Imperfection modes determining the buckling resistance of pressurized spherical caps

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Abstract

The paper deals with determination of primary bifurcation modes of spherical caps loaded by the external pressure. These modes can be used to generate the worst imperfection modes which are then the basis for the determination of the lowest critical pressure of slender, elastic shells. A program developed by the author and based on the finite element method was used in the numerical examples presented herein. The buckling mode strongly depends on the semi-angle of a cap and the R/t ratio.

Keywords: spherical shell, external pressure, geometrically nonlinear analysis, finite element method, equilibrium path, bifurcation point, bifurcation mode, worst imperfection mode, buckling resistance.

1. Introduction

Structural elements in the form of a full spherical shell or segment of a spherical shell (a cap) are structural elements encountered very often in the engineering practice. Shells in the form of spherical caps create covers of monumental, historical buildings. In such applications, they are usually very thick and the stress limit state is decisive in the designing procedure. Spherical caps also occur as closures of cylindrical or conical pressure vessels and it is probably the most frequent engineering case of using shallow spherical shells.

It is a natural designing tendency that the shells are thinner and thinner and in such cases the buckling limit state seems to be the most important.

The linear buckling problem of a full sphere subjected to the external pressure was solved by Zoelly [16]. The same solution was obtained independently by Leibenson in Russia in 1917. In the sixth decade of the twenties century the first works, in which the geometrically nonlinear approach was adopted, appeared. Works of Weinitschke [14] and Huang [5] were probably the most important. In subsequent years, many other authors contributed to the clarification of stability phenomenon of spherical shells. The buckling behaviour of spherical pressurized shells is the subject of interest of many contemporary researchers as well. Works of Deml & Wunderlich [3], Wunderlich & Albertin [15], Blachut & Galletly [1] are just examples and evidence of a continuous interest of many authors in the mechanical behaviour of spherical shells. The work of Grigolyuk & Lopanitsyn [4], in which stability problem of a spherical shell was solved by the Ritz method adopted for nonlinear, Marguerre type equilibrium equations of a shell, is also worth mentioning.

The problem is important also from the point of view of engineering practice. Recommendations [2] which refer to the safe design of shell structures with respect to buckling criteria require the knowledge of the worst imperfection modes. A configuration corresponding to the first bifurcation point (primary bifurcation point) is often the worst imperfection mode. The primary bifurcation point should be determined by employing a linear elastic but geometrically nonlinear analysis (GNA).

It is worth emphasizing that the linear buckling analysis gives too high values of critical loads in a case of shallow shells loaded laterally. It is due to the fact that in the prebuckling state nonlinear deformations appear. In such cases the critical load corresponding to the primary bifurcation point should be determined as a result of the geometrically nonlinear analysis. The linear buckling analysis implemented in many commercial packages gives wrong results with reference to the critical load and also with reference to the buckling mode.

The main objective of this paper is the presentation of the numerical procedure leading to localization of primary bifurcation points on the nonlinear equilibrium path and then determination of the deformation mode corresponding to this bifurcation point. The determined mode of bifurcation can be treated as the worst imperfection mode. Using it and adopting a proper amplitude of initial deformations one can determine the lowest critical pressure.

To this end, the author's numerical program based on the finite element method was used. Three different illustrative examples are presented and all of them refer to spherical caps clamped on their base circles and loaded by the external pressure. Linear elastic and geometrically nonlinear analyses of these selected spherical shells were performed and results of these analyses are presented in the paper.

2. Calculation of equilibrium paths

The geometrically nonlinear analysis requires calculation of nonlinear equlibrium paths in the load–displacement space. A numerical approach based on the finite element method is used in this research. The author's computer program (comp. works of Marcinowski and Hadid [8] and Marcinowski [6]) previously exploited and tested on shells of various geometries was applied. The eight node, quadrilateral finite element adequate for thin and thick shells subjected to big displacements within the elastic range was used in this program. Arbitrary displacements can be taken into account, but rotations have to remain moderate.

It is worth mentioning that only conservative loads are considered in the analyses and the whole loading process has a quasi static character. As far as material properties are concerned, only a linearly elastic material is taken into account. Elastic–plastic effects are excluded from considerations presented in this research.

In the detailed analyses presented below only the displacement control technique was used with the algorithm of automatic selection of the best displacement control parameter.

It is obvious that in many cases besides the fundamental path, there usually also exist bifurcation paths and their determination is even more difficult than the calculation of the fundamental path in a particular case. First of all, the location of the bifurcation point must be established. It is the point at which the equilibrium path splits, or more strictly speaking, the bifurcation path takes its origin on the fundamental path.

In the program the so called stability indicator (SI) was defined and was monitored all the time while tracing the equilibrium path. The stability indicator is defined as the number of negative parameters on the diagonal of the tangent stiffness matrix after its triangularisation. One can prove that if all those diagonal parameters are positive, the equilibrium configuration under analysis is stable. If at least one parameter is negative, the equilibrium configuration is unstable. The level of instability is determined by the number of negative diagonal parameters. It is worth mentioning that the SI always changes its value after passing the consecutive bifurcation point on the fundamental path or on the bifurcation path. In this manner, the SI indicates locations of bifurcation points. Minimising the value of the displacement control parameter one can determine the location of the bifurcation point very accurately. Knowing the location of the bifurcation point on the fundamental equilibrium path it is relatively easy to switch into the bifurcation path by using the well known load disturbance procedure.

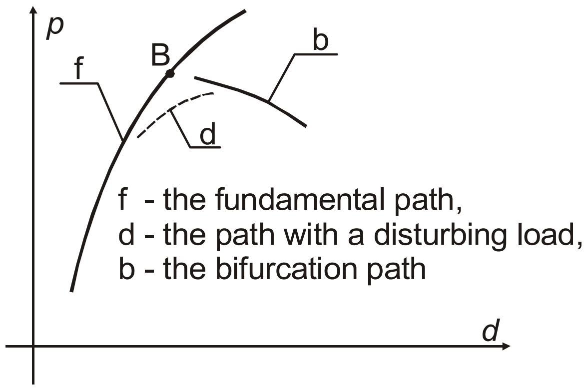

Figure 1. The switching path procedure

This idea is illustrated in Figure 1. Knowing the location of the bifurcation point *B* on the fundamental path the path switching procedure starts beneath this very point after adding an additional small disturbing load. As a result the path denoted as *d* is obtained. Then the disturbing force is removed in one or in several steps and the first configuration on the bifurcation path is determined. Afterwards the tracing of the bifurcation path is continued in the one or in the other direction. The deformation mode corresponding to configurations on this path is the primary bifurcation configuration which was looked for.

3. Primary bifurcation modes of selected spherical caps

The procedure outlined above was adopted for three different spherical caps. The finite element mesh used in calculations is shown in Figure 2. It was very important to create such a mesh which is nearly ideally axisymmetric and in a case of quadrilateral finite elements it was not an easy task.

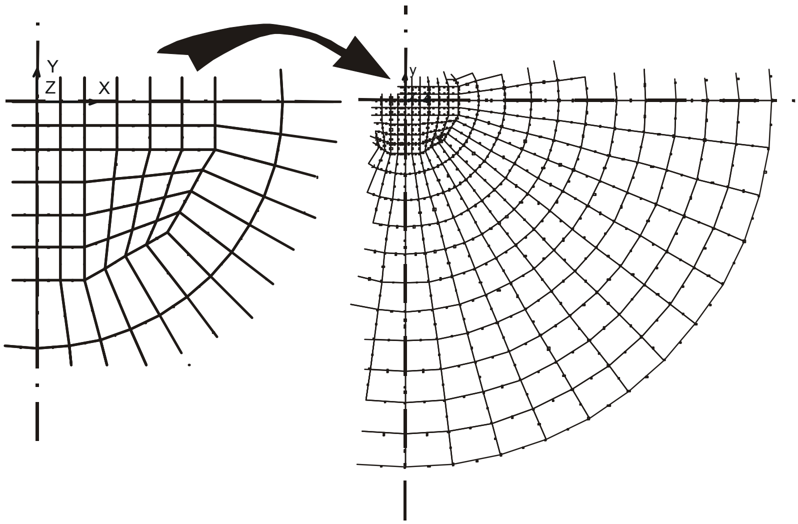

Figure 2. The FE mesh used in the presented calulations

The data for considered caps were taken from works of Marcinowski [9] (cap A), Nolte and Makowski [13] (cap B) and Wunderlich and Albertin [15] (cap C) and are presented below in Figure 3.

Cap	R [m]	t [mm]	R/t	φ [rad]	E [MPa]	ν	p_{B1} [MPa]	p_z [MPa]	p_{B1}/p_z
A	19.244	5.0	3849	0.0520 $=2.98^o$	205000	0.3	12.85E-03	16.75E-03	0.767
B	179.35	400	448	0.2094 $=11.99^0$	200	0.3	9.084E-4	1.204E-03	0.75
C	1.4732	29.46	50	1.0472 $=60^o$	210000	0.3	76.65	101.68	0.75

Figure 3. Geometrical and material data.

The slenderness of the cap can be measured by the geometrical parameter λ introduced by Weinitschke [14] and defined as follows

$$\lambda = [12(1-\nu^2)]^{\frac{1}{4}} \frac{a}{\sqrt{Rt}} \,. \tag{1}$$

Values of this parameter for the shells under consideration are 5.86, 7.83 and 11.13, respectively.

The symbol p_z denotes the critical pressure for the full sphere and its value follows from the Zolly-Leibenson formula:

$$p_z = \frac{2E}{\sqrt{3(1-\nu^2)}} \left(\frac{t}{R}\right)^2 . \tag{2}$$

3.1. Cap A

This shell was analysed in detail in the work of Marcinowski [9] for a slightly different material (E =210 GPa, was adopted in that work) and using a coarse finite element mesh. The shell of similar slenderness was considered independently by Grigolyuk and Lopanitsyn [4].

The initial segment of the nonlinear equilibrium path, the localization of the primary bifurcation point and the deformation mode corresponding to the bifurcation path splitting from the fundamental path at this point are presented in Figure 4.

On the ordinate of the plot, the ratio p/p_z is depicted, where

$$p_z = \frac{2E}{\sqrt{3(1-\nu^2)}} \left(\frac{t}{R}\right)^2 = 16{,}75 \text{ kPa} \,. \tag{3}$$

On the abscissa of the plot, the ratio w_c/t is depicted, where w_c is the vertical deflection of the central node of the shell.

The bifurcation mode shown in Figure 4 is very similar to the primary bifurcation mode determined by Grigolyuk and Lopanitsyn [4] and Marcinowski [9].

It is worth mentioning that four other bifurcation points were detected above the primary bifurcation point B1 before the limit point was reached. This is a typical case of clustered bifurcation points. It is obvious that the most important one is the lowest (B1) with a properly chosen deformation amplitude. Using this deformation mode as the imperfection shape, one can obtain the lowest critical pressure which may be decisive in the assessment of the elastic buckling resistance.

3.2. Cap B

This shell was analysed originally by Mescall [12] and then by Nolte and Makowski [13] and Marcinowski [10]. The initial segment of the nonlinear equilibrium path, the localization of the primary bifurcation point and the deformation mode corresponding to the bifurcation path splitting from the fundamental path at this point are presented in Figure 5.

On the ordinate of the plot, the ratio p/p_z is depicted, where

$$p_z = \frac{2E}{\sqrt{3(1-\nu^2)}}\left(\frac{t}{R}\right)^2 = 1204{,}2 \text{ kPa} . \tag{4}$$

It is worth mentioning that nine other bifurcation points were detected above the primary bifurcation point B1 before the limit point was reached. This is again a typical case of clustered bifurcation points.

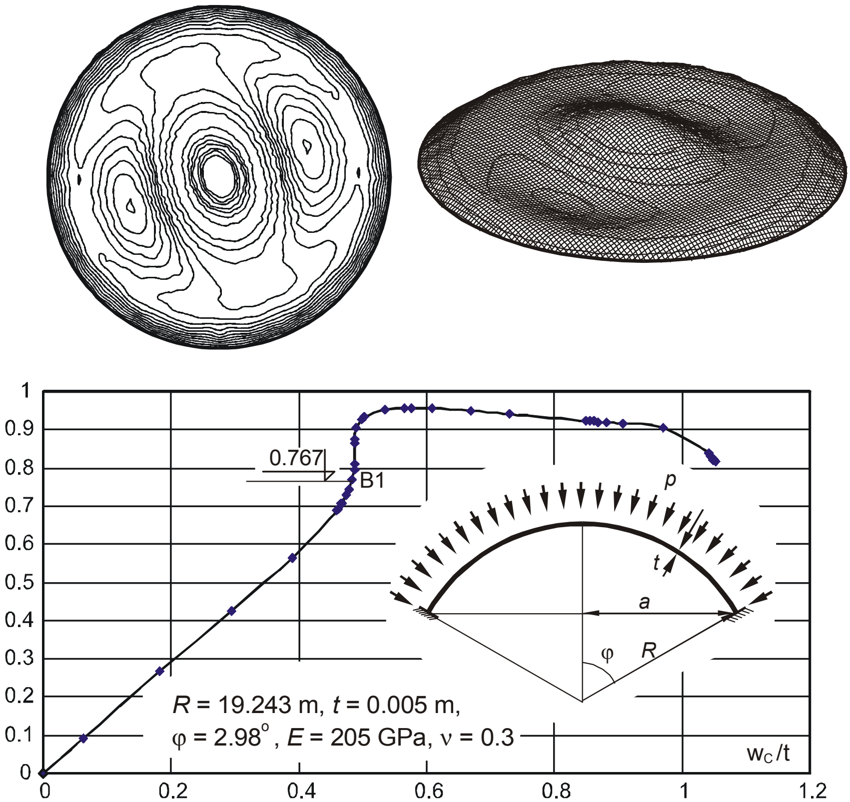

Figure 4. Equilibrium path and the first bifurcation mode for the cap A

3.3. Cap C

This shell was the subject of a detailed analysis performed by Wunderlich and Albertin [14]. The initial segment of the nonlinear equilibrium path, the localization of the primary bifurcation point and the deformation mode corresponding to the bifurcation path splitting from the fundamental path at this point are presented in Figure 6. p_z used to depict the ratio p/p_z is equal to 101,68 MPa.

In this very case fifteen other bifurcation points were detected above the primary bifurcation point B1 before the limit point (94,33 MPa) was reached. It would be a very laborius task to determine all bifurcation paths having their origins at these points. Some of them terminate on the descending part of the fundamental equilibrium path and some on other bifurcation paths (compare solutions presented in the work of Marcinowski [9]). Of

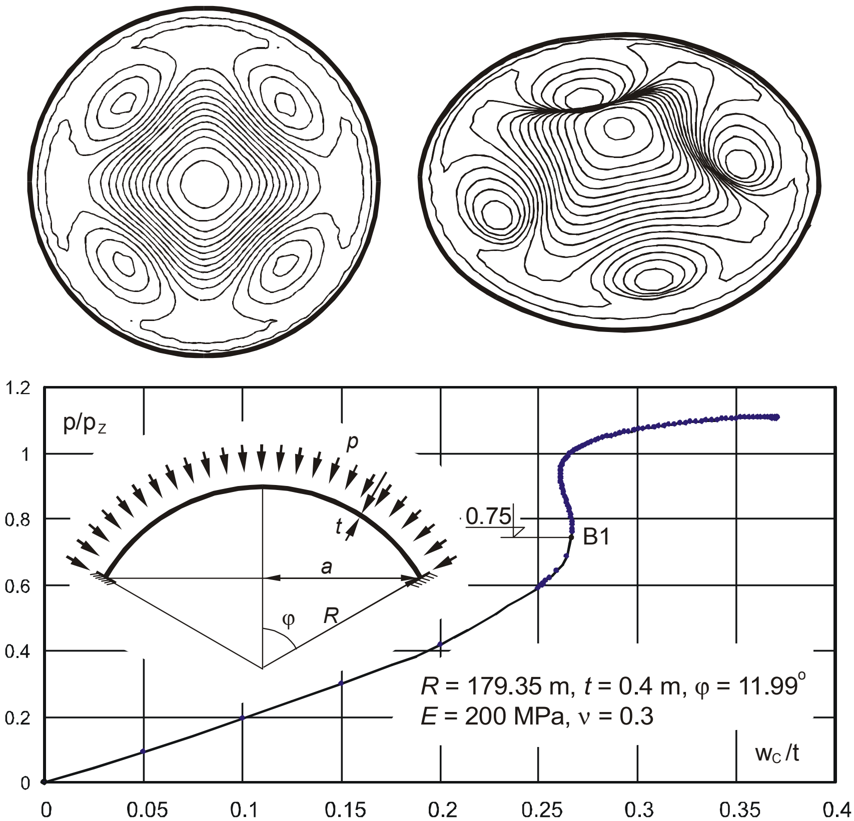

Figure 5. Equilibrium path and the first bifurcation mode for the cap B

course, the first bifurcation path and the corresponding mode of bifurcation are the most important as far as the buckling resistance is concerned.

It is worth mentioning that the buckling mode shown in Figure 6 corresponds to the buckling mode obtained in the work of Wunderlich and Albertin [15]. The critical pressure obtained in the present work (p_{B1}/p_z=0,75) is little bit smaller than the one presented in the work of Wunderlich and Albertin [15] (p_{B1}/p_z=0,8).

4. Recapitulation

It is obvious that the buckling resistance of a shell is determined by the worst imperfection mode. In most common cases of structural members the configuration corresponding to the first bifurcation point (primary bifurcation point) is at the same time the worst imperfection mode at least as far as linearly elastic analysis is concerned.

The effective procedure leading to determination of the buckling mode corresponding to the primary bifurcation point was presented in the paper in reference to spherical shells in a form of a cap clamped along the basic circle and loaded by the external pressure. Three different spherical shells were considered as illustrative examples. Buckling modes were obtained as a result of geometrically nonlinear analysis in which not only fundamental paths were obtained but bifurcation points laying below the limit point as well. Using the

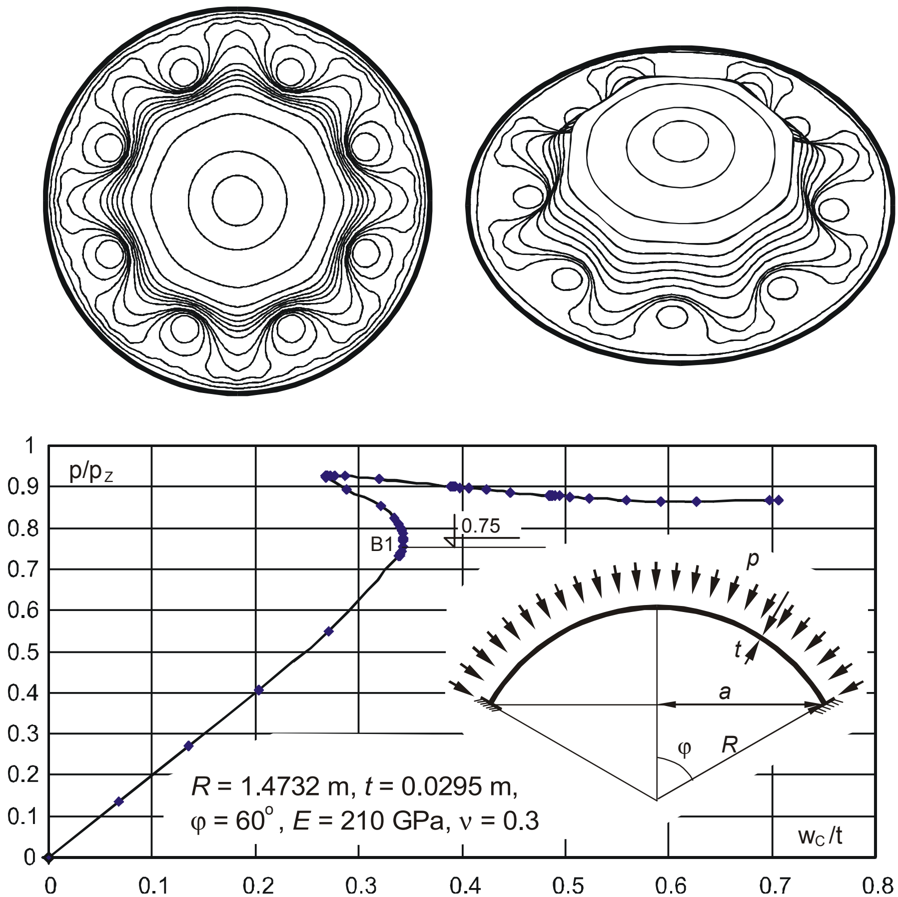

Figure 6. Equilibrium path and the first bifurcation mode for the cap C

load perturbation technique the branching paths corresponding to the primary bifurcation points and accompanying deformation modes were determined as well. The complexity of these deformation modes depends on the semi angle φ and R/t ratio of the cap. Deformation patterns obtained in this manner can be treated as the worst imperfection modes for considered shells.

The elastic critical pressure which can be measure of the buckling resistance can be obtained for the geometry being result of the superposition of the original geometry and the deformation pattern defined by the amplitude which is dependent on an fabrication quality parameter (cf. Recommendations [2]).

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