

# Triangles and Quadrangles in Space

Koryo MIURA\*

\*Institute of Space and Astronautical Science, JAXA  
3-9-5 Tsurukawa, Machida, Tokyo 195-0061  
miurak@gakushikai.jp

## Abstract

In some space missions, the highest priority of the structural design is given to the geometry of the surface which has to be packaged and then deployed in space. Large space antennas and large solar cell arrays are such examples. It is interesting to note that basic geometric forms play a major role in conceiving novel concepts of these space structures. In this presentation, the author introduces two concepts of deployable space structures that are symbolized with groups of triangles and quadrangles, respectively.

**Keywords:** conceptual design, space structure, membrane structure, deployable structure, antenna, origami, solar cell.

## 1. Introduction

Under the zero-gravity space environment, the “Structure” loses its traditional meaning of the word as the load carrying device. For instance, if the purpose of a space mission is to collect electro-magnetic wave, its functional surface is the main structure even if it only carries the load of solar pressure. Therefore, the highest priority of the structural design of the mission is given to the geometric property of the surface which has to be packaged and then deployed in space. Ironically, the fabricated space structures have to be proof tested their behaviors on the ground. It is especially tough requirement for large gossamer space structures since there is no way to provide perfect zero-gravity compensation.

Under such serious difficulties, the following words by Professor Torroja are still vividly alive in our minds.

***“Before and above any calculation, comes the idea, to mould the material into a resistant shape, ready to accomplish a functional”.***

As a disciple of Prof. Torroja, the author shall introduce his own examples of conceiving the “idea” or “concept” on structures. The title of this presentation is due to the fact that their essential parts are either triangles or quadrangles.

## 2. Triangles for parabolic reflectors

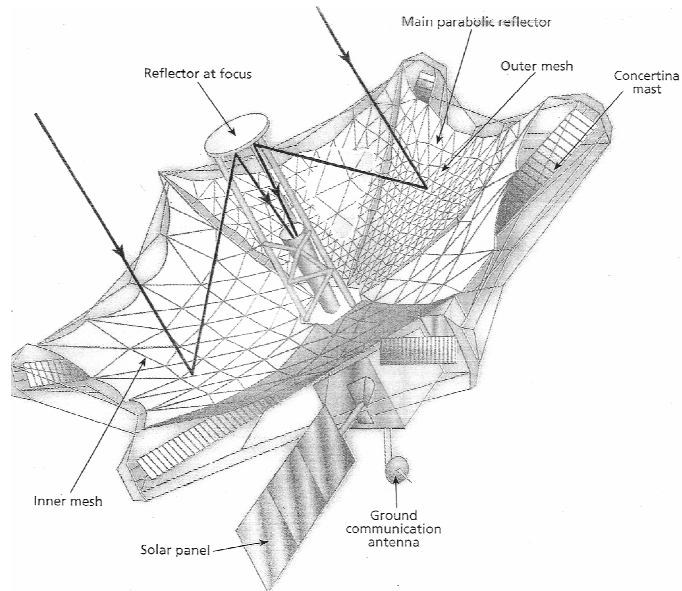


Figure 1: Large deployable antenna for HALCA (Hirabayashi et al. [1])

Figure 1 shows the large deployable antenna for Space VLBI (very long baseline interferometry) mission (1997). Radio astronomers in particular needed reflectors of large size, 10-40 m in diameter, and high surface accuracy, less than 0.5 mm rms error. Structural engineers may be interested in the fact that it is a membrane structure which facilitates the shape of accurate parabolic surface, that is, the surface of positive Gaussian curvature. The trick of the structural concept lies in the triangular cable net shown in the figure.

The design of a large deployable antenna with such a demanding accuracy requirement poses many difficult problems, such as how to package a large structure into a small volume, how to control the deployment of a very flexible structure considering that the presence of gravity when the structure is tested on earth may considerably change its behaviour; how to guarantee deployment reliability; and so forth. Previous concepts for large antennas had envisaged a mesh surface supported on centrally-mounted radial ribs, as in umbrellas, or on radial cable structures connected to a deployable ring, but to provide high surface accuracy in a large aperture such schemes would require an impractically large number of radial elements.

A central problem is how to form an *accurate, furlable reflector surface* and hence the inherent difficulty is the so-called “pillowing effect”, as shown in Figure 2.

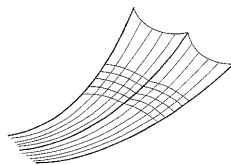


Figure 2: Pillowing of mesh reflector (Miura and Pellegrino [7])

The geometric surface of the reflector has to be a paraboloid of revolution, i.e. a surface of positive Gaussian curvature. However, to realize such a surface with a series of membrane elements that are equally tensioned in all directions, for example, would require each element to form a minimal surface. Hence, in a surface consisting of membrane elements stretched across a series of parabolic ribs, as in an umbrella, each element has principal curvatures of opposite sign. There is no way of avoiding this fundamental problem unless a pressure loading is applied on the surface.

Miura and Miyazaki [5] proposed a novel concept to form a parabolic reflector surface using a tension structure. Figure 3 shows the rough of the concept.

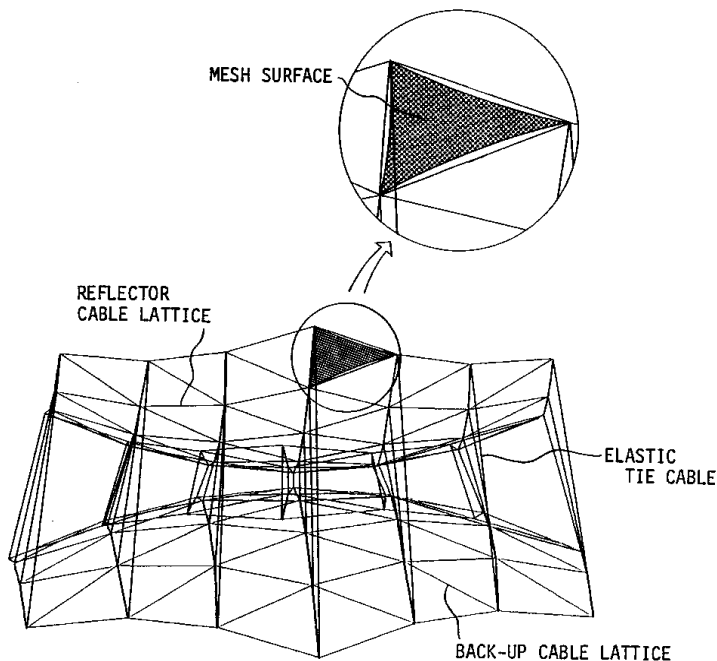


Figure 3: Concept of tension truss antenna (Miura and Miyazaki [5])

This concept is based on the following three points:

\**Local Structure.* The parabolic surface is divided into many “triangular facets”, each consisting of a mesh surface and a boundary cable. As the result, if all of the boundary cables are properly tensioned, each mesh facet is kept flat ( $K=0$ ) and hence the pillow effect is eliminated.

\**Global structure.* The resulting cable net is pretensioned by applying forces at each joint, approximately normal to the surface. This net forms a rigid truss structure, as shown in Figure 3. This leads to the general outcome that one is able to form a synclastic curvature,  $K>0$ , with tensile elements which tend to form anticlastic surfaces,  $K<0$ .

\**Support structure.* The rear net with similar geometry to the front active net provides a series of attachment points for the elastic tie cable.

Figure 4 is a photo of the first conceptual model made of chains, rubber bands, nodes, and plywood.



Figure 4: Conceptual model of tension truss antenna

The concept is now called *tension truss* or *tensioned cable truss* but, of course, it is not limited to space antenna.

Figure 5 shows the assembling scene of the 10 m diameter reflector of HALCA. Note that six radial masts form the structure that deploys and prestresses the tension truss.



Figure 5: Assembling of tension truss antenna for HALCA

The concept of tension truss antenna lies in its functional parabolic surface. Since the functional surface is thin and invisible, one can not identify the triangulated facet in most of applications. Figure 6 shows the in orbit deployment of modified tension truss antenna for ETS-VIII satellite. It looks like an assemble of 14 hexagonal modules, however, it is an integrated single antenna. Only who understand the concept will see the triangles.

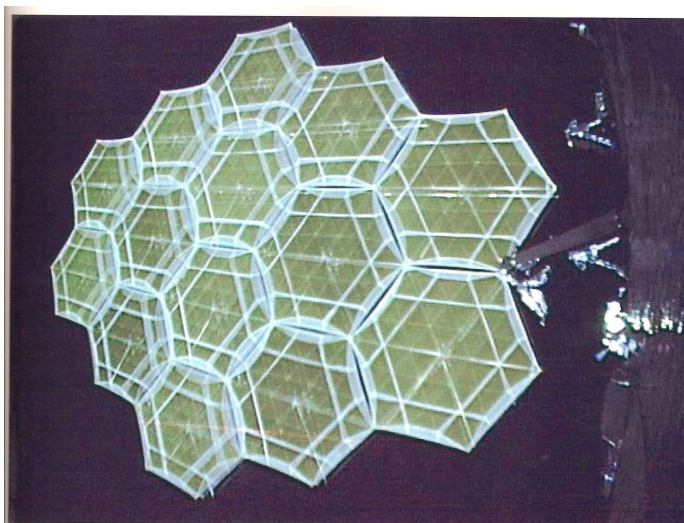


Figure 6: ETS-VIII deployment in orbit (Meguro et al. [2])

### 3. Quadrangles for planer deployable structures

Figure 7 shows the 2-D Array Experiment on board the Space Flyer Unit in space proposed by Miura and Natori [6]. The purpose of this experiment is to demonstrate the possibility of solar power satellite that will collect solar energy in space and send it to the Earth. For such a purpose, a system of packaging and deployment of the large planar structure has to be developed.

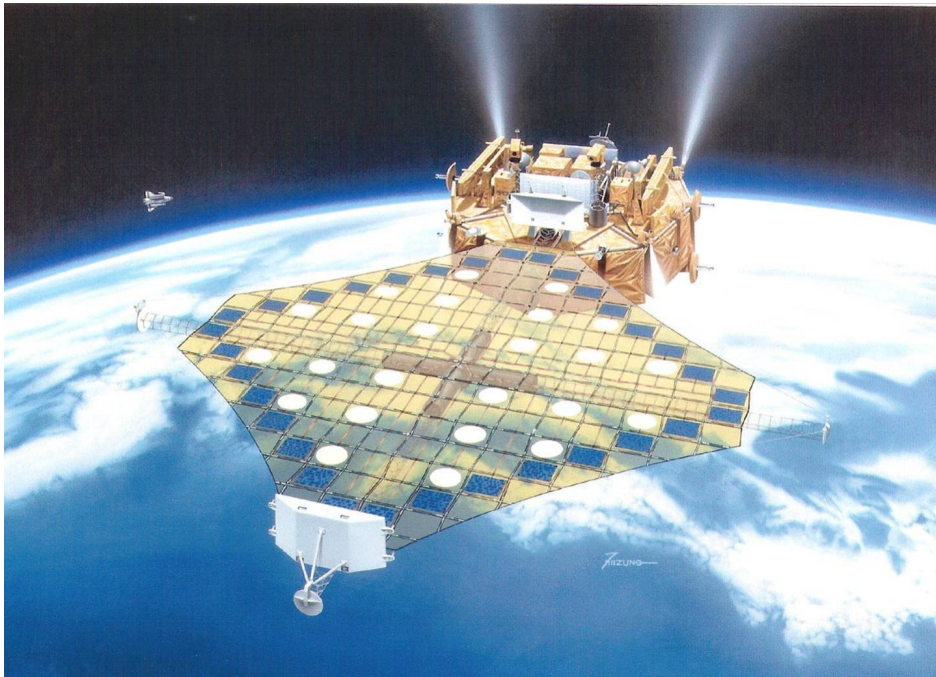


Figure 7: 2-D Array experiment onboard the Space Flyer Unit

Relevant to the subject, at IASS Symposium 1970, Miura [4] presented a paper on a polyhedral shell which looks like Yoshimura-pattern of buckled thin cylindrical shells. One of the interesting properties of the shell is its foldability. It is known that, at the post-buckling state, the shell sustains the substantial rigidity in the axial direction. Thus, even if the fold lines are assumed to be hinges, the shell is stiff enough to hold substantial loading.

However, if we consider a sector part of the shell instead of the whole shell, the behaviour will be drastically changed. The sector can be folded in the manner that both the axial length and the macroscopic radius decrease.

Figure 8 shows the virtual simulation of folding a sector of Yoshimura-pattern starting from the original shell to the completely folded state. It is the virtual simulation because the interference and intrusion among elements are neglected.

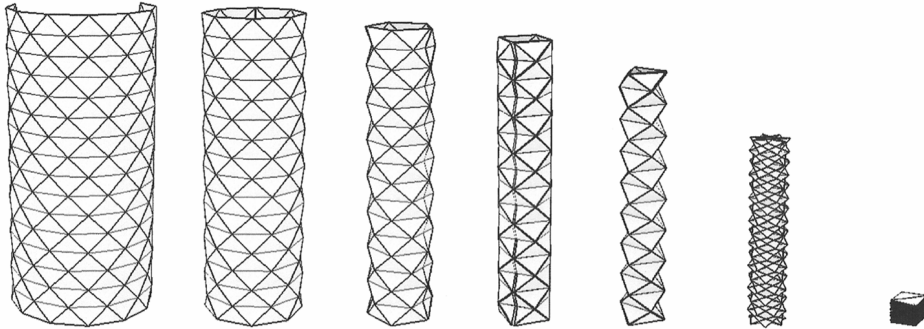
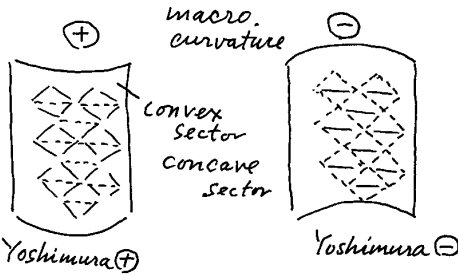


Figure 8: Folding of a sector of Yoshimura-pattern (Courtesy of Tachi, T.)

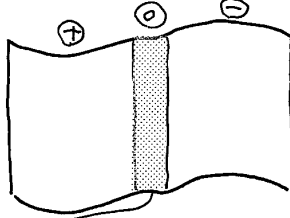
Even though the above simulation is the virtual one, it provides us important information on which the further strategy can be constructed. We know both the cylindrical and plane surfaces belong to the same category of zero Gaussian curvature, and these are mutually translatable without extension. Thus, we can say with fair certainty that the similar phenomenon of foldability exists for the plate case with the similar boundary condition.

The following Figure 9 shows the rough of working hypothesis formulated at the crucial stage of this research.

Yoshimura sector = "gene" of 2D-foldable



Find a compatible (origami) sector at the inflection point.



Yoshimura (0) for this zone will keep gene of 2D-foldable

"Multiplication" to produce a tessellation

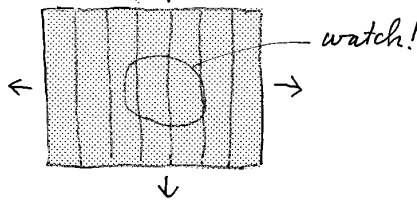


Figure 9: Rough of working hypothesis on folding a thin plate

Following the process described in Figure. 9, we are able to construct Yoshimura (0) sector shown in Figure 10. In this plan, the dotted lines indicate mountain folds and the broken lines indicate valley folds. It is found that the Yoshimura (0) consists of a zigzag array of identical parallelograms.



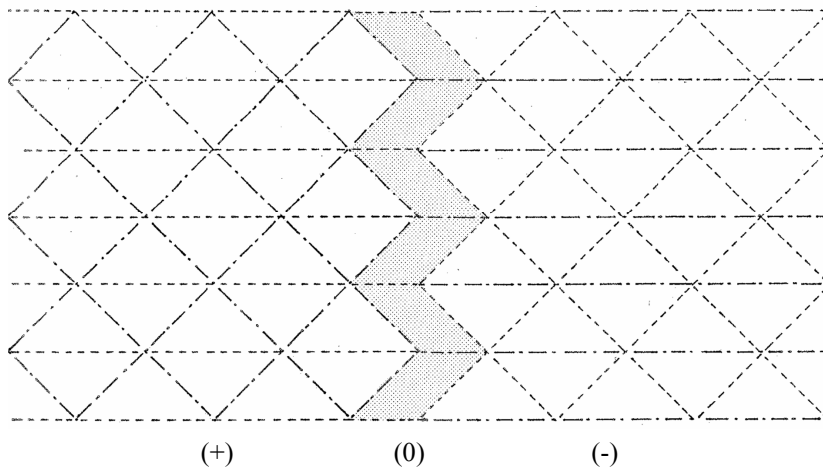


Figure10: Joining Yoshimura (+) to Yoshimura (-) to fsearch Yoshimura (0)

The following Figure 11 shows the sequence of multiplication of Yoshimura (0) in the lateral direction. Resultantly, we have a 2-dimensional tessellation P1 of the fundamental region consists of four identical parallelograms (shaded area)

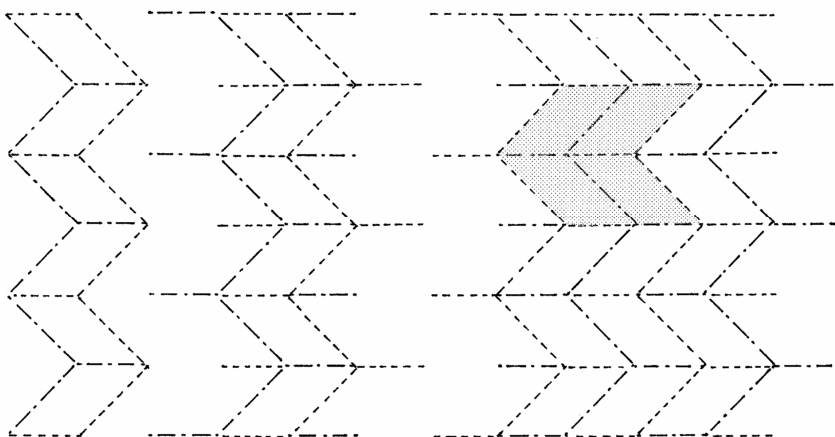


Figure 11: Multiplication of Yoshimura (0)

The resulting shape of the bi-axial folding of a flat surface is clearly shown with a paper model in Figure 12.

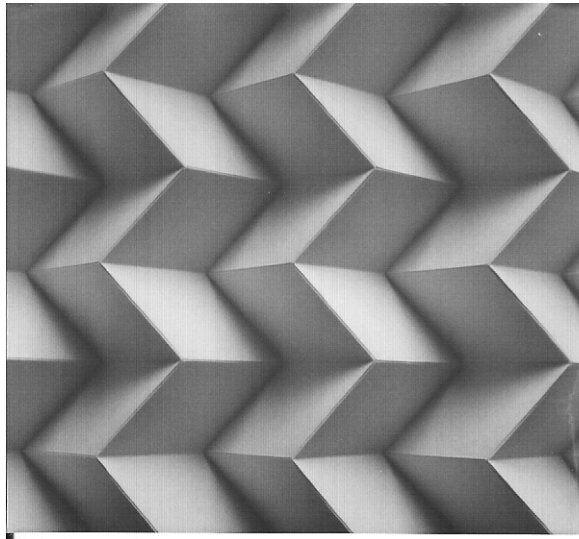


Figure 12: Paper model of bi-axial folding of a flat surface

Without using the model, one can hardly describe the dynamic behaviour of the surface. Thus it is supplemented by a computer simulation of deployment and retraction of the surface as shown in Figure 13.

The primary features of the surface are summarized in the following.

- \*It deploys and retracts simultaneously in orthogonal directions. The deployment and retraction follow the identical path.

- \*It possesses a single degree of freedom of motion no matter how large the array.

Conclusively, the validity of the idea was confirmed. It is indeed surprising that the surface obtained with the deductive approach is exactly the same as the following rigorous analysis. Tanizawa and Miura [8] proved that it is exactly the optimum solution among many other plausible solutions. The surface was initially named the developable double corrugation (DDC) surface, however, is more frequently referred as Miura-ori (“ori” comes from origami). In a recent study, Mahadevan and Rica [3] have succeeded to obtain a beautiful picture of the surface under the carefully controlled experiment.

This is a story why a particular array of parallelograms was launched to an orbit in 1995 as shown in Figure 7.

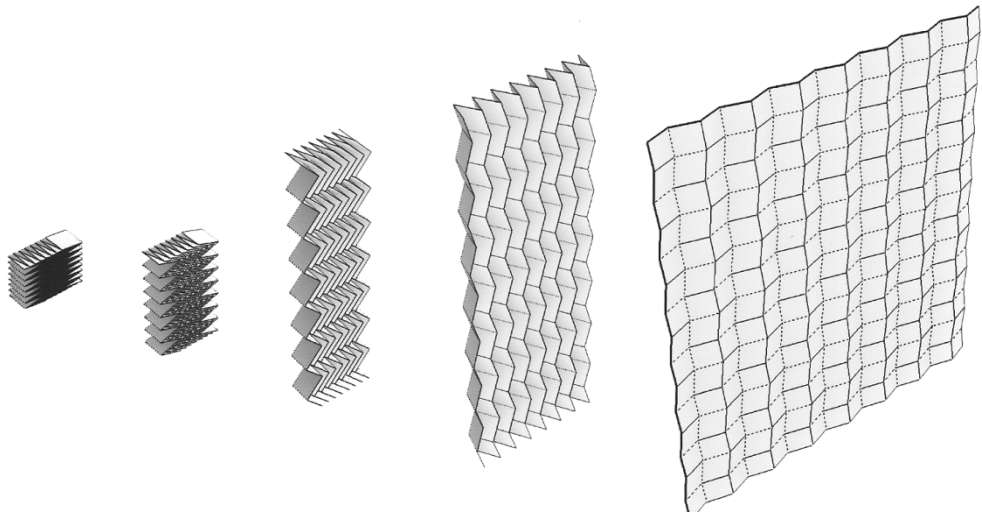


Figure 13: Deployment of Miura-ori surface (Courtesy of Tachi T.)

## Acknowledgements

It is my great honor to be able to give presentation at the 50<sup>th</sup> Anniversary of IASS founded by Prof. Eduardo Torroja. His book “Philosophy of Structures” was the starting point of my research.

## References

- [1] Hirabayashi H., Inoue W., and Kobayashi H., (edit) *Frontier of VLBI*. Tokyo University Press Inc, 1991.
- [2] Meguro A., et al., In orbit deployment characteristics of large deployable antenna reflector onboard Engineering Test Satellite VIII, *Acta Astronautica*, 2009, doi: 10.1016/j.
- [3] Mahadevan L., and Rica S., Self-organized origami, *Science*, 2005, 37, 1740.
- [4] Miura K., Proposition of pseudo-cylindrical concave polyhedral shells. *IASS Symposium on Folded Plates and Prismatic Structures*, Krapfenbauer R (ed.), Goldmann-Druck, 1970.

- [5] Miura K., and Miyazaki Y., Concept of the tension truss antenna, *AIAA Journal*, 1988, 28, 1098-1104.
- [6] Miura K., and Natori M., 2-D Array experiment on board Space Flyer Unit. *Space Solar Power Review*, 1985, 5, 345-356.
- [7] Miura K., and Pellegrino S., Structural concepts and their theoretical foundations, (to be published).
- [8] Tanizawa K., and Miura K., Large displacement configurations of bi-axially compressed infinite plate. *Transaction of the Japan Society for Aeronautical and Space Sciences*, 1978, 20, 177-187.