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Response Spectrum Analysis Methods for Spatial Structures

Yongfeng LUO*, Lei WANG

Department of Building Engineering, Tongji University
1239 Siping Road, Shanghai, China, 200092
yfluo93@tongji.edu.cn

Abstract

The current response spectrum method, based on the principle of superposition, is the principal method for seismic analysis of structures. It has been widely adopted by seismic design codes all over the world, although it is limited to linear systems. The progresses of the method are summarized in this paper. The limitations and the shortcomings of the method are pointed out. In current response spectrum method, the initial stiffness matrix of a structure is adopted in traditional modal analysis, but the effects of the structure deformations on the stiffness matrix are not considered yet. In fact, behavior of long span spatial structures under dynamic loads is higher nonliear and much complex. As an example, an long span arch structure is analyzed for revealing the effects of the geometrical nonlinearity and comparison in the paper. The numerical results show that there are obvious differences between two groups of vibration modes and corresponding periods obtained from the deformed and the undeformed structure configurations. Therefore, it is theoretically irrational to describe overall dynamic responses of a long span spatial structure by means of linear vibration mode theory. And also only a general damping ratio of a structure is usually employed in the troditional spectrum response analysis, instead of multi-material damping ratios, in almost all seismic design codes. If a structure is composed of different materials or/and installed more than one type of dampers, there are of couse several different damping ratios in the structure. The damping matrix in motion equations is no longer a classical damping matrix. As a result, the free vibration characters will be obviously different from one of the structure composed of the same material or/and without dampers. Apparently, the traditional response spectrum method fails to cope with the dynamic problems of a structure composed of different materials or/and installed dampers. The current develop situation of the nonlinear normal mode theory and its application are also introduced in this paper. The current effective analysis method for nonlinear dynamic equations is the well-known step by step integration method, named time-history method. However, it is difficult for wide application because of unacceptable time consumption, especially in analysis of large-scale buildings or spatial structures. The nonlinear mode theory is simple and effective for interpreting the complex nonlinear dynamic phenomenon. Then the nonlinear mode theory is still a developing vibration theory now. Few references about it can be found nowadays. If the nonlinear normal mode theory is adopted trying to solve the nonlinear structural dynamic problems, one of the crucial technique is to solve the geometrical nonlinear eigen equations. Futher investigation about an improved theoretical method for analysis of geometrical nonlinear dynamic behavior of long span spatial structures has been conducted now. The nonlinear mode theory and the response spectrum method are combined in the improved method. The research focus on 5 main steps. A future research frame of the method is introduced briefly.

Keywords: response-spectrum method, nonlinear normal modes, geometrical nonlinearity, long span spatial structures.

1. Introduction

Development of seismic design methods can be divided into three stages, the stage of static analysis method, the stage of response spectrum method and the stage of dynamic method (Hu [4]). The static analysis method was initiated in Italy and moved forward by Japanese scholars. Omori Fusakichi, a famous Japanese seismologist, put forward that the seismic force P of a building can be written as

$$P = \frac{W}{g}\alpha_{\max} = kW \tag{1}$$

Where W is the self-weight of the building; $k=\alpha_{max}/g$ is a seismic coefficient.

In fact, every building will deform more or less when suffering loads. Therefore, the application of the static analysis method is obviously limited because of its hypothesis of rigidity of structures. A simple and concise conception of the response spectrum had been proposed by Biot.M, Benioff.H and Hournser until 1940s after getting strong earthquake records. The response spectrum theory focues on how to express the dynamic relationship between structural dynamic behavior and ground motion characteristics under earthquake. Meanwhile, the seismic force P of a structure can be written similar to eq(1), as follows

$$P = k\beta(T)W = \alpha(T)W \tag{2}$$

Where, k is the seismic coefficient, describing the intensity of ground motion; $\beta(T)$ is the dynamic coefficient, i.e. acceleration magnification of the building; $\alpha(T)=k\beta(T)$ is the seismic influence coefficient.

The response spectrum method has been worldwide employed as the principal seismic analysis method for its clear conception and convenient operation. With the development of computer techniques and numeric analysis methods, the solution procedure of step by step integration of kinetic equations can be realized. The time history analysis can be adopted for obtaining more accurate results. Then the time history analysis will spend more solution time than the response spectrum method does. Meanwhile, the computation results directly depend on selected earthquake records and may have larger discreteness. Current China code for seismic design of buildings suggests that the time history analysis should be adopted as supplementary checking computation for the seismic design of complicated or important buildings (Xu [14]).

2. Basic principle of the response spectrum method (Lin *et al.* [5])

The equation governing the motion of a Single Degree of Freedom(SDF) system is

$$m\ddot{y} + c\dot{y} + ky = -m\ddot{x}_g(t) \tag{3}$$

Where *m*, *c* and *k* are mass, damping coefficient and stiffness of SDF system respectively.

Introducing the critical damping ration $\xi = \dfrac{c}{2m\omega_0}$ and circle frequency $\omega_0 = \sqrt{\dfrac{k}{m}}$, eq(3) is changed to

$$\ddot{y} + 2\xi\omega_0\dot{y} + \omega_0^{\ 2} y = -\ddot{x}_g(t) \tag{4}$$

According to the response spectrum method, the solution of eq(4) is

$$y = \alpha \frac{g}{\omega_0^{\ 2}} \tag{5}$$

The equation governing the motion of a Multiple Degree of Freedom(MDF) system is

$$[M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{y\} = -[M]\{E\}\ddot{x}_g(t) \tag{6}$$

Where, [*M*], [*C*] and [*K*] is mass matrix, damping coefficient matrix and stiffness matrix of MDF system respectively; {*E*} is a direction vector of inertia forces.

The corresponding free vibration equation of MDF system of eq(6) is

$$\left\|[K] - \omega^2 [M]\right\|\{y\} = 0 \tag{7}$$

Where, ω is the natural frequency of vibration.

Solving eq(7), the natural frequencies ω_j(j=1,2,3, …, N) and corresponding vibration mode vectors $[\Phi]=[\{\phi_1\},\{\phi_2\},\ldots,\{\phi_N\}]$ can be obtained. The solution displacement vector of eq(6) can be composed of the vibration modes:

$$\{y(t)\} = [\Phi]\{u(t)\} \tag{8}$$

Where {*u*(*t*)} is a vector of mode coordinates.

With assumption about the classical damping matrix [*C*], i.e. mode matrix $[\Phi]=[\{\phi_1\},\{\phi_2\},\ldots,\{\phi_N\}]$ is orthogonal about damping matrix [*C*], substitute eq(8) into eq(6) and left premultiplied by $[\Phi]^T$, *N* motion equations about {*u*(*t*)} are obtained.

$$\ddot{u}_j + 2\xi_j\omega_j\dot{u}_j + \omega_j^{\ 2}u_j = -\gamma_j\ddot{x}_g(t) \tag{9}$$

Where, $\gamma_j=\{\phi\}^T_j[M]\{E\}$ (j=1,2,3, …, N) is defined as *j*th mode participation factor.

Compare eq(4) and eq(9), solution of eq(9) is given by

$$u_j = \gamma_j \alpha_j \frac{g}{\omega_j^{\ 2}} \tag{10}$$

Substituting eq(10) into eq(8), obtain

$$\{y(t)\}_j = u_j(t)\{\phi\}_j \tag{11}$$

Selecting *k*th interesting element of $\{y(t)\}$, a vector$\{d_k\}=[d_{k1}, d_{k2}, , \ldots, d_{kN}]^T$ is composed, in which the number of elements is equal to the number of vibration modes. The response of the structural displacement, y_k ,can be calculated by following eq(12).

$$y_k = \sqrt{\{d_k\}^T [\rho]\{d_k\}} \tag{12}$$

Where $[\rho]$ is the correlation matrix denoting the correlation between vibration modes. All diagonal elements are 1.0.

The method to get overall displacement response is called SRSS combination, when $\rho_{ij}=0$ $(i \neq j)$; and CQC combination, when

$$\rho_{ij} = \frac{8\sqrt{\xi_i \xi_j}(\xi_i + r\xi_j) r^{\frac{3}{2}}}{(1-r^2)^2 + 4\xi_i \xi_j r(1+r^2) + 4(\xi_i^2 + \xi_j^2) r^2} (i \neq j)$$

The response spectrum method is also called mode superposition method. The crucial technique of the method is to decouple motion equations of MDF system using the linear normal modes of free vibration of a linear conservative system. Then motion equations of SDF system can be obtained. The general displacement response can be calculated by solving equations of SDF system and consequently applying principle of superposition. The general response is viewed as combination of the contribution of each mode when using response spectrum method. The major characters of the method are:

(1) All modes are independent each other and orthogonal about [M]and [K];

(2) General responses of a structure can be obtained by superposition of the contribution of all modes.

3. The limitation of the response spectrum method

If Rayleigh damping mechanism is suitable and the damping ratio ξ<10%, the response spectrum method seems perfect for seismic design of multi-storey buildings and has been also approved in practice too. However, accuracy of the response spectrum method should be further discussed if adopted in seismic design of spatial structures. Let’s observe the application scope of the response spectrum method by analyzing a typical arch.

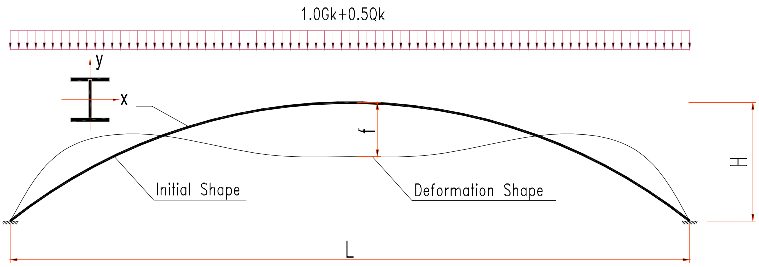


Figure 1: An arch structure

A typical arch is shown in Figure 1. *L* and *H* are span and rise of the arch. *f* is vertical deflection of the arch middle section when suffering equivalent load. *Gk* is dead load. *Qk* is live load. It is assumed here that load level and rise to span ratio keep constant no matter how the arch span changes. The equivalent load, 1.0*Gk*+0.5*Qk*, is 30kN/m. The rise to span ratio is 1/5. Basic information of the arch is shown in Table 1.

Model	L(m)	H(m)	Section	q_{cr}(kN/m)	q/q_{cr}	Steel
A1	10.00	2.00	H120×55×5×6	50	0.6	Q345
A2	30.00	6.00	H350×150×6×6	50	0.6	Q345
A3	60.00	12.00	H600×240×10×10	50	0.6	Q345

Table 1 : Basic information of the arch

In table 1, the in-plane elastic critical stability load of the arch without hinges is $q_{cr} = \alpha_1 \frac{EI_x}{L^3}$. Where, α_1 is the in-plane critical stability load factor and here reaches 90.4; *E* is Young's modulus of steel; I_x is the section inertia moment.

Model	L(m)	H(m)	linear	nonlinear
A1	10.00	2.00	1.30	1.30
A2	30.00	6.00	18.70	19.20
A3	60.00	12.00	216.00	325.00

Table 2 : Vertical deflections, f(mm), under equivalent load

It is shown in table 2 that the geometrical nonlinear effects of the arch increase dramatically when span increases. However, the response spectrum method introduced above is still adopted in current seismic design of large span structures similar to the arch. It can be found from eq(1) that the general stiffness matrix [*K*] is developed before structure deformation. In fact, large displacements will appear in large span structures under equivalent load. The configuration of the structure changes much more from initial shape.

Therefore, an accuracy mode analysis should at least be based on deformed configuration of the structure.

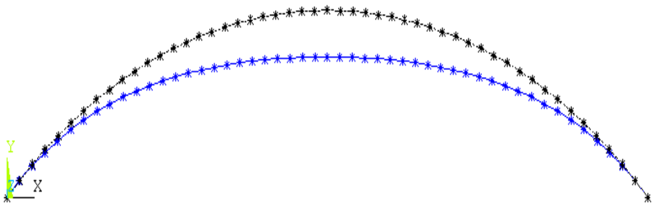
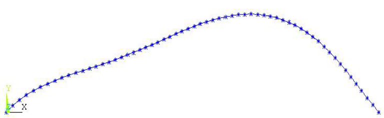


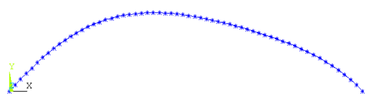
Figure 2: Arch deformation under the equivalent load

Model	Configuration	T1	T2	T3	T4	T5
A1	initial	0.07	0.06	0.03	0.02	0.02
	deformed	0.07	0.06	0.03	0.02	0.02
A2	initial	0.61	0.33	0.17	0.17	0.11
	deformed	0.63	0.34	0.18	0.17	0.11
A3	initial	2.42	1.29	0.70	0.49	0.33
	deformed	3.70	1.82	0.79	0.52	0.35

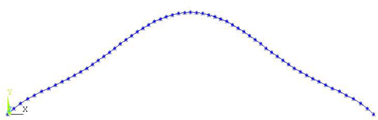
Table 3： Comparison of periods(s)

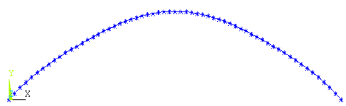
Dynamic characteristics of the arch are shown in Figure 3 and Table 3, which are obtained from the motion equations and corresponding stiffness matrices based on the initial and the deformed configuration respectively. It can be found from Figure 3 and Table 3 that obvious differences exist in first two modes and corresponding periods obtained from the different structure configurations. Therefore, computation error may not be estimated beforehand if traditional response spectrum method is directly adopted.





Mode 1 of the initial configuration of A3   Mode 1 of the deformed configuration of A3





Mod e2 of the initial configuration of A3   Mode 2 of the deformed configuration of A3

Figure 3: Mode shape comparison between initial configuration and deformed onfiguration

When fixing arch span and increasing equivalent load step by step, as we know, the geometrical nonlinear effects will become obvious, i.e. the geometrical nonlinear effects depend also on load level. Although the mass of the structure keeps constant, the equivalent load of the structure always keep consistent change with its acceleration excited by wind, earthquake or other random dynamic load. The geometrical nonlinear effect is also function of time. Therefore it is theoretically irrational to describe overall dynamic responses of a structure by means of linear vibration modes at any specific time point.

Only a general damping ratio of a structure is employed in the spectrum response analysis in almost all seismic design codes. If a building is installed more than one type of dampers, the free vibration characters will be different from one of the building without dampers. They can not be represented each other. The free vibration equation of a damped MDF system is

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=\{0\} \qquad (13)$$

In order to understand the damping mechanism clearly, a general damping can be divided into two parts, structural damping and component damping, respectively. If the geometrical nonlinear effect is considered here at the same time, eq(13) can be also written in form of

$$[M]\{\ddot{y}\}+\left([C]_s+[C]_c\right)\{\dot{y}\}+[K]_T\{y\}=\{0\} \qquad (14)$$

Where, $[C]_s$, $[C]_c$ and $[K]_T$ are structural damping matrix, component damping matrix and tangent stiffness matrix respectively.

Apparently, the traditional response spectrum method fails to cope with such dynamic problems described by eq(13) or eq(14).

4. Existing problems of dynamic analysis of long span spatial structures

Matter of fact, behavior of long span spatial structures under dynamic loads is much complex. Firstly, the damping matrix should not be removed from the free vibration equation when dampers are installed. In addition, the damping matrix is no longer a classical damping matrix. The normal and orthogonal mode vectors can not be obtained. Secondly, the restoring character of a structure member may behave strongly nonlinear and the restoring force depends on not only initial conditions but also instantaneous dynamic load level. The correct solution of the motion equation can be obtained by updating the stiffness matrix step by step during iteration. Then it can not be realized by the current response spectrum analysis method. If the current response spectrum method is adopted in analysis of long span spatial structures, the geometrical nonlinear eigen equation should be solved. Therefore, it is important to found a strategy for solving the nonlinear eigen problems, which can accurately describe dynamic characteristics of long span spatial structures.

4.1. Mode analysis of non-classical damping systems

Given a MDF dynamic system under earthquake excitation, including non-classical damping and without considering geometrical nonlinearity, the motion equation is

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=-[M]\{\gamma\}\ddot{X}_g(t) \tag{15}$$

There are three primary methods to analyze non-classical damping vibration, approximate real mode method, complex mode method and approximate complex mode method(Li *et al.* [6]).

The approximate real mode method makes full use of charaters of real mode method. At the beginning of dynamic analysis, real vibration modes are obtained based on the assumption of free vibration of a conservative system without damping. Then eq(15) is decoupled into a set of motion equations of SDF. Some approximate decoupling methods(Clough *et al.* [1] ,Thomson *et al.* [10] , Traill-Nash *et al.* [11] ,Warburton *et al.* [12] , Wilson *et al.* [13]) have been reported, which are almost the same as the traditional method. The main steps of the methods are as following:

1) Solve the eigen-equation $|[K]-\omega^2[M]|=0$, and obtain corresponding frequency matrix $[\Lambda]$and mode matrix $[\Phi]$;

2) Introduce the modal coordinate vector $\{u\}$ and expand nodal displacement vector according vibration modes $\{y\}=[\Phi]\{u\}$;

3) According to the orthogonality of $[\Phi]$ about mass matrix $[M]$ and stiffness matrix $[K]$, substitute $\{y\}=[\Phi]\{u\}$ into eq(15) and left premultiplied by $[\Phi]^T$, then coupling motion equation changes to

$$[E]\{\ddot{u}\}+\left[\tilde{C}\right]\{\dot{u}\}+[\Lambda]\{u\}=-[\Phi]^T[M]\{\gamma\}\ddot{X}_g(t) \tag{16}$$

Where $\left[\tilde{C}\right]=[\Phi]^T[C][\Phi]$ is a non-diagonal damping matrix.

4) Look for a rational diagonal damping matrix $\left[\tilde{C}\right]^D$ instead of $\left[\tilde{C}\right]$ for decoupling eq(16). The principle of substitution is minimal error of solution.

The approximate real mode method will lead to dramatic error when the damping matrix is higher coupling. Then the complex mode method may be a better choice to obtain precise dynamic characters of the non-classical damping system. However, the complex eigen equation with 2N-dimension needs to be solved. There are a large quantity of freedoms in a long span spatial structure. Solution of the complex eigen equation may take much computation time. It is difficult for actual use in spatial structures.

The approximate complex mode method, based on complex mode method, is a relative simple method for operation and application. Both subspace complex mode method proposed by Mau(Mau *et al.*[7]) and Lanczos method(Nour-Omid *et al.* [8]) belong to the approximate complex mode methods. The methods are able to reduce rank of the motion equation from 2N to M (M<<2N). The large-scale computation of complex eigen equation can be avoided. Meanwhile the solution can reach an expected accuracy.

All methods mentioned above are very valuable to cope with linear dynamic systems with non-classical damping. However, any existing building is possible to suffer strong dynamic loads. As a result, installed dampers may dissipate energy. And some structural members may yield yet. It may lead to some different extent of nonlinearity. Hence the linear theory is not suitable here.

4.2. Nonlinear time-history analysis methods

If the modal superposition method is adopted, the structure behavior should keep linear during the dynamic analysis. Nonlinearity of the structure matrix will result in analysis failure. In fact, the coefficient matrices cannot always keep constant. If a member yields, the elastic modulus reduces. The stiffness matrix should be modified in computation. Long span spatial steel structures can keep elastic in most cases, but change of member axial forces or joint positions may induce sharp change of stiffness matrix and geometrical nonlinearity. Current effective analysis method for nonlinear dynamic equations is the well-known step by step integration method. Its main operation procedure begins from the increment motion equation (Clough [2]).

$$m\Delta\ddot{v} + c_0\Delta\dot{v} + k_0\Delta v = \Delta p \tag{17}$$

Given a definite acceleration increment, expression of velocity increment and displacement increment can be obtained from eq(17). After derivation and simplication, a quasi equilibrium increment equation can be obtained.

$$\hat{k}\Delta v = \Delta p \tag{18}$$

Where, $\hat{k} = k_0 + \frac{3}{\tau}c_0 + \frac{6}{\tau^2}m$; $\Delta p = \Delta p + c_0\left(3\dot{v}_0 + \frac{\tau}{2}\ddot{v}_0\right) + m\left(\frac{3}{\tau}\dot{v}_0 + 3\ddot{v}_0\right)$; τ is integral time step; c_0 and k_0 are damping and stiffness respectively.

Since parameter c_0 and k_0 should be updated in every time step, the time-history method is difficult for wide application because of time consumption, especially in analysis of large-scale buildings or spatial structures. Therefore, a concise and accurate technique to handle geometrical nonlinear dynamic problems is required for design of long spatial structures.

4.3. Brief introduction of the theory of nonlinear normal modes

In general, the dynamic characteristics of a nonlinear system are closer to those of an actual structure than a simplified linear system. However, the linear mode theory has been widely accepted and used for its clear conception, convenient operation and concise computation procedure, so as to be adopted directly to analyze the dynamic response of a nonlinear system without any doubt. Then the theoretical deficiency of the linear mode theory has not been improved in its wide application till now, especially in analysis of long span spatial structures. And it has not been proved that the linear mode theory may be also suitable for nonlinear systems, such as the responses of the long span spatial structures under strong earthquake or typhoon. Therefore, an effective and refined technique is needed to deal with nonlinear dynamic problems. The nonlinear mode theory is useful and effective for interpreting the complex nonlinear dynamic phenomenon. Rosenberg proposed the nonlinear mode theory and defined the nonlinear normal modes in 1960s, nonlinear normal modes is consistent vibration of a system(Rosenberg[9]). Rand, Manevitch and Mikhlin undertook further studies in 1970s. At the beginning of 1990s, there were some great theoretical progresses and development about the nonlinear mode theory, contributed by outstanding work of Vakakis, Shaw and Pierre(G. Kerschen *et al.* [3]). Compared with the linear mode theory, there are both advantages and disadvantages in current nonlinear mode theory.

The advantages of the nonlinear normal modes for effective description of nonlinear vibration are:

(1) Nonlinear normal modes can exhibit complex nonlinear behavior that linear normal modes cannot, such as jumping, bifurcation, internal resonance and modal interaction etc.;

(2) Nonlinear normal modes can be used to analyze low-order lumped mass models.

Some main limitations of the Nonlinear normal modes are:

(1) The principle of superposition is not applicable;

(2) Since lack of orthogonality relations between Nonlinear normal modes, decoupling of motion equations may be much difficult.

4.4. New conception of geometrical nonlinear dynamic analysis of long span spatial structures

The response spectrum method has become the most popular and effective seismic design method for ordinary multistory buildings, but there are few references about how to use the method correctly for geometrical nonlinear dynamic analysis of long span spatial structures. In recent years, the nonlinear mode theory has improved rapidly. Its application involves many fields, such as astronavigation, military industry and mechanical industry etc. The nonlinear mode theory can describe structural vibration accurately, although the limitation of superposition. Futher investigation about a new theoretical method for analysis of geometrical nonlinear dynamic behavior of long span spatial structures has been conducted now. The nonlinear mode theory and the response spectrum method are combined in the new method. The research frame focus on following 5 steps:

(1) Determine the nonlinear intensity of a nonlinear system;

(2) Given initial conditions. Introduce nonlinear high-order items of restoring forces in stiffness matrix of a system using the current coordinate method. Select a rational damping ratio and derive motion equations;

(3) Obtain the typical vibration equation of nonlinear normal modes;

(4) Set up a mode rectification-forecasting system. Conduct segment linearization of the Nonlinear normal modes;

(5) Set up a combination method of Nonlinear normal modes and the corresponding appraisal benchmark.

5. Conclusions

Based on the linear mode theory, the mode superposition method is adopted in the classical response spectrum theory to obtain general responses of a structure. The response spectrum theory is only suitable for linear systems. In China seismic design code, the elasto-plastic dynamic analysis of a building is conducted by means of reduced spectrum. Then it is not suitable for the geometrical nonlinearity of long span spatial structures. The nonlinear mode theory is still a developing vibration theory now. The influences of different damping mechanism and nonlinear restoring forces on vibration modes can be analyzed with it. In other words, the vibration status of a nonlinear system can be discribed more approximately. According to existing seismic design methods and research reports, a new conception and a reaearch frame are proposed in this paper. The conception combines the nonlinear mode theory and the response spectrum method together for dynamic analysis of nonlinear systems.

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