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A Hadamard product involving inverse-positive matrices

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Abstract: In this paper we study the Hadamard product of inverse-positive matrices. We observe that this class of matrices is not closed under the Hadamard product, but we show that for a particular sign pattern of the inverse-positive matrices A and B, the Hadamard product $A \circ B^{-1}$ is again an inverse-positive matrix.

MSC: 15A09,15B48

1 Introduction

In economics as well as other sciences, the inverse-positivity of real square matrices has been an important topic. A nonsingular real matrix A is said to be inverse-positive if all the elements of its inverse are nonnegative. An inverse-positive matrix being also a Z-matrix is a nonsingular M-matrix, so the class of inverse-positive matrices contains the nonsingular M-matrices, which have been widely studied and whose applications, for example, in iterative methods, dynamic systems, economics or mathematical programming are well known. However, there are inverse-positive matrices that are not M-matrices.

The concept of inverse-positive is preserved by multiplication, left or right positive diagonal multiplication, positive diagonal similarity and permutation similarity. So we may assume, without loss of generality, that all diagonal entries are equal to 1 when they are positive.

The Hadamard (or entry-wise) product of two $n \times n$ matrices $A = (a_{ij})$ and $B = (b_{ij})$ is $A \circ B = (a_{ij}b_{ij})$.

The class of inverse-positive matrices is not closed under the Hadamard product. It is easy to observe that matrices

$$A = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & 2 \\ 3 & -1 \end{pmatrix}$$

are inverse-positive. However, $C = A \circ B$ is not an inverse-positive matrix.

Several authors have investigated about the Hadamard product of different types of matrices. For example, recently Fallat and Johnson considered in [1] the Hadamard powers of totally positive matrices and Wang et al. in [6] studied the behaviour of the inverse M-matrices under this product. Fan in [2] noted that if the sign pattern is properly adjusted the Hadamard product of two M-matrices is again an M-matrix and Johnson in [3] showed that for any pair A, B of $n \times n$ M-matrices, the Hadamard product $A \circ B^{-1}$ is an M-matrix. This result does not hold in general for inverse-positive matrices. For example, if we take matrices of the previous example, $A \circ B^{-1}$ is not an inverse-positive matrix.

In this paper we analyze the Hadamard product and the inverse-positive concept for a particular type of pattern: the checkerboard pattern.

An $n \times n$ real matrix $A = (a_{ij})$ is said to have a checkerboard pattern if $a_{ij} = 0$ or $\mathrm{sign}(a_{ij}) = (-1)^{i+j}$, $i, j = 1, 2, \ldots, n$.

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For any $n \times n$ matrix A, we denote the submatrix lying in rows α and columns β, in which $\alpha, \beta \subseteq N = \{1, \dots, n\}$, by $A[\alpha|\beta]$, and the principal submatrix $A[\alpha|\alpha]$ is denoted by $A[\alpha]$. Similarly, $A(\alpha|\beta)$ denotes the submatrix obtained from A by deleting rows α and columns β and $A(\alpha|\alpha)$ is denoted by $A(\alpha)$.

In this paper, we analyze when the Hadamard product, $A \circ B^{-1}$ or $A \circ A^{-1}$, is an inverse-positive matrix for any inverse-positive matrices A, B with checkerboard pattern.

For matrices of size 2×2, it is easy to prove the following result.

Proposition 1.1. *Let A, B be 2×2 inverse-positive matrices with* sign$(\det A)$ = sign$(\det B)$*. Then, $A \circ B^{-1}$ is an inverse-positive matrix.*

Unfortunately, in general this result does not hold for matrices of size $n \times n$, $n \geq 3$, so $A \circ B^{-1}$ is not always an inverse-positive matrix.

Example 1. The following matrix

$$A = \begin{pmatrix} -2 & 1 & -1 \\ 1 & -1 & 2 \\ -0.5 & 1 & -2 \end{pmatrix},$$

is inverse-positive, but $A \circ A^{-1}$ is not an inverse-positive matrix.

Example 2. The matrices A and B

$$A = \begin{pmatrix} 2 & -2 & 5 \\ -1 & 1 & -2 \\ 4 & -2 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & -2 \\ 1 & -1 & 3 \end{pmatrix}$$

are inverse-positive, but $A \circ B^{-1}$ is not an inverse-positive matrix.

By embedding the matrices A and B as a principal submatrix and putting 1's on the main diagonal and zeros on the remaining positions, we produce inverse-positive matrices, of size $n \times n$, $n \geq 3$, with checkerboard pattern, for which $A \circ B^{-1}$ (Example 2) or $A \circ A^{-1}$ (Example 1) are not inverse-positive matrices.

However, for matrices of size 3×3 we can establish the following result.

Proposition 1.2. *Let A and B be 3×3 inverse-positive matrices with checkerboard pattern and* $\det A > 0$, $\det B > 0$*. Then, $A \circ B^{-1}$ is an inverse-positive matrix.*

Proof: Consider

$$A = \begin{pmatrix} a_{11} & -a_{12} & a_{13} \\ -a_{21} & a_{22} & -a_{23} \\ a_{31} & -a_{32} & a_{33} \end{pmatrix} \text{ and } B = \begin{pmatrix} b_{11} & -b_{12} & b_{13} \\ -b_{21} & b_{22} & -b_{23} \\ b_{31} & -b_{32} & b_{33} \end{pmatrix}$$

where $a_{ij} \geq 0$ and $b_{ij} \geq 0$. We can assume, without loss of generality, that $\det A = 1$ and $\det B = 1$. We denote

$$A^{-1} = \begin{pmatrix} A_{11} & -A_{21} & A_{31} \\ -A_{12} & A_{22} & -A_{32} \\ A_{13} & -A_{23} & A_{33} \end{pmatrix} \geq 0, \quad B^{-1} = \begin{pmatrix} B_{11} & -B_{21} & B_{31} \\ -B_{12} & B_{22} & -B_{32} \\ B_{13} & -B_{23} & B_{33} \end{pmatrix} \geq 0$$

where A_{ij} and B_{ij}, $i, j = 1, 2, 3$, are the determinants of the submatrices $A(\{i\}|\{j\})$ and $B(\{i\}|\{j\})$, respectively, and

$$C = A \circ B^{-1} = \begin{pmatrix} a_{11}B_{11} & a_{12}B_{21} & a_{13}B_{31} \\ a_{21}B_{12} & a_{22}B_{22} & a_{23}B_{32} \\ a_{31}B_{13} & a_{32}B_{23} & a_{33}B_{33} \end{pmatrix}.$$

We know that $a_{11} \geq 0$, but we can prove that $a_{11} \neq 0$ since in other case we could obtain a contradiction with $\det A \neq 0$. With a similar reasoning it is easy to obtain that $a_{ii}, b_{ii}, A_{ii}, B_{ii}$ are greater than zero, for $i = 1, 2, 3$.

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Now, if we denote by C_{ij} the determinant of the submatrix $C(\{i\}|\{j\})$, $i, j = 1, 2, 3$, we are going to prove that $C_{33} > 0$. Since $b_{33} = B_{11}B_{22} - B_{12}B_{21}$, we have

$$\begin{aligned} C_{33} = a_{11}B_{11}a_{22}B_{22} - a_{12}B_{21}a_{21}B_{12} &= a_{11}a_{22}b_{33} + (a_{11}a_{22} - a_{12}a_{21})B_{12}B_{21} \\ &= a_{11}a_{22}b_{33} + A_{33}B_{12}B_{21} > 0. \end{aligned}$$

An analogous proof allows to assure that $C_{ii} > 0$, $i = 1, 2, 3$. So, by applying Gauss elimination we have

$$\det C = \frac{1}{a_{11}B_{11}}(C_{22}C_{33} + C_{32}C_{23}).$$

In order to prove $\det C > 0$ we need to analyze the sign of C_{32} and C_{23}. Since $-b_{23} = -(-B_{11}B_{32} + B_{12}B_{31})$, we have

$$\begin{aligned} C_{32} = a_{11}B_{11}a_{23}B_{32} - a_{13}B_{31}a_{21}B_{12} &= -a_{11}a_{23}b_{23} + (a_{11}a_{23} - a_{13}a_{21})B_{31}B_{12} \\ &= -a_{11}a_{23}b_{23} - A_{32}B_{31}B_{12} \le 0. \end{aligned}$$

Analogously, $C_{23} \le 0$. Therefore, $\det C > 0$.

Let $C^{-1} = (d_{ij})$ be the inverse of C. We are going to prove that $d_{ij} \ge 0$, $i, j = 1, 2, 3$. We know that d_{11}, d_{22}, d_{33} are greater than zero. Taking into account that $\det C > 0$, in order to prove $d_{12} \ge 0$ we only need that $-(a_{12}B_{21}a_{33}B_{33} - a_{13}B_{31}a_{32}B_{23}) \ge 0$. Since $-b_{12} = B_{21}B_{33} - B_{31}B_{23}$, we have

$$\begin{aligned} a_{13}B_{31}a_{32}B_{23} - a_{12}B_{21}a_{33}B_{33} &= a_{13}a_{32}b_{12} + (a_{13}a_{32} - a_{12}a_{33})B_{21}B_{33} = \\ &= a_{13}a_{32}b_{12} + A_{21}B_{21}B_{33} \ge 0. \end{aligned}$$

Following a similar reasoning we can prove that $d_{ij} \ge 0$, $i, j = 1, 2, 3$.

In the next section we are going to analyze the Hadamard product of upper (lower) triangular inverse-positive matrices, with checkerboard pattern.

2 Triangular inverse-positive matrices with checkerboard pattern

In this section we work with upper triangular inverse-positive matrices, with checkerboard pattern. So, we can assume, without loss of generality, that all diagonal entries are equal to 1.

The following condition is inspired in the *PP*-condition introduced by Johnson and Smith in [4] for non-negative matrices.

Definition 2.1. *Let $A = (a_{ij})$ be an $n \times n$ upper triangular matrix. A satisfies the P-condition if*

$$a_{ij} \le a_{ik}a_{kj}, \quad i < k < j.$$

We need the following technical lemma in order to get the main result of the paper.

Lemma 2.1. *Let A be an $n \times n$ nonsingular upper triangular matrix, with checkerboard pattern, that satisfies the P-condition. Then* $\det A[\{i-1, i, \dots, n-1\}|\{i, i+1, \dots, n\}] = 0$ *or*

$$sign(\det A[\{i-1, i, \dots, n-1\}|\{i, i+1, \dots, n\}]) = (-1)^{n+i-1}, \quad i = 2, 3, \dots, n.$$

Proof. The proof is by induction on n. For $n = 2$ the result is trivially satisfied. For $n = 3$, the matrix A with $a_{i-1,i} \le 0$ and $a_{i-1,i+1} \ge 0, \forall i$ has the form

$$A = \begin{pmatrix} 1 & a_{12} & a_{13} \\ 0 & 1 & a_{23} \\ 0 & 0 & 1 \end{pmatrix}.$$

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For $i = 3$, $\det A[\{2\}|\{3\}] = a_{23} \leq 0$, and for $i = 2$, $\det A[\{1,2\}|\{2,3\}] = a_{12}a_{23} - a_{13} \geq 0$, from the P-condition.

Suppose that the result holds for $(n-1) \times (n-1)$ matrices and we are going to prove it for matrices of size $n \times n$. We assume that n is odd (for n even we proceed in analogous way). For $i = n$, $\det A[\{n-1\}|\{n\}] = a_{n-1,n} \leq 0$. For $i = n-1, n-2, \ldots, 3$ we apply the hypothesis of induction for the submatrix $A[\{2,3,\ldots,n\}]$. Finally, for $i = 2$,

$$\begin{aligned}\det A[\{1,2,\ldots,n-1\}|\{2,3,\ldots,n\}] = \\ &= a_{12} \det A[\{2,3,\ldots,n-1\}|\{3,4,\ldots,n\}] \\ &- \det A[\{1,3,\ldots,n-1\}|\{3,4,\ldots,n\}].\end{aligned}$$

By applying the P-condition we can prove that

$$\det A[\{1,3,\ldots,n-1\}|\{3,4,\ldots,n\}] \leq 0,$$

and by the hypothesis of induction applied to submatrix $A[\{2,3,\ldots,n\}]$ we have $\det A[\{2,3,\ldots,n-1\}|\{3,4,\ldots,n\}] \leq 0$, (recall that n is an odd number). So, $\det A[\{1,2,\ldots,n-1\}|\{2,3,\ldots,n\}] = 0$ or $sign(\det A[\{1,2,\ldots,n-1\}|\{2,3,\ldots,n\}]) = (-1)^{n+1}$. □

Theorem 2.1. *Let A be an $n \times n$ nonsingular upper triangular matrix with checkerboard pattern, that satisfies the P-condition. Then, A is an inverse-positive matrix.*

Proof. The proof is by induction on n. For $n = 2$ the result holds since A is an M-matrix. Suppose that the results holds for $(n-1) \times (n-1)$ matrices and we are going to prove it for $n \times n$ matrices. For $n \geq 3$ a matrix of size $n \times n$ can be partitioned as

$$A_n = \begin{pmatrix} A_{n-1} & v_{n-1} \\ 0 & 1 \end{pmatrix},$$

where A_{n-1} is the upper triangular submatrix $A[\{1,2,\ldots,n-1\}]$ and v_{n-1} is the submatrix $A[\{1,2,\ldots,n-1\}|\{n\}]$. We can observe that

$$A_n^{-1} = \begin{pmatrix} A_{n-1}^{-1} & -A_{n-1}^{-1}v_{n-1} \\ 0 & 1 \end{pmatrix},$$

where

$$\begin{aligned}&-A_{n-1}^{-1}v_{n-1} = \\ &= \left[(-1)^{n+1} \det A[\{1,\ldots,n-1\}|\{2,\ldots,n\}], \ldots, (-1)^{2n-1} \det A[\{n-1\}|\{n\}]\right]^T.\end{aligned}$$

The hypothesis of induction and Lemma 2.1 allow us to assure that $A_n^{-1} \geq 0$. □

In general, the converse of this result does not hold, but we can establish the following result.

Theorem 2.2. *Let A be a 3×3 nonsingular upper triangular matrix with checkerboard pattern. Then A satisfies the P-condition if and only if A is an inverse-positive matrix.*

In order to obtain a necessary condition in the general case, we introduce the following notation (see [6]). Given an $n \times n$ matrix A and the positive integers i and k, we denote

$$a_{i,i+1,\ldots,i+k} = (-1)^{k+1} \det A[\{i, i+1, \ldots, i+(k-1)\}|\{i+1, i+2, \ldots, i+k\}]. \tag{1}$$

We can establish the following preliminary result.

Lemma 2.2. *Let A be an $n \times n$ nonsingular upper triangular matrix with checkerboard pattern. If $A^{-1} = (b_{ij})$ then,*

$$b_{ij} = -a_{i,i+1,\ldots,j}.$$

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Proof. By definition,

$$b_{ij} = (-1)^{i+j} \det A[\{1,2,\ldots,j-1,j+1,\ldots,n\}|\{1,2,\ldots,i-1,i+1,\ldots,n\}],$$

and taking into account that A is upper triangular, we have

$$\det A[\{1,2,\ldots,j-1,j+1,\ldots,n\}|\{1,2,\ldots,i-1,i+1,\ldots,n\}] = \det A[\{i,i+1,\ldots,j-1\}|\{i+1,i+2,\ldots,j\}]$$

and by (1)

$$detA[\{i,i+1,\ldots,j-1\}|\{i+1,i+2,\ldots,j\}] = (-1)^{i+j+1} a_{i,i+1,\ldots,j}.$$

Then,

$$b_{ij} = (-1)^{i+j}(-1)^{i+j+1} a_{i,i+1,\ldots,j} = -a_{i,i+1,\ldots,j}.$$

□

The next result is an immediate consequence of this lemma.

Theorem 2.3. *Let A be an $n \times n$ nonsingular upper triangular matrix with checkerboard pattern. Then, A is an inverse-positive matrix if and only if for any positive integers i and k*

$$a_{i,i+1,\ldots,i+k} \le 0. \tag{2}$$

Now, we are going to analyze when the class of upper triangular inverse-positive matrices, with checkerboard pattern, is closed under the Hadamard product of type $A \circ B^{-1}$.

In order to prove the main result of this work, we need the following technical result which follows from Lemma 2.2.

Corollary 2.1. *Let $A = (a_{ij})$ and $B = (b_{ij})$ be $n \times n$ nonsingular upper triangular matrices. If $(B)^{-1}_{ij}$ denotes the (i,j) entry of B^{-1}, we have*

$$(B)^{-1}_{ij} = -b_{i,i+1,\ldots,j}, \qquad (B)^{-1}_{i,i+1} = -b_{i,i+1} \tag{3}$$

and, being $C = A \circ B^{-1} = (c_{ij})$,

$$c_{ij} = -a_{ij} b_{i,i+1,\ldots,j}, \quad \text{for all } i < j. \tag{4}$$

Theorem 2.4. *Let A, B be $n \times n$ upper triangular nonsingular matrices, with checkerboard pattern, satisfying the P-condition. Then $A \circ B^{-1}$ is an inverse-positive matrix.*

Proof. The proof is by induction on n. For $n = 2$, A and B are M-matrices and the result is known.

Now, let A, B be upper triangular matrices of size $n \times n$, $n \ge 3$. These matrices can be partitioned as,

$$A = \begin{pmatrix} A_{11} & \bar{a}_{12} \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} B_{11} & \bar{b}_{12} \\ 0 & 1 \end{pmatrix}.$$

Note that, A_{11} and B_{11} are nonsingular upper triangular matrices of size $(n-1) \times (n-1)$, with checkerboard pattern and 1's in the main diagonal, satisfying the P-condition.

$$C = A \circ B^{-1} = \begin{pmatrix} A_{11} \circ B_{11}^{-1} & -\bar{a}_{12} \circ B_{11}^{-1} \bar{b}_{12} \\ 0 & 1 \end{pmatrix} = (c_{ij})_{1 \le i,j \le n}$$

and

$$C^{-1} = \begin{pmatrix} (A_{11} \circ B_{11}^{-1})^{-1} & [r_{1n}, r_{2n} \ldots, r_{n-1,n}]^T \\ 0 & 1 \end{pmatrix},$$

where, by (3), $r_{jn} = (C)^{-1}_{jn}$, $j = 1,2,\ldots,n-1$.

By using the hypothesis of induction, in order to assure that C is inverse-positive we only need to prove that $r_{jn} \ge 0$, $j = 1,2,\ldots,n-1$.

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For $j = n-1$, by (4) we have

$$r_{n-1,n} = -c_{n-1,n} = -a_{n-1,n}(-b_{n-1,n}) \geq 0.$$

Now,

$$r_{n-2,n} = -c_{n-2,n-1,n} = \det C[\{n-2, n-1\}|\{n-1, n\}] = c_{n-2,n-1}c_{n-1,n} - c_{n-2,n}$$

and by (4)

$$r_{n-2,n} = (-a_{n-2,n-1}b_{n-2,n-1})(-a_{n-1,n}b_{n-1,n}) - (-a_{n-2,n}b_{n-2,n-1,n}).$$

By reordering in an adequate form, and taking into account that A and B have checkerboard pattern and satisfy the P-condition, we obtain

$$r_{n-2,n} = (a_{n-2,n-1}a_{n-1,n} - a_{n-2,n})(b_{n-2,n-1}b_{n-1,n}) + a_{n-2,n}b_{n-2,n} \geq 0.$$

In a similar way,

$$\begin{aligned} r_{n-3,n} &= -\, c_{n-3,n-2,n-1,n} = -\det C[\{n-3, n-2, n-1\}|\{n-2, n-1, n\}] \\ &= -\, c_{n-3,n-2}r_{n-2,n} + c_{n-3,n-1}c_{n-1,n} - c_{n-3,n}. \end{aligned}$$

Since C has checkerboard pattern, we have

$$-c_{n-3,n-2}r_{n-2,n} \geq 0$$

and, by using the P-condition and the checkerboard pattern of A and B, we obtain

$$c_{n-3,n-1}c_{n-1,n} - c_{n-3,n} = (a_{n-3,n-1}a_{n-1,n} - a_{n-3,n})(b_{n-3,n-2,n-1}b_{n-1,n}) + +a_{n-3,n}(-b_{n-3,n-2}b_{n-2,n} + b_{n-3,n}) \geq 0.$$

Therefore, $r_{n-3,n} \geq 0$. In a similar way, we prove that $r_{jn} \geq 0$, for $j = n-4, n-5, \ldots, 2$. Finally, we are going to prove that $r_{1n} \geq 0$.

$$\begin{aligned} r_{1n} =& (C)^{-1}_{1n} = (-1)^{n+1} \det C[\{1, 2, \ldots, n-1\}|\{2, 3, \ldots, n\}] \\ =& (-1)^{n+1} \Big[c_{12} \det C[\{2, 3, \ldots, n-1\}|\{3, 4, \ldots, n\}] - c_{13} \det C[\{3, 4, \ldots, n-1\}|\{4, 5, \ldots, n\}] \\ &+ c_{14} \det C[\{4, \ldots, n-1\}|\{5, \ldots, n\}] + \cdots + (-1)^{n-1} c_{1,n-1}c_{n-1,n} + (-1)^n c_{1n} \Big]. \end{aligned}$$

By using a similar reasoning as $r_{n-3,n}$, if n is even, we have

$$\begin{aligned} r_{1n} =& -\, c_{12}r_{2n} + \big(c_{13} \det C[\{3, 4, \ldots, n-1\}|\{4, 5, \ldots, n\}]\big) - \big(c_{14} \det C[\{4, \ldots, n-1\}|\{5, \ldots, n\}]\big) \\ &+ \cdots + (c_{1,n-1}c_{n-1,n} - c_{1n}) \geq 0. \end{aligned}$$

Finally, by using a similar reasoning as $r_{n-2,n}$, if n is odd, we have

$$\begin{aligned} r_{1n} =& \big(c_{12}r_{2n} - c_{13} \det C[\{3, 4, \ldots, n-1\}|\{4, 5, \ldots, n\}]\big) + \big(c_{14} \det C[\{4, \ldots, n-1\}|\{5, \ldots, n\}] \\ &- c_{15} \det C[\{5, \ldots, n-1\}|\{6, \ldots, n\}]\big) + \cdots + (c_{1,n-1}c_{n-1,n} - c_{1,n}) \geq 0. \end{aligned}$$

We only need to prove in order to assure the previous inequalities that $(c_{1,n-1}c_{n-1,n} - c_{1,n}) \geq 0$, for all n.

Note that:

$$\begin{aligned} c_{1,n-1}c_{n-1,n} - c_{1,n} =& (-a_{1,n-1}b_{1,2,\ldots,n-1})(-a_{n-1,n}b_{n-1,n}) + (a_{1,n}b_{1,2,\ldots,n}) = \\ =& (a_{1,n-1}a_{n-1,n}b_{1,2,\ldots,n-1}b_{n-1,n}) + (a_{1,n})(-1)^n \det B[\{1, 2, \ldots, n-1\}|\{2, 3, \ldots, n\}] \\ =& (a_{1,n-1}a_{n-1,n} - a_{1,n})(b_{n-1,n}b_{1,2,\ldots,n-1}) + (a_{1,n})(-1)^{2n+2} b_{1,2,\ldots,n-2,n} \\ =& (a_{1,n-1}a_{n-1,n} - a_{1,n})(b_{n-1,n}b_{1,2,\ldots,n-1}) + \ldots + a_{1,n}b_{1,n} \geq 0, \end{aligned}$$

by using the P-condition and the checkerboard pattern of matrices A and B.

Therefore, C is an inverse-positive matrix. □

The results obtained in this work are straightforward satisfied by lower triangular matrices.

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