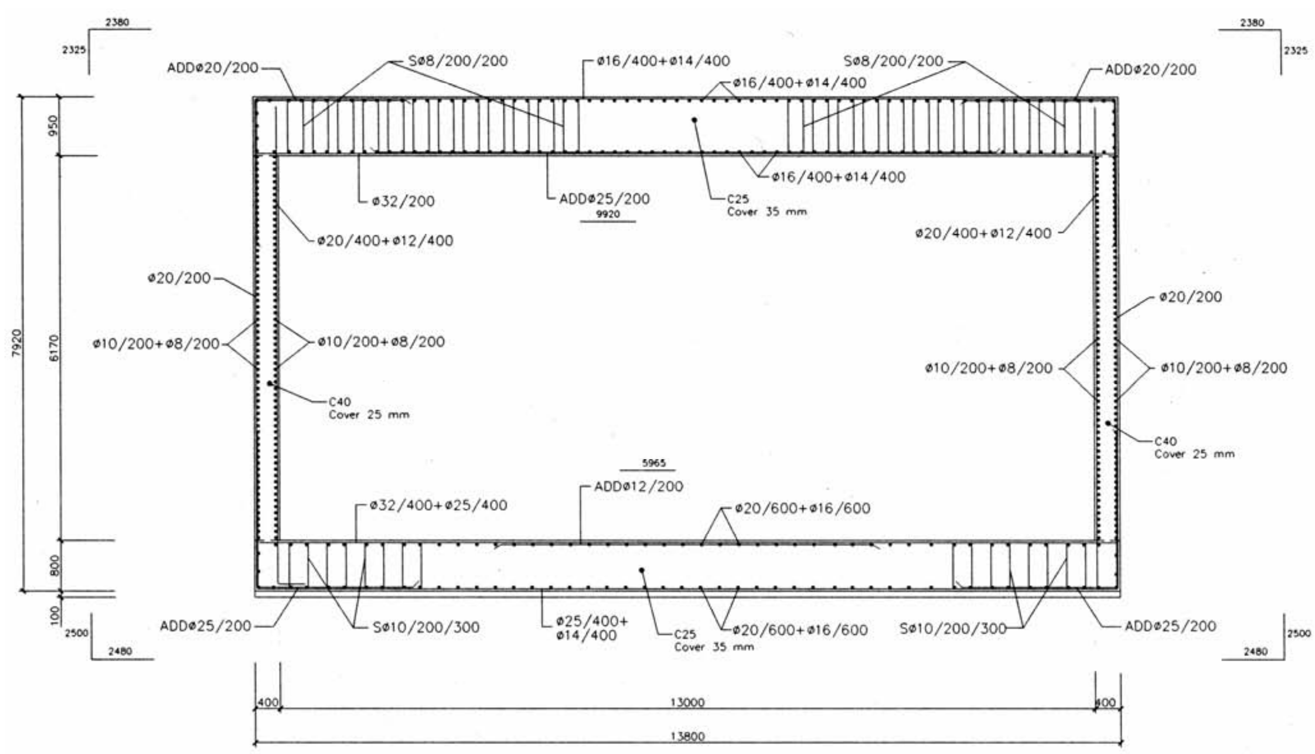

Fig. 1—Design example of RC frame bridge

problem. Unit prices considered for the RC frame bridges are given in Table 1 and are obtained from Spanish contractors of road construction.

The proposed optimization model was described in detail in a previous publication[28]. Figure 2 specifies the 50 variables considered in this study, including three geometrical values (the depth of the walls and slabs), three different grades of concrete for the three types of elements and 44 types of reinforcement bars and bar lengths following a standard set-up. All variables are discrete in this analysis. Steel grade corresponds to EHE Spanish B500S in all cases (yield stress of 500 MPa). Flexural bars include variable length corner reinforcement bars and positive bending reinforcement in the top and bottom slabs. Shear reinforcement includes two zones of variable definition in slabs (corner and mid-span) and three variable zones in walls (bottom, middle and top). The depth of the top slab can vary between a minimum of 400 mm and a maximum of 1650 mm in steps of 50 mm. As regards concrete grades, they can vary between 25 and 50 MPa cylinder compressive strength in steps of 5 MPa. Finally, steel bars can vary between 6 and 40 mm following a standard sequence. The key parameters are the horizontal free span, the vertical free span, the earth cover, the traffic loads, the earth fill properties, the ballast coefficient of the bearing and the partial safety coefficients. Structural constraints considered followed standard Spanish provisions for the design of this type of structure[32,33], which requires checks into the service and ultimate limit states of flexure and shear for the stress envelopes given the traffic loads and the earth fill. Traffic loads considered are a uniformly distributed load of 4 kN/m^2 and a heavy vehicle of 600 kN. Stress resultants and reactions were calculated by an internal matrix method program using a 2-D mesh with

Table 1—Basic prices of the cost function of the box road frames

Unit	Cost (€)
kg of steel (B-500S)	0.583
m^2 of lower slab formwork	18.030
m^2 of wall formwork	18.631
m^2 of upper slab formwork	30.652
m^3 of scaffolding	6.010
m^3 of lower slab concrete (labour)	5.409
m^3 of wall concrete (labour)	9.015
m^3 of upper slab concrete (labour)	7.212
m^3 of concrete pump rent	6.010
m^3 of concrete HA-25	45.244
m^3 of concrete HA-30	49.379
m^3 of concrete HA-35	53.899
m^3 of concrete HA-40	58.995
m^3 of concrete HA-45	63.803
m^3 of concrete HA-50	68.612

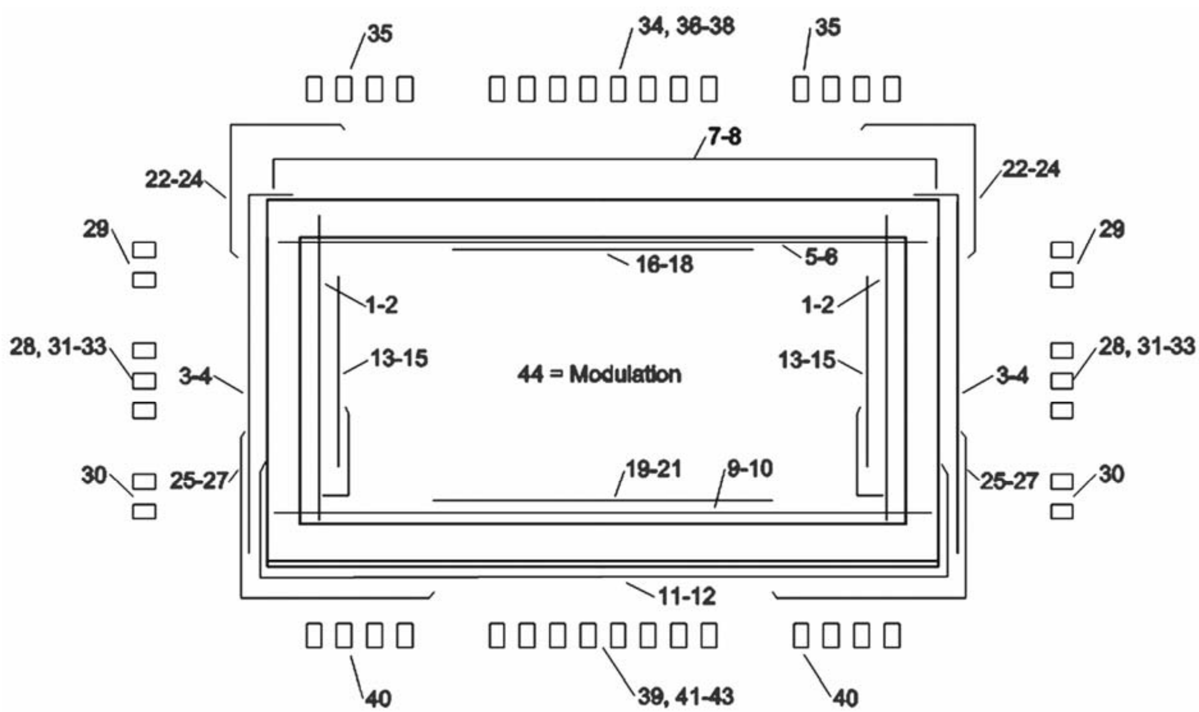

Fig. 2—Reinforcement related variables of the RC frame bridge

40 elements and 40 sections (out-of-plane bending moments had to be assumed as a practical one-fifth proportion of in-plane bending moments). Deflections were limited to 1/250 of the free span for the quasi-permanent combination. The calculation includes the tension stiffening effect following Eurocode 2 provisions[34]. Although concrete and steel fatigue is rarely checked in road structures, it was considered in this study since this ultimate limit state cannot be neglected, as it has been reported elsewhere[28]. The loading considered was a 468 kN heavy vehicle prescribed for fatigue by the Spanish loading code for bridges[32], and the stresses were checked against Eurocode 2 limitations for fatigue[34].

Proposed Threshold Accepting Strategy

The search method used in this research is the threshold accepting (TA henceforth), a method which was proposed by Dueck and Scheuer[35] as an alternative to the simulated annealing algorithm[36]. The algorithm starts with a randomly generated feasible solution and a high initial threshold accepting value. The initial working solution is changed by a small random move in the values of certain variables. The new current solution is evaluated in terms of cost. Greater cost solutions are accepted when the cost increment is less than the current threshold accepting value. The current solution is then checked against structural constraints and if it is feasible, it is adopted as the new working solution. The initial threshold level value is decreased geometrically by means of a coefficient k. A number of iterations called cycles are allowed at each step of the threshold accepting value. The algorithm stops when the threshold accepting value is a small percentage of the initial value (typically 1% or 1-2 cycles). The TA method is able to surpass local optima at high-medium threshold values and gradually converges as the threshold value drops to zero. The TA method requires calibrating the initial threshold accepting value, the length of the cycles and the reducing coefficient. The initial threshold value was that indicated by Medina[37]. Computer runs were repeated 25 times to obtain minimum, mean and deviation of the results with regard to the best result.

The TA algorithm was programmed in Compaq Visual Fortran Professional 6.6.0. The algorithm was calibrated using a bridge box road frame with 13 m horizontal free span, 6.17 m vertical free span and 1.5 m earth cover (additional parameters are 10 MN/m^3 stiffness modulus of the foundation, specific weight of the fill of 20 kN/m^3, a 30° internal friction angle of the fill and partial safety coefficients of 1.5 for loading and 1.5-1.15 for concrete-steel as materials). The TA algorithm required 1000 iteration cycles and a reducing coefficient of 0.95. The most efficient move found for the TA algorithm was a random variation of up to 5 variables of the 50 of the problem. A typical computer run lasted about 110 s.

Key Factors Affecting Optimum Frame Bridges

There are seven key factors affecting the design: (i) the horizontal free span, (ii) vertical free span, (iii) earth cover, (iv) traffic loads, (v) earth fill properties, (vi) ballast coefficient of the bearing, and (vii) partial safety coefficients. The horizontal free span of the parametric study below concentrates on the most common traffic intersections, which usually vary from a minimum of 8 m to a maximum of 16 m clearance. The 8 m clearance is typical of narrow secondary roads measuring 6 m in width plus two additional 1 m shoulders. The 16 m span is typical of roads with a 12 m platform plus two additional shoulders of 2 m. The vertical free span for these traffic intersections generally varies between 5 and 6 m, since the vertical clearance is between 4.5 to 5 m. The present study fixes this vertical free span at 5.5 m. Results for 5-6 m in height are quite similar, and the adopted value is considered a representative typical value for this type of intersections.

The earth cover usually varies between a minimum of 0.5 m and a maximum of 5 m. This is so because most embankments usually measure up to about 10-12 m in height, which, together with the vertical free span and the thicknesses of the slabs, makes earth covers of up to 5 m the most typical measurements in road construction. The characteristics of the earth fill have a moderate influence in the frame design. Typical types of fill considered in practice can be found elsewhere[30], where three types of fill are reported: F_1, F_2 and F_3 with 35°, 30° and 24° of internal friction angle and 22, 20 and 18 kN/m^3 of specific weight. The F_1 fill corresponds to a high quality, coarse granular fill while the F_3 fill corresponds to low quality fills of fine soils with low plasticity and the F_2 fill corresponds to typical intermediate fills of granular soils with more than 12% being fines. We assume that the fill is an intermediate type F_2 with an internal friction angle of 30° and a specific weight of 20 kN/m^3. Accordingly, the active and resting coefficients of earth pressure adopted are 0.33 and 0.50, respectively.

The traffic loads depend on the code of bridge loads adopted and vary from country to country. As mentioned earlier, the present study adopts the Spanish code of loads in road bridges[32], which prescribes a uniformly distributed load of 4 kN/m^2 and a heavy vehicle of 600 kN. The load on the top slab is made of the self weight of the slab, the earth cover load and the traffic loads; thus, as the earth cover increases, the relevance of the traffic loads diminishes. Hence, results for a high earth cover of 5 m are expected to be quite similar regardless of the loading code used. The Spanish loading code is rather similar to Eurocode 1 loading[38], which prescribes a uniformly distributed load of 9 kN/m^2 and a tandem vehicle of 600 kN in the most heavily loaded lane. The difference in the uniform load of 5 kN/m^2 is equivalent to 0.25 m of extra earth cover and the sizes of the heavy vehicles are quite similar. The Spanish heavy vehicle is made of 6 point loads spaced 1.50 m in the traffic direction and 2 m in the transverse direction, whereas the Eurocode heavy vehicle is a four point loads spaced 2×2 m. Therefore, the Eurocode heavy vehicle is slightly smaller in plan and, thus, it gives rise to somewhat larger vertical pressures. As a rough and tentative generalization, the Eurocode traffic loads may be assimilated to the Spanish traffic loads with an additional 1 m of extra earth cover, e.g., results for 2 m of earth fill for the Eurocode loading may be considered as equal to the 3 m of earth cover for Spanish traffic loading.

The ballast coefficient of the bearing mainly affects the design of the bottom slab. Better coefficients imply fewer stress resultants in the bottom slab and, hence, less thickness and fewer materials. This study takes a conservative approach by adopting a sufficiently low value of 10 MN/m^3 which appears to be low enough to represent a lower-bound of the foundation stiffness in most cases. Finally, the partial safety factors adopted are 1.35-1.50 for permanent-live loading and 1.50-1.15 for concrete-steel as materials. These coefficients might be reduced with an intense level of quality control, but again the present study takes a conservative approach in this respect.

Parametric Study of RC Road Frame Bridges

A parametric study for varying horizontal spans and earth covers with the TA optimization model is presented. Five horizontal spans of 8, 10, 12, 14 and 16 m were considered. Six earth covers were considered ranging from 0.50 to 5m; namely 0.5, 1, 2, 3, 4 and 5 m. The vertical free span is kept constant at 5.5 m. Hence, a total of 30 bridge frames were analysed. For this parametric study, certain practical limitations were imposed on the optimization process: neither extra reinforcement in the interior face nor shear reinforcement in the walls was allowed; and the concrete grade was fixed at 25 MPa, the typical value for the entire box frame. The primary economic, geometric and reinforcement characteristics are examined. The results of the parametric study lead to practical rules for the preliminary design of optimum structures. The results are discussed together with the results of a regression analysis. The corresponding functions are valid approximations within the range of the studied parameters and therefore careful consideration is required when extrapolation is carried out.

Analysis of the total cost

Figures 3 and 4 show the variation in the total cost of the frame bridges for the selected geometrical dimensions. Figure 3 illustrates the cost variation with regard to the horizontal span for different earth covers while Fig. 4 shows the cost variation as a function of the earth cover for distinct horizontal span values. The total cost evolution as a function of the horizontal span leads to a very good quadratic correlation. The cost increments of the frame bridges are due to higher material costs, necessary to resist increased member forces and to satisfy deflection requirements. Additionally, formwork, falsework and material costs rise since the geometric variation results in linear variation of the shear force, quadratic variations of the bending moment and biquadratic change of the deflections. The total costs increase on the average by a factor of 2.5 when the horizontal span increases from 8 m to 16 m. Note that the R^2 regression coefficients in Fig. 3 are almost 1, which indicates a nearly functional relation.

With varying earth covers for constant horizontal spans, a linear correlation is found for the total cost (see Fig. 4). Again, the high correlation factor of nearly one indicates an almost functional relation. The reason for this can be found in the somewhat linear influence of the earth cover on the member forces and deflections and, correspondingly, on the design cost. Given an artificial tunnel with a longitudinal earth cover development, the above chart can be used to determine the optimum number of different sections and the earth covers for which these sections must be designed. Designing each segment of a tunnel

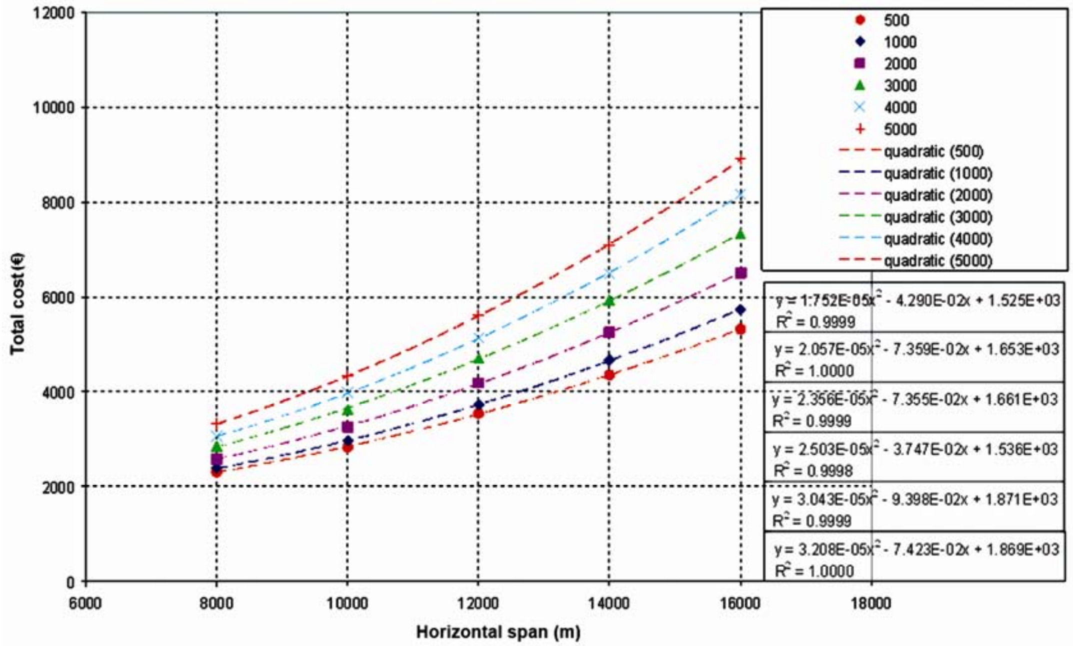

Fig. 3—Total cost versus horizontal span

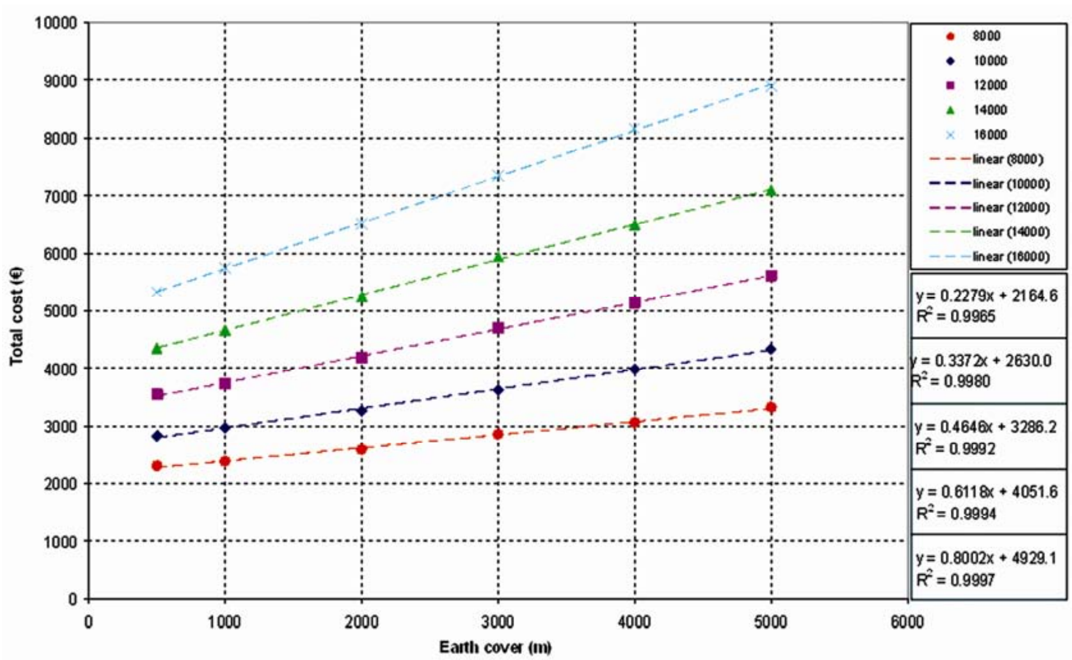

Fig. 4—Total cost versus earth cover

individually is not optimal not only because of the increased design and organization costs but also because of the unattained benefits of repetition in the construction process.

Analysis of the total concrete cost

The development of the total concrete cost is shown in Figs 5 and 6, in an analogous way to the total cost charts, i.e., Fig. 5 depicts the total concrete cost for a range of horizontal spans and Fig. 6 depicts the total concrete cost for a series of earth covers. Very good adjustment is achieved using a quadratic function for Fig. 5. The mean concrete cost increases by a factor of 2.95 if the horizontal span increases from 8 m to 16 m, which is a higher increment than

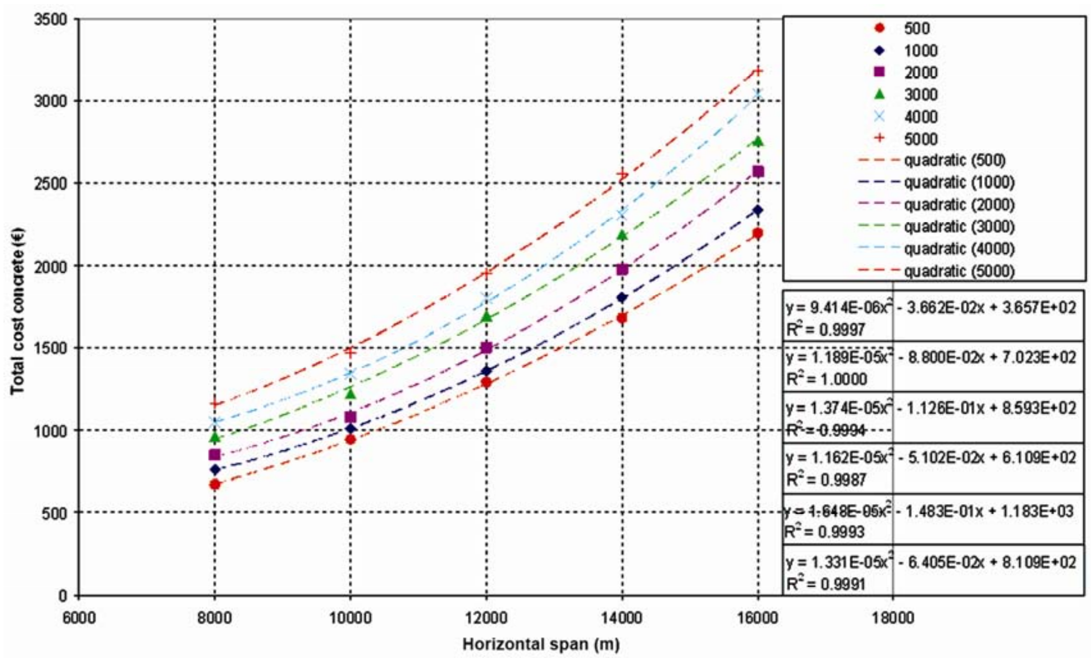

Fig. 5—Total concrete cost versus horizontal span

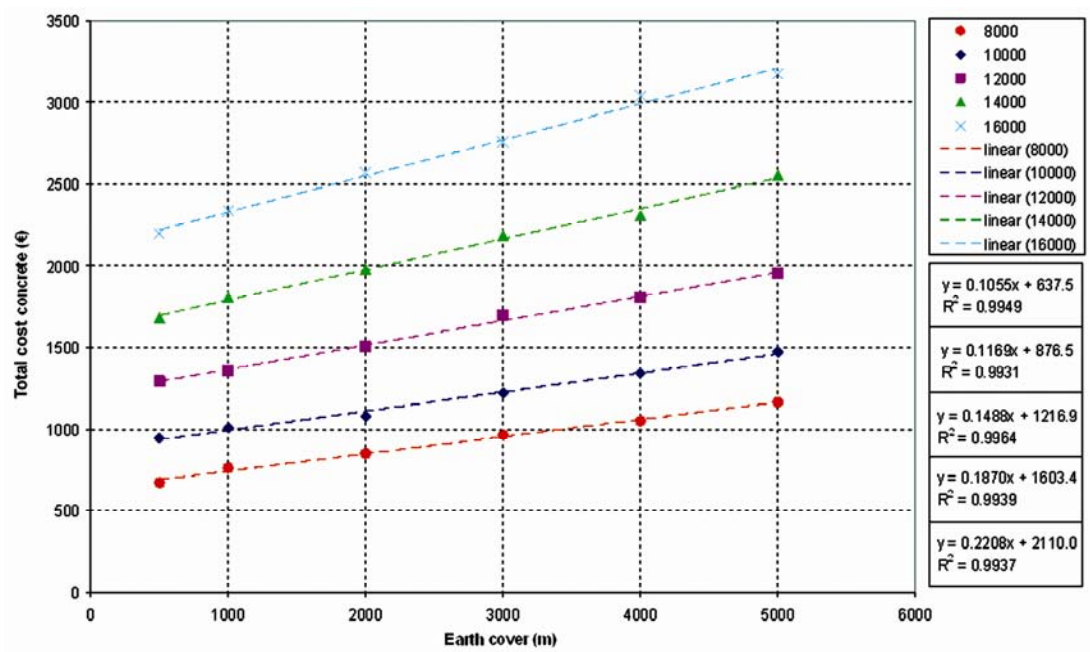

Fig. 6—Total concrete cost versus earth cover

that for the total cost, which increased by a factor of 2.5. However, one should take into account that the total costs include the costs of formwork and falsework which varied by less than a factor of 2. It is worth noting that total costs in Figs 5 and 6 are for a concrete class of C25, whose unit cost is about 57.56 €/m^3 (see Table 1). Hence, Figs 5 and 6 can be used to obtain the approximate measurement of concrete per unit length, i.e., m^2/m of concrete.

For the different earth covers, a linear relationship fits the data adequately. The total concrete costs increase with a change in the earth cover from 1 m to 5 m by a factor of 1.4 on average. For a frame with a span of 8 m and 16 m increasing the earth cover by

1 m increases the total concrete cost by 110 € and 220 €, respectively, as noted in the inclination variation of the graphs and in the regression functions.

Analysis of the total steel cost

In this section, the variation of the total steel cost is analysed. Figure 7 shows the variation as compared to the horizontal span for different earth covers, and Fig. 8 illustrates the variation as a function of the earth cover for varying horizontal span values. Again, it is worth noting that total costs in Figs 7 and 8 are calculated for a steel class of B500S (yield stress of 500 MPa); whose unit cost is 0.583 €/kg (see Table 1). Therefore, Figs 7 and 8 can be used to determine the measurement of steel kilograms per unit length, i.e., kg/m of steel. Thus, expressions in Figs 5-8 may be used to obtain the basic

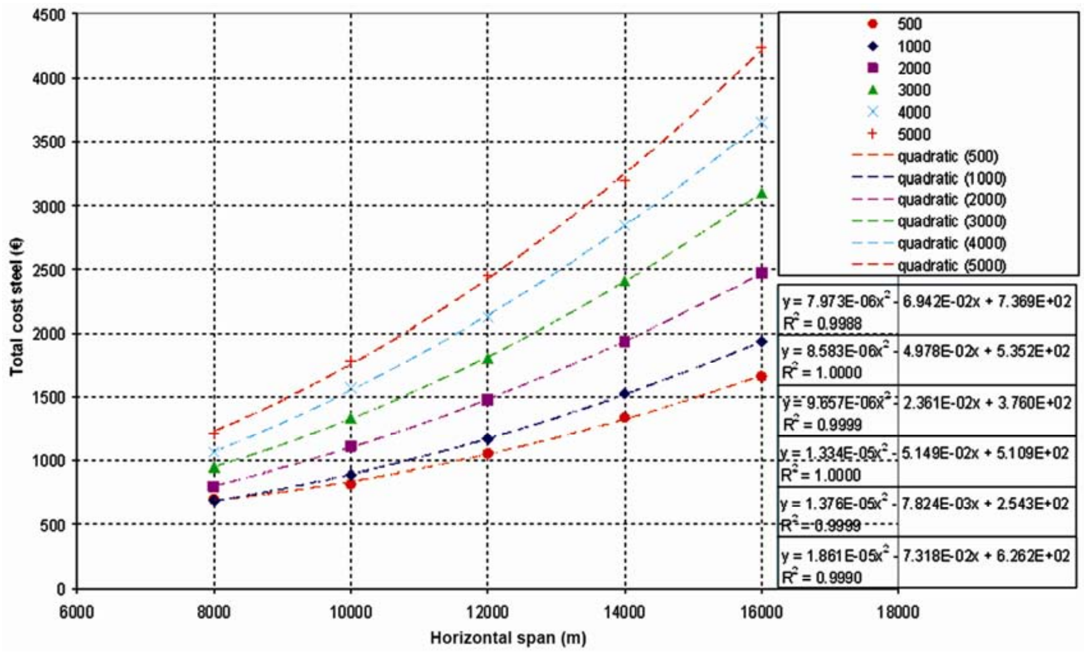

Fig. 7—Total steel cost versus horizontal span

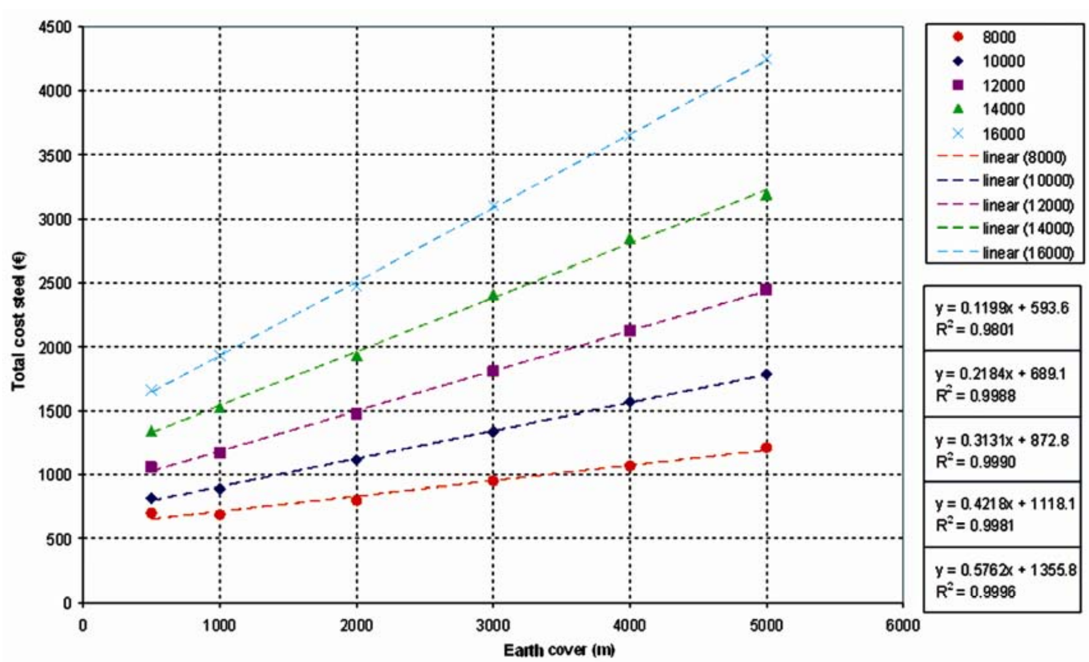

Fig. 8—Total steel cost versus earth cover

measurements of structural materials and, hence, to estimate the total cost of the frame materials.

Concrete costs are found to be similar to those of steel, which increased by a factor of 3.2 for distinct horizontal spans from 8 m to 16 m. This is a higher increase than for the total concrete cost, which changed by a factor of 2.95. For the specified horizontal spans, the increase in the steel cost differs greatly. In this case for the 8 m and 16 m span, an increment of 120 € and 576 € per additional meter of earth cover is detected, compared to the 110 € and 220 € costs in the concrete.

Analysis of the upper slab depth and mid-span reinforcement area

The correlation quality is in general inferior when analysing single variables like the wall depth or the slab depth. One reason for this is the discrete nature of the variables, which results in a dependency of the values of one variable on those of other variables. Figures 9 and 10 illustrate the variation in the upper slab depth and in the mid-span reinforcement with the horizontal span. The span to thickness ratio for frames with 8 m of horizontal span ranges from a maximum of 20 for 0.50 m of earth cover to a minimum of 11.27

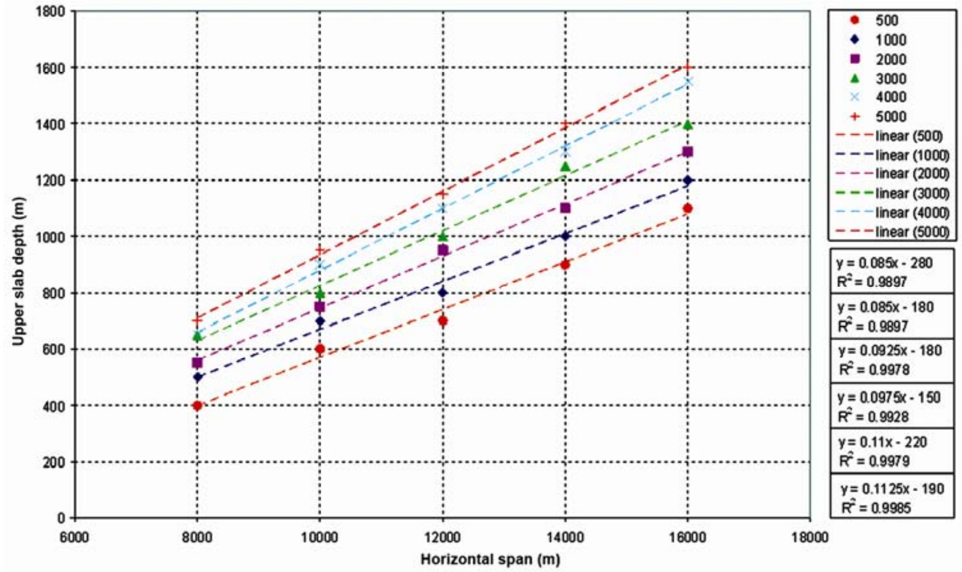

Fig. 9—Upper slab depth versus horizontal span

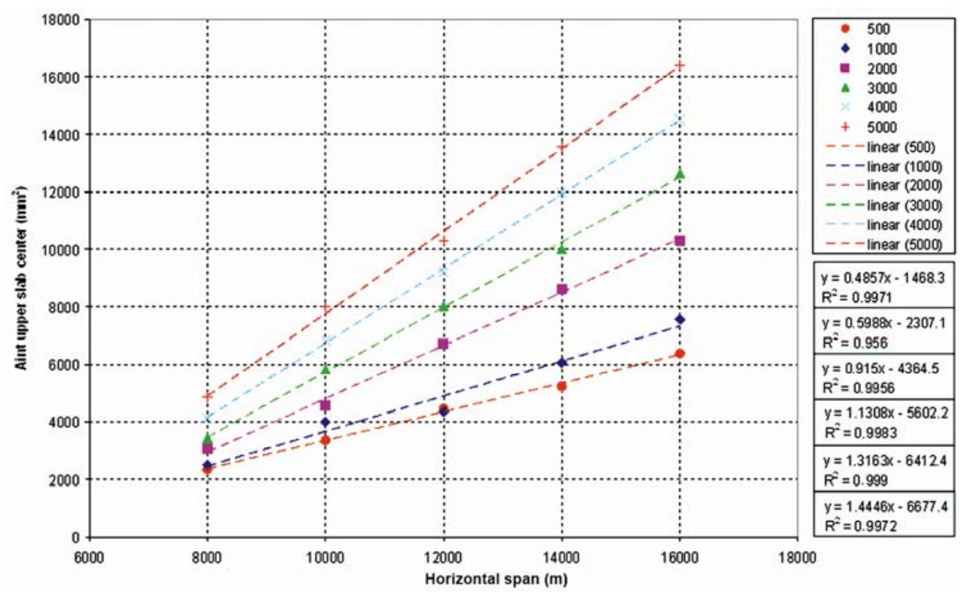

Fig. 10—Mid-span interior reinforcement area versus horizontal span

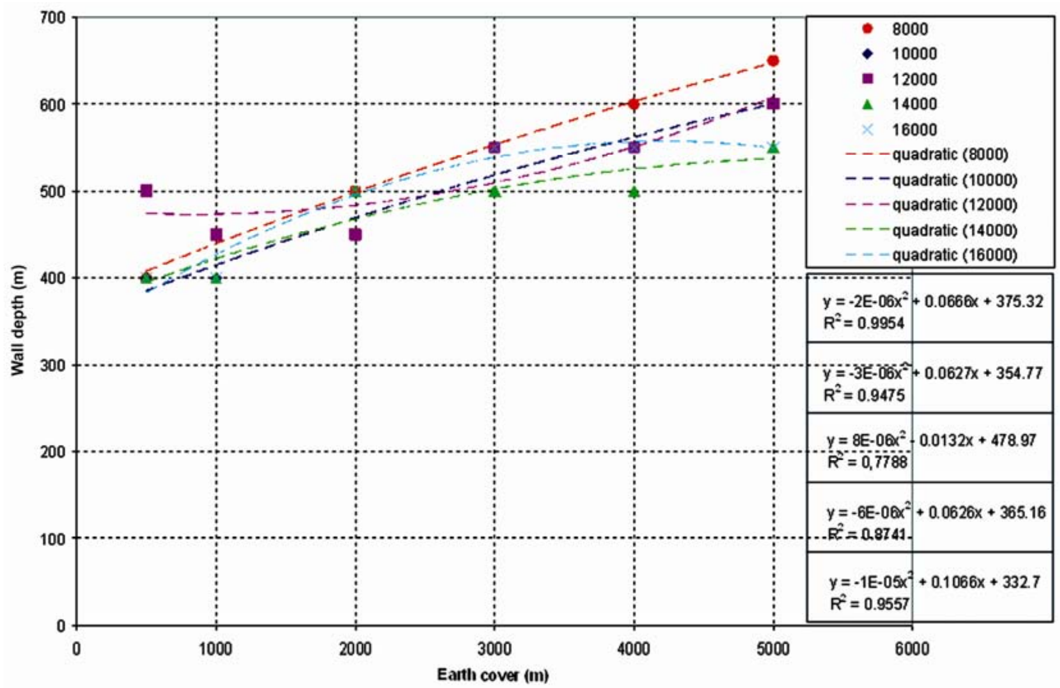

Fig. 11—Wall depth versus earth cover

for 5 m of earth cover. In the case of frames with 16 m of horizontal span, these values vary from a maximum of 14.81 to a minimum of 9.94. As derived from the expressions, the upper slab depth augments by a factor of 2.4 with an increase in the horizontal span from 8 to 16 m. The interior area of reinforcement augments by a factor of 2.9 for the 8-16 m increase in the horizontal span. A comparison between the upper and lower slab depths shows that the lower slab depth is on average 80% of the upper slab depth. The mid-span reinforcement of the lower slab is approximately 60% of the upper slab mid-span reinforcement area. It is worth noting that the amount of steel variables is very large and that due to paper size limitations we only provide as an example of steel sizing. Note that this omission is of secondary importance since structural practitioners have alternative methods to design the steel reinforcement once the geometry is known. It is important to note that these practitioners methods are not objective since the design can be achieved by many subjective choices of steel reinforcement. In contrast, the present model does not design the reinforcing steel in a subjective way, but checks the reinforcing steel, which is an objective procedure.

Analysis of the wall depth

The second geometric variable is the wall depth. Compared to the earth cover, the average wall depth increases by 40% (see Fig. 11). In general, the wall depth is slender with span-to-depth ratios ranging from as high as 14 to 8.5. The wall depth has a nonlinear response, which is caused by its dependency on other variables like the upper and the lower slab depth. To arrive at an optimal design the practising engineer can select the wall depth to be 400 mm plus 4% of the earth cover as an average value and check if shear reinforcement is required in the wall. If this is the case the wall depth has to be increased, as the optimum structures do not have shear reinforcement in the walls as mentioned earlier.

Influence of steel cost on steel quantity

To determine the influence of the steel cost on the optimum structure characteristics, the 30 optimum structures of the parametric study were optimized for two different steel costs (0.896 €/kg and 0.294 €/kg). One expects that a change in the unit prices would result in a modification of the optimum structure. Steel quantity is examined and findings indicate that an increase in the steel cost fairly reduces the steel quantity in the optimum structures. As noted in Table 2, the change in the steel quantity is moderate falling only from 79.60 to 74.51 kg/m^3 for a 50% increase in the steel cost. Reducing the steel cost to 50% of the actual price results in an increase in the steel quantity to 85.41 kg/m^3. It thus may be tentatively concluded that variations in the steel cost

Table 2—Steel quantity depending on steel cost

Steel cost (€/kg)	Steel quantity (kg/m^3)
0.294	85.412
0.583	79.604
0.896	74.511

result in moderate changes in the frame characteristics.

Conclusions

From this study, the following conclusions may be derived:

(i) RC bridge frames can potentially use heuristic algorithms for the advanced automatic design of real concrete structures. It is essential to note that the present model eliminates the need for experience-based rules of design.

(ii) The total cost, the steel cost and the concrete cost may be estimated with a high degree of accuracy. Quadratic and linear relationships are observed for the horizontal span and for the earth cover, respectively.

(iii) Single variables like the depth of the walls and slabs lead to more variation. However, the data figures may be used for the preliminary optimum geometry design.

(iv) The characteristics of the optimum structures are fairly influenced by the steel to concrete cost.

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References

1 Jones M T, *Artificial Intelligence Application Programming* (Charles River Media, Hingham-Massachusetts), 2003.
2 Sriram R, *Adv Eng Inform,* 20 (2006) 3-5.
3 Fletcher R, *Practical Methods of Optimization* (Wiley, Chichester), 2001.
4 Hernandez S & Fontan A, *Practical Applications of Design Optimization* (WIT-Press, Southampton), 2002.
5 Glover F & Laguna M, *Tabu Search*, (Kluwer Academic Publishers, Boston), 1997.
6 Coello C A, *Comput Method Appl Mech Eng*, 191 (2002) 1245-1287.
7 Dorigo M & Stüzle T, *Ant colony optimization* (MIT Press, Cambridge), 2004.
8 Dreo J, Petrowsky A, Siarry P & Taillard E, *Metaheuristics for hard optimization. Methods and case studies* (Springer, Berlin Heidelberg), 2006.
9 Eiben A E & Smith J E, *Introduction to Evolutionary Computing* (Springer, Berlin Heidelberg), 2007.
10 Yepes V & Medina J R, *J Transp Eng-ASCE*, 132(4) (2006) 303-311.
11 Cohn M Z & Dinovitzer A S, *J Struct Eng-ASCE*, 120(2) (1994) 617-649.
12 Sarma K C & Adeli H, *J Struct Eng-ASCE*, 124(5) (1998) 570-578.
13 Jenkins W M, *Struct Eng*, 69(24/17) (1991) 418-422.
14 Rajeev S & Krisnamoorthy C S, *J Struct Eng-ASCE*, 118(5) (1992) 1233-1250.
15 Coello C A, Christiansen A D & Santos F, *Eng Comput*, 13 (1997) 185-196.
16 Leite J P B & Topping B H V, *Adv Eng Software*, 29(7-9) (1998) 529-562.
17 Rafiq M Y & Southcombe C, *Comput Struct*, 69(4) (1998) 443-447.
18 Rajeev S & Krisnamoorthy C S, *Comput-Aided Civil Infrastruct Eng*, 13 (1998) 63-74.
19 Leps M & Sejnoha M, *Comput Struct*, 81 (2003) 1957-1966.
20 Lee C & Ahn J, *J Struct Eng-ASCE*, 129(6) (2003) 762-774.
21 Camp C V, Pezeshk S & Hansson H, *J Struct Eng-ASCE*, 129(1) (2003) 105-115.
22 Sahab M G, Ashour A F & Toporov V V, *Eng Struct*, 27(3) (2008) 313-322.
23 Govindaraj V & Ramasamy J V, *Comput Struct*, 84 (2005) 1957-1966.
24 Srinivas V & Ramanjaneyulu K, *Adv Eng Softw*, 38(7) (2007) 475-487.
25 Barakat S A & Altoubat S, *Eng Struct*, 31(2) (2009) 332-334.
26 Carbonell A, Martinez F, Yepes V, Hospitaler A & Gonzalez-Vidosa F, in *Proceedings of the tenth international conference on computer aided optimum design in engineering*, edited by Hernandez S & Brebbia C A (WIT Press, Southampton) (2007) 141-150.
27 Gonzalez-Vidosa F, Yepes V, Alcala J, Carrera M, Perea C & Paya-Zaforteza I, in *Simulated Annealing*, edited by Tan C M (I-Tech Education and Publishing, Vienna) (2008) 307-320.
28 Perea C, Alcala J, Yepes V, Gonzalez-Vidosa F & Hospitaler A, *Adv Eng Software*, 39(8) (2008) 676-688.
29 Paya-Zaforteza I, Yepes V, Hospitaler A & Gonzalez-Vidosa F, *Comput-Aided Civil Infrastruct Eng*, 23 (2008) 596-610.
30 Yepes V, Alcala J, Perea C & Gonzalez-Vidosa F, *Eng Struct*, 30(3) (2008) 821-830.
31 Paya-Zaforteza I, Yepes V, Gonzalez-Vidosa F & Hospitaler A, *Eng Struct*, 31(7) (2009) 1501-1508.
32 M. Fomento, *IAP-98. Code about the actions to be considered for the design of road bridges (in Spanish)*, (M. Fomento, Madrid), 1998.
33 M. Fomento, *EHE Code of Structural Concrete (in Spanish),* (M Fomento, Madrid), 1998.
34 CEN, *Eurocode 2. Design of Concrete Structures. Part 2: Concrete Bridges*, (EN, Brussell), 1996.
35 Dueck G & Scheuer T, *J Comput Phys*, 90 (1990) 161-175.
36 Kirkpatrick S, Gelatt C D & Vecchi M P, *Science*, 220(4598) (1983) 671-680.
37 Medina J R, *J Waterw Port Coast Ocean Eng-ASCE*, 127(4) (2001) 213-221.
38 CEN, *Eurocode 1. Basis of design and actions on structures. Part 3: Traffic loads on bridges*, (EN, Brussells), 1995.