FDMs and Transformation Methods for nonlinear BS equations

Abstract

Vera Egorova started her position in ITN Strike as Early-Stage-Researcher (ESR4) on September 1, 2013. The work is under the supervision of Prof. Lucas Jódar and Prof. Rafael Company. This Project is focused on numerical analysis and computing of finite difference schemes for the nonlinear Black-Scholes (BS) model. The American style option is the starting point of interest and the front-fixing method is a useful and efficient tool for the free boundary value problem associated to the American option. There exist several front-fixing transformations that are considered. A new, novel, transformation is applied to the American call option pricing problem. An analysis of properties for the numerical solutions of option pricing models as the positivity, stability and consistency of the solutions is done.

Front-fixing method

The idea of the front-fixing method is to transform the problem into a new non-linear partial differential equation where the free boundary is a new variable of the PDE. An explicit Finite Difference Method (FDM) is constructed for the numerical solution.

First of all, the method is applied to American put option pricing. An explicit finite difference method (FDM) is constructed for the numerical solution. Consistency and stability of the numerical scheme with the PDE problem was established (see Fig. 1).

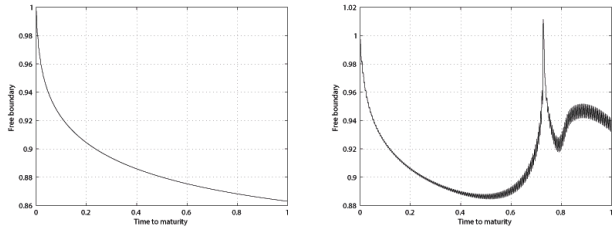

Figure 1: *Optimal stopping boundary. Left: satisfying the stability condition. Right: stability condition is broken.*

The positivity and monotonicity of the solution was proved under the stability conditions. All theoretical results were confirmed by the numerical tests. Details can be found in the papers:

- *Numerical Solution of American Option Pricing Models Using Front-Fixing Method* by V. Egorova, R. Company and L. Jódar, chapter 30, pp. 311-319, in book: *Mathematical Modeling in Engineering and Social Sciences*, Nova Science Publishers, Inc. New York, 2014. ISBN: 978-1-63117-339-4.
- *Solving American Option Pricing Models by the Front Fixing Method: Numerical Analysis and Computing* by R. Company, V.N. Egorova, and L. Jódar in Abstract and Applied Analysis, Volume 2014 (Article ID 146745).

The American call option with dividends is determined by the linear BS equation with an additional term. Therefore it is more complicated by the point of view of the numerical analysis. The same transformation was also applied to the multi-asset American option that allows the reducing of dimension (tends to a 1-dimensional problem), such as exchange options. These results were presented at ECMI-2014, Taormina.

- *A Positive, stable and consistent front-fixing numerical scheme for American Options*, by R. Company, V.N. Egorova, and L. Jódar, Subm. to Proc. ECMI-2014.

An numerical analysis was done for this method. There appear additional conditions for the stability due to the dividend term. There exist parameters of the problem for which the monotonicity of the solution couldn't be guaranteed. This drawback was eliminated by using the new transformation. Under this transformation a nonlinear PDE with homogeneous boundary conditions, independent of the free boundary, is obtained. This fact simplifies the numerical analysis of the finite difference scheme.

The proposed explicit finite difference scheme preserves theoretical properties of the solution. Dealing with prices it is important to guarantee that the proposed numerical solutions are non-negative. Our scheme guarantees this property, as well as monotonicity of the free boundary and the option price. Numerical experiments show that the method is efficient and accurate in comparison with other implicit methods.

The explicit scheme is conditionally stable and consistent with the second order in space and the first order in time. Convergence of the method is demonstrated numerically.

The explicit FDM was compared to the unconditionally stable, implicit, scheme. This last method needs additional calculations of the inverse Jacobian matrix in each iteration. It was shown that for the same step sizes the explicit method is faster than the implicit one.

- *Constructing Positive Reliable Numerical Solution for American Call Options: A New Front-Fixing Approach*, by R. Company, V.N. Egorova, and L. Jódar. Accepted for publication in Journal of Computational and Applied Mathematics.

Outlook and collaboration

The next step of the study is to consider nonlinear BS equations, e.g., the Barles and Soner's model.

The numerical method based on the use of characteristics (known as semilagrangian) schemes will be applied to evaluate American options under nonlinear option pricing models. The idea is to implement a suitable semilagrangian scheme to the option pricing problem, after carefully analysing the financial problem and the mathematical model. Collaboration with Prof. Carlos Vázquez Cendón (University of A Coruña, Spain) is foreseen in this.

Implicit finite difference methods, such as Newton-like methods, can be used for the numerical solution in order to compare it with explicit methods. This is aim of the joint work with Prof. Choi-Hong Lai and ESR10 Shih-Hau Tan of the University of Greenwich, London, UK.

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