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Computational morphogenesis of free form shells: Filter methods to create alternative solutions

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Abstract

Actual trends in numerical shape optimal design of structures deal with handling of very large dimensions of design space. The goal is to allowing as much design freedom as possible while considerably reducing the modelling effort. As a consequence, several technical problems have to be solved to get procedures which are robust, easy to use and which can handle many design parameters efficiently. The paper briefly discusses several of the most important aspects in this context and presents many illustrative examples which show typical applications for the design of light weight shell and membrane structures.

Keywords: Shape optimization, shells, membranes

1. Introduction

Shape optimal design is a classical field of structural optimization. Applied to the design of free form shells and membranes or, more generally, light weight structures, it is of big importance in architecture, civil engineering or various applications of industrial metallic or composite shells as e.g. in automotive or aerospace industries [**1**-**3**]. In the "old" days of the pre-computer age optimal shapes had been found by experiments such as inverted hanging models or soap film experiments. Still, those shapes or of great importance for practical design as they define structures of minimal amount of bending which, in turn, are as stiff as possible. As a consequence, "stiffness" is one of the most important design criteria one can think of. The methods discussed in the sequel refer to this design criterion in various ways.

A standard approach of optimal shape design is to discretize the structure and to use geometrical discretization parameters as design variables, e.g. nodal coordinates. As optimization is a mathematical inverse problem it exhibits typical pathological properties which in particular become obvious or even dominant if the number of design parameters becomes large. In particular, one has to deal with questions like irrelevant degrees of freedom tangential to the surface, highly non-convex design spaces, and mesh dependency, just to mention the most important. The state-of-the-art answer to those problems is to use CAGD methods for the discretization of geometry: The success of that approach, however, is a consequence of the reduced number of design parameters rather than a consequent elimination of the source of deficiencies. In other words, if the number of CAGD parameters used for structural optimization is increased, the pathological properties become obvious, again.

If geometrical parameters of a fine discretization are used, as e.g. the coordinates of a finite element mesh, strategies have to be developed to stabilize the original deficiencies of the inverse problem.

2. Shape control

Consider a discretized solid body as shown in Figure 1. It is obviously clear that moving a surface node will modify the body's shape only if it is moved in normal direction **n**. In contrast, movements along the surface tangents $\mathbf{t}_1$ and $\mathbf{t}_2$ will affect the mesh but not the shape. Consequently, movements of discretization nodes are distinguished to be either "shape relevant" or shape "irrelevant". In turn, there are two or three shape relevant move directions at edges or vertices, respectively. Modifications of nodes inside the body are shape irrelevant in all three dimensions. It is intuitively clear that if a design parameter is linked to a shape irrelevant direction the optimization problem will be singular with respect to that parameter. As for several reasons that kind of parameters cannot be eliminated, instead, the problem formulation must be enhanced by additional stabilization terms.

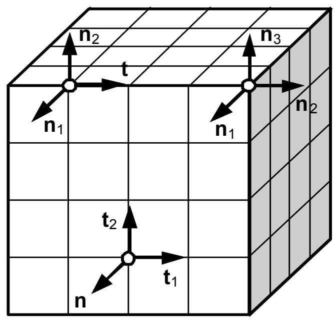

Figure 1: Surface characteristics of a solid body.

3. Minimal surfaces

As it is well known the mechanical analogy to the geometrical definition of minimal surfaces is the equilibrium shape of a surface stress field of given constant magnitude (soap film analogy). The principle of virtual work applies in terms of prescribed Cauchy surface stresses $\boldsymbol{\sigma}$, derivatives of virtual displacements **u** and additional surface loads **q** such as pressure. Thickness t is assumed to remain constant during deformation:

$$-\delta w = \underbrace{t\int_a \boldsymbol{\sigma} : \frac{\partial(\delta \mathbf{u})}{\partial \mathbf{x}} da}_{internal} - \underbrace{\int_a \mathbf{q} \cdot \delta \mathbf{u} da}_{external} = 0 \tag{1}$$

Consequently, if the problem is discretized with respect to all three spatial coordinates of surface nodes it appears that the resulting system of equations is singular with respect to the shape irrelevant degrees of freedom. Typically, they cannot be eliminated because the discretization mesh must tangentially be adapted if the edges are flexible. A typical application is form finding of textile membranes supported by edge cables.

The problem can be reformulated in terms of 2nd Piola Kirchhoff stresses **S** and the deformation gradient **F** which refers to appropriate reference geometries:

$$-\delta w_{int} = \mathbf{U} \cdot \delta \mathbf{u} = t\int_A (\mathbf{F} \cdot \mathbf{S}) : \delta \mathbf{F} \, dA \tag{2}$$

If, now, **S** is prescribed instead of **σ** the system of equation appears to be non-singular and can be solved. The reason is that **S** refers to the undeformed reference geometry. Of course, the solution differs from the true minimal surface. Repeating the procedure with adapted reference geometry the solutions will converge to the true solution. That's why the method is called Updated Reference Strategy, URS. Irrelevant degrees of freedom are stabilized and element shape can be controlled by proper choice of the reference base vectors. Also, anisotropic stress fields can be prescribed as it is the case in many practical applications. Reducing URS to cable nets leads to the force density method which is well established for the design of pre-stressed cable nets [**4-6**].

4. Plateau regularization; tangential mesh control

URS can be generalized to control mesh quality on any surface other than minimal surfaces. Now, an artificial surface stress field **S** is assumed which now is referring to the shape irrelevant degrees of freedom only. All normal directions, Figure 1, are assumed to be fixed for this artificial loading [**7**].

Consider design objective f to be a function in terms of the three coordinates of each discretization node as well as the first derivative ∇f with respect to those coordinates at a certain node. The derivative vanishes with respect to the shape irrelevant directions. It will be regularized by considering the artificial "URS" contribution **U** and projection **P** onto the surface:

$$\begin{aligned} \nabla f_{mod} &= \nabla f + \mathbf{P} \cdot \mathbf{U} \\ \mathbf{P} &= \mathbf{I} - \mathbf{n} \otimes \mathbf{n} && ; \ \textit{surface} \\ \mathbf{P} &= \mathbf{t} \otimes \mathbf{t} && ; \ \textit{edge} \\ \mathbf{P} &= \mathbf{0} && ; \ \textit{vertex} \\ \mathbf{P} &= \mathbf{I} && ; \ \textit{interior} \end{aligned} \tag{3}$$

The above formulae refer to the derivative and its regularization at a node with respect to all geometric parameters (usually spatial coordinates in 3D). In (3) URS or Plateau regularization **U** refers to the specific nodal contribution.

Note, that an appropriate control of reference metric can be generated to create r-refined meshes.

5. Normal mesh control; shape derivative

As a consequence of the inverse nature of optimization many problems appear to be highly non-convex with respect to shape relevant parameters. Without further modifications optimization strategies converge to arbitrary, highly oscillating solutions depending on the choice of initial values and algorithmic constants. Furthermore, shape oscillations which originally are physically explained may lead to severely distorted meshes which then create additional spurious locking effects and, therefore, wrong results. As an example, think of the non-unique problem of how to create stiffening beads in thin metal plates.

As a recipe against this effect the function derivatives with respect to shape relevant parameters must be filtered within a certain diameter. Choices of diameter and regularizing filter function are design decisions and intrinsic part of problem modelling. Usually, no natural principle exists which may guide in that regard.

Smoothness is further improved if first order sensitivity data or even higher derivatives (if available) are filtered. Even simple hat functions on a radial basis as filter function are successful. Considering discretized problems sensitivity data at all nodes within the filter radius has to be considered. Now, D_i is the value of the filter function at node i within the filter radius and sensitivity at a certain node is modified with respect to all neighbouring nodes within the filter radius:

$$\left(\frac{\partial f}{\partial s}\right)_{mod} = \frac{\sum_{i=1}^{n} D_i \cdot \left(\frac{\partial f}{\partial s}\right)_i}{\sum_{i=1}^{n} D_i} \tag{4}$$

Here, s is supposed to be a shape relevant parameter with respect to the normal direction.

All together, at a surface node the resulting shape derivative with respect to all parameters at this node is generated as:

$$\nabla f_{mod} = \frac{\sum_{i=1}^{n} D_i^p \left(\nabla f_i \cdot \mathbf{n}_i\right)}{\sum_{i=1}^{n} D_i^p}\mathbf{n} + \mathbf{U} \cdot \mathbf{P} \tag{5}$$

Typically, for a 3D problem there are three geometric parameters at each node [1].

6. Adjoint sensitivity analysis

In principle, the coordinates of all nodes of a finite element model are candidates to be design parameters. Consequently, the dimension of the optimization problem is very large. It is straight forward to reduce numerical effort and, therefore, to apply adjoint methods to determine system sensitivity data as prerequisite for gradient based solution methods. It appears that simple descent methods are very robust and effective.

Taking design variables **s** and state variables **u** as independent the following optimization problem can be stated:

$$f(\mathbf{s},\mathbf{u}) \rightarrow min$$
$$\mathbf{G}(\mathbf{s},\mathbf{u}) = 0 \tag{6}$$

Typically, state equations **G** are partial differential equations, as e.g. equilibrium conditions in structural optimization. Without considering additional constraints at this point of discussion and introducing adjoint variables **λ** the Lagrange function L and its variation are defined as:

$$L = f(\mathbf{s},\mathbf{u}) + \boldsymbol{\lambda}^T \mathbf{G}(\mathbf{s},\mathbf{u})$$
$$\delta L = \left(f,_{\mathbf{s}} + \boldsymbol{\lambda}^T \mathbf{G},_{\mathbf{s}}\right)\delta\mathbf{s} + \left(f,_{\mathbf{u}} + \boldsymbol{\lambda}^T \mathbf{G},_{\mathbf{u}}\right)\delta\mathbf{u} + \delta\boldsymbol{\lambda}^T \mathbf{G} = 0 \tag{7}$$

where $(\cdot),_{\mathbf{s}}$ means partial derivative with respect to **s**. After solving for the adjoint variables

$$\boldsymbol{\lambda} = -\mathbf{G},_{\mathbf{u}}^{-1} \mathbf{G},_{\mathbf{s}} \tag{8}$$

finally, the total derivative of f with respect to design parameters is given as

$$f;_{\mathbf{s}} = f,_{\mathbf{s}} + \boldsymbol{\lambda}^T \mathbf{G},_{\mathbf{s}} \tag{9}$$

and will be used for further processing.

7. Semi analytical sensitivity analysis

A typical bottleneck in sensitivity analysis is the effort to get derivatives of element stiffness matrices with respect to design parameters **s**. In particular, for sophisticated elements as e.g. shell elements the straight forward development of the chain rule results is cumbersome and error prone. Also, evaluating all the additional terms generated by the chain rule in total takes more time than evaluating the element stiffness matrix itself. Therefore, it is effective and efficient in terms of coding as well as execution to consider finite difference approximations for the element stiffness derivatives. However, additional covariant terms have to be considered explicitly as derivatives with respect to geometrical parameters have to be taken. Those terms are identified by the rigid body rotation (RBR) test. Basic research in this direction had been undertaken by Mljenek, Lund and van Keulen which has been consolidated by own results in the following form.

Consider a specific finite element stiffness matrix **k** and rigid body rotation modes ϕ_i to be given. Then, it holds:

$$\mathbf{k}\,\boldsymbol{\phi}_i = 0$$
$$\boldsymbol{\phi}_i^T\,\mathbf{k},_{\mathbf{s}}\,\boldsymbol{\phi}_j = 0 \quad i,j = \{1,2,3\} \quad in\ 3D \tag{10}$$

As, typically, approximations $\Delta\mathbf{k}$ of the stiffness matrix derivatives fail the RBR test $\Delta\mathbf{k}$ is enhanced by additional dyadic products:

$$\Delta\mathbf{k}_{mod} = \Delta\mathbf{k} + a_{ij}\phi_i \otimes \phi_j \qquad (11)$$

Again applying the RBR test 6 factors a_{ij} are identified as:

$$a_{ij} = -\phi_i\,\Delta\mathbf{k}\,\phi_j \qquad (12)$$

It remains to identify orthogonal RBR modes which can easily and efficiently be done for any finite element formulation without modification or even knowledge of related code. All together, this approach is extremely efficient and accurate [8].

8. Illustrative examples

8.1 Pre-stressed surfaces

These examples, Figures 2-5, present the direct application of URS for the design of pre-stressed surfaces due to isotropic (minimal surfaces) and anisotropic surface stresses. Note, that even ideal minimal surfaces can easily be determined which is a challenge for many available structural form finding methods. Note also the high quality of the final mesh as a result of the tangential regularization technique. The implemented procedure is able to treat form finding under additional effects as there are additional surface loads (e.g. pressure), interior edge cables (needs additional formulation of constraints on cable length) and consideration of stiffening members in bending and compression (kind of tensegrity structures). For further information refer to www.membranes24.com.

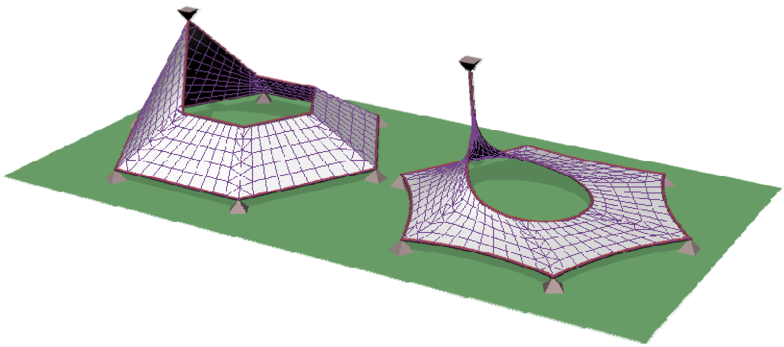

Figure 2: Form finding: Textile membrane with isotropic surface stress, outer and inner edge cables

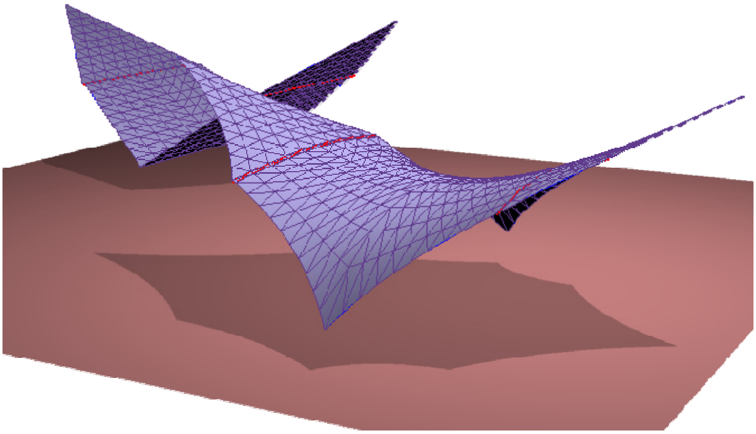

Figure 3: "Bat Wing", Form finding of hybrid structure: Isotropic surface stress, edge cables, spokes in compression and bending

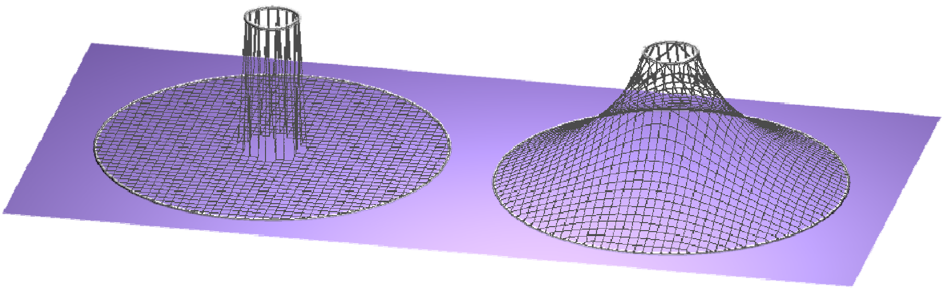

Figure 4: Anisotropic pre-stressed cable net

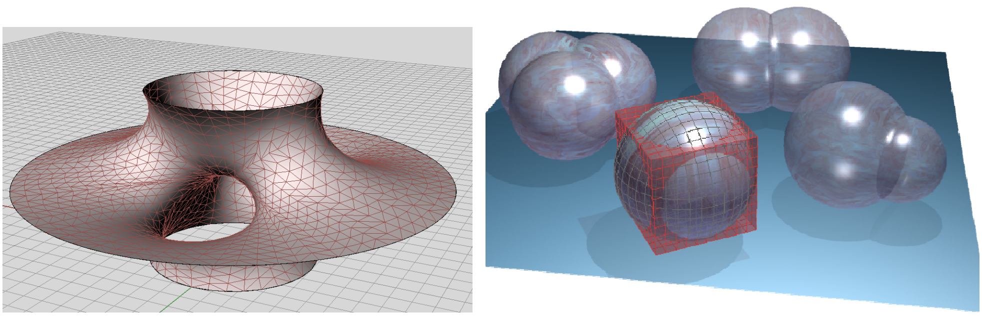

Figure 5: Costa's multiple-connected minimal surface and ideal spherical soap bubbles

8.2 Bead design of plates and shells for single loads

A bend cantilever made of a thin (metal) sheet is loaded as shown, Figure 6. A filter radius as large as the width of support is used. The model consists of appr. 5.000 shape variables. The optimal shape (most right) is reached after 19 iteration steps.

Another example demonstrates the mesh independence of the method, Figure 7. A quadratic plate is loaded in the centre and supported at the corners. The question is to find the optimal topology of stiffening beads. A filter radius is chosen as large as half of the width of support. Additionally, a constraint on the maximum bead depth is given. As shown, the optimal solution is characterized by the filter but it is mesh independent. The choices of filter type and size are additional degrees of design freedom which may be used to explore the design space. Note the smooth final surface although local radial filters are applied.

Single loads on shells cause local bending. Maximizing stiffness of cylindrical shells by applying the local filter technique typically modifies shape in the vicinity of the point of load application. That confirms St. Venant's principle, Figure 8. Again, the filter technique creates smooth surfaces. Tangential regularization was not applied as can be seen. The number of iterations appears always to be not more than 20 for every problem size.

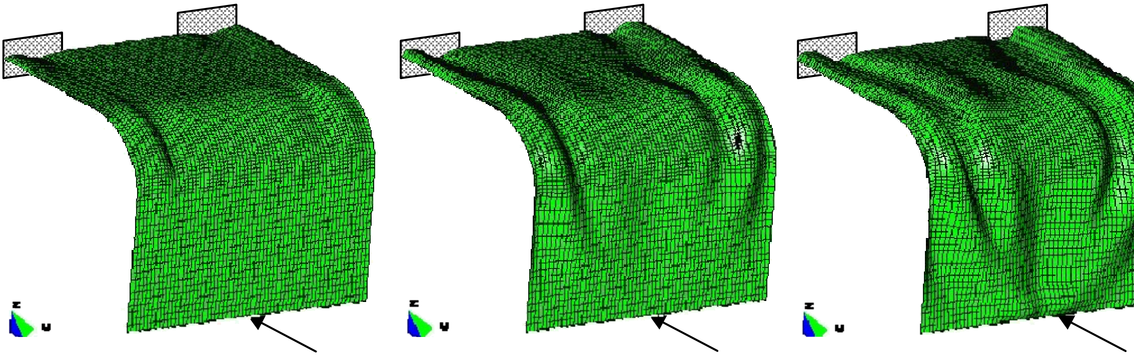

Figure 6: Shape optimization of cantilever shell.

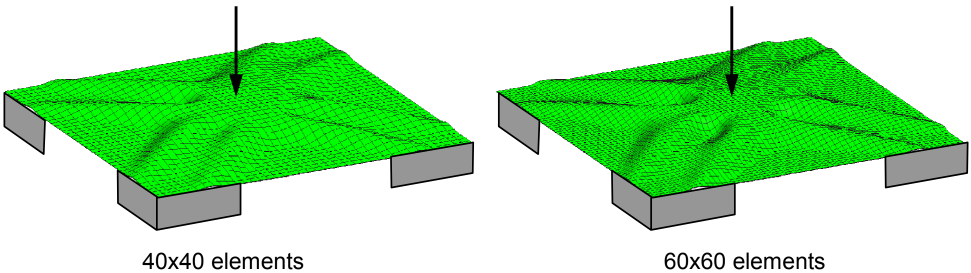

Figure 7: Optimal bead design of initially plane sheet.

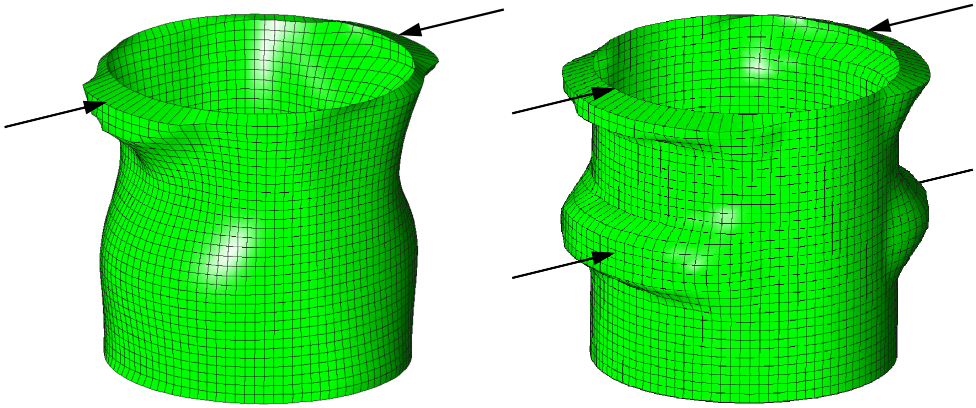

Figure 8: Effect of local load application on optimally stiffened cylinders.

8.3 Optimal design of a pressure bulkhead

This example demonstrates how the different techniques of form finding and shape optimization can be combined to find suitable best solutions. For the present case we are dealing with an oval bulkhead under pressure, a typical structure as it is used as closure of pressurized vessels, Figure 9. As the optimal shape will be similar to a "balloon" the first step of shape design is done as to find the optimal form of a membrane under pressure with fixed supports, Figure 9a. In a second step the shape is modified due to the true supports which are of roller type. Now, the filter technique is applied to directly maximize the shell stiffness, Figure 9b. The additional shape modification is shown in Figure 9c as the difference of a) and b). The main effect is to adapt the cross edge tangent inclination. It appears that the tangent is vertical where the edge curvature is small whereas in regions with larger edge curvature the tangent is not that steep. That reflects the mechanical relation between the curvature of a curved beam and its lateral loading. The solution is affected by the filter choice which can be seen from the contour lines of shell elevation. The designer may deal with the centre elevation and the filter as additional degrees of freedom to find that solution which satisfies all those additional conditions which are not directly part of the shape optimization procedure. Typically, structural stability must be checked additionally. An actual research project is dealing with the question of directly including structural stability during optimization.

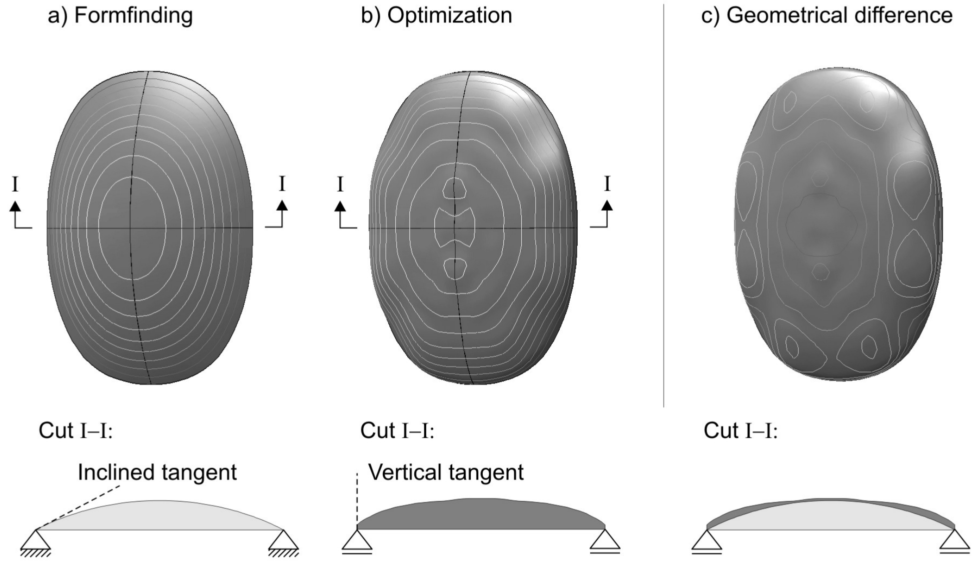

Figure 9: Two stage optimal design of pressure bulk head

8.4 MIT reloaded

This example deals with a reference example which has previously been solved by a CAGD based procedure and simulation of a hanging model [9,10], Fig 10. It is motivated by the question about how to modify the famous shell on the MIT campus due to stiffness considerations. Now the problem is solved by directly dealing with the finite element mesh applying all presented techniques. Filter radii are chosen smaller towards the shell's edges to foster negative surface curvature to develop. The shell is supported at the tips of its toes, load is self weight, and edges are allowed to be modified in vertical direction only. The initial structure is a plate within the boundaries of the projected original structure. As can be seen from Figure 10 a large variety of local minima can be found when the filter is modified. Again, notice the smooth surface although filters have only local support. Solutions include the well know case b) with negative curved edges where cases a) and c) obviously work without. The latter solutions can also be found by hanging models when cutting patterns are modified as additional degrees of design freedom. Finally, case c) has some similarity with Candela's shell designs which are generated from globally negative curved surfaces.

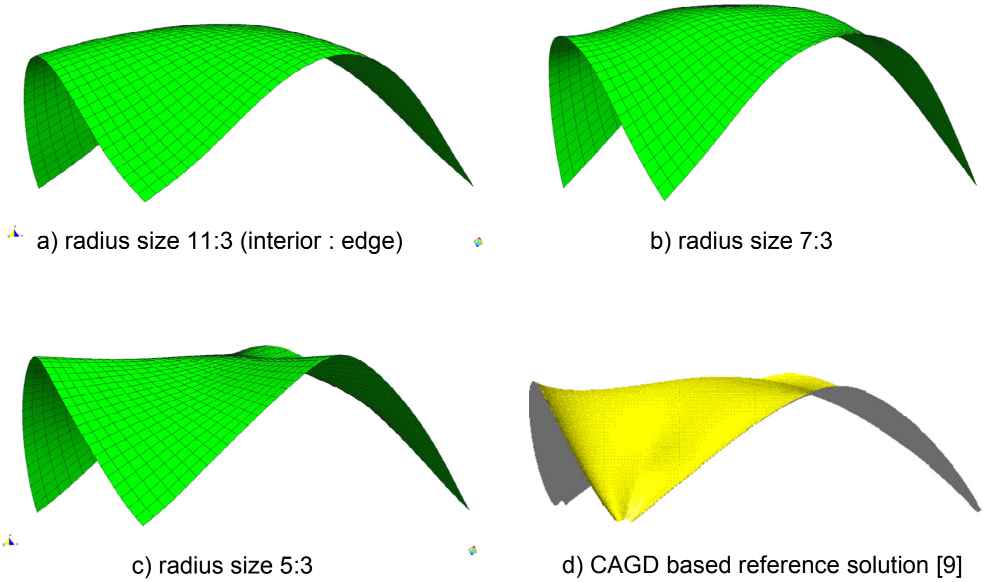

Figure 10: Further solutions of the MIT reference example.

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