

Eccentric discharge buckling of a very slender silo

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Abstract

Metal silos used to store granular solids often take the form of a cylindrical shell with an aspect ratio in the range $2 < H/D < 6$. It has long been recognized that the most serious load case for silos is the condition of eccentric discharge of its stored solid, and more failures have occurred under this condition than any other. Two of the chief reasons for this high failure rate are the difficulty in characterizing the pressure distribution caused by eccentric solids flow, and the difficulty in understanding the pattern of stresses that develops in a shell wall under such unsymmetrical pressure regimes. The nonsymmetric behavior of a shell structure under such a loading condition is not at all well described in the voluminous shell structures literature, and only a few studies have explored the mechanics leading to high local stresses which in turn lead to buckling failure under eccentric discharge.

In this study, the pressures caused by eccentric discharge are characterized using the new rules of the European Standard EN 1991-4 [5] that defines the Actions in Silos and Tanks. Using this new improved description of unsymmetrical pressures, it is now possible to perform relatively realistic calculations relating to this common but complicated shell buckling condition. The calculations described here are part of a wider study believed to be the first of its kind and are undertaken using geometrically and materially nonlinear analyses in accordance with the European Standard EN 1993-1-6 [6] on Strength and Stability of Shells. The paper explores the structural behavior leading to buckling during eccentric discharge, including the critical effects of changes of geometry and imperfection sensitivity.

Keywords: Thin shell structures, solids flow, structural stability, nonlinear computer analysis, shell buckling.

1. Introduction

The most serious load case for silos has long been recognized to be the condition of discharge of its stored granular solid, due to the consequent increase in wall normal pressures, and this is the condition for which most metal silos are designed. The most common form of failure in slender circular thin-walled silos is buckling due to axial compressive membrane stresses, caused by both normal pressures and frictional tractions

on the silo wall. When the silo is eccentrically discharged (see Fig. 1), a very unsymmetrical pressure pattern can arise which exacerbates the problem of buckling, and this, for the first time, is codified in the new European Standard EN 1991-4 [5].

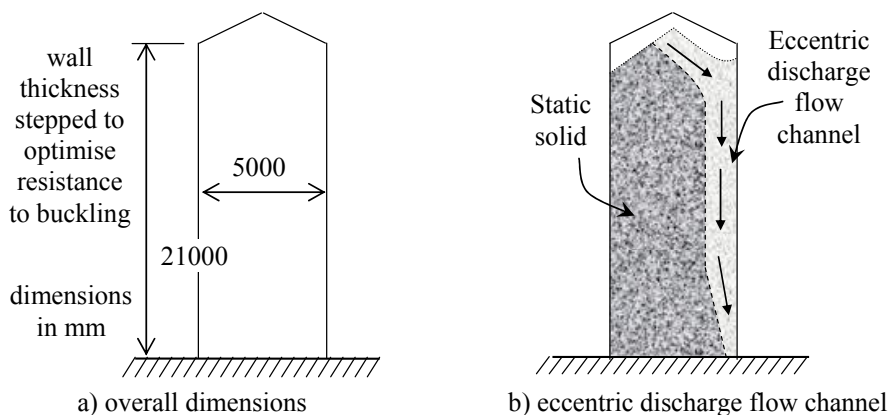


Figure 1: Illustration of eccentrically discharging flow channel and geometry of the example design silo

In this paper it is shown that the failure mode under eccentric discharge is by buckling in the elastic material range under local axial compressive membrane stresses induced by this highly unsymmetrical flow regime. This explanation follows that of Rotter [15], [17] and [18], but counter to those previously offered by Jenike [9], Wood [24], Robert & Ooms [14] and others, whose common misconception that failure in circular metal silos under eccentric discharge is governed by yielding due to circumferential bending and tension. This misconception led silo designers to treat the shell as a simple ring. Though such a treatment may be appropriate for thick-walled reinforced concrete silos, thin-walled metal silos behave very differently.

2. Pressure patterns in silos

The increased normal pressures that occur on the silo wall during concentric mass flow discharge fluctuate very erratically and are difficult to characterize with simple equations (Rotter [15], Nielsen [11] & Ooi *et al.* [12]). Nonetheless, this increase is traditionally accounted for in design through simple multiplication factors based on concepts from quite simple theories (e.g. Arnold *et al.* [1], Jenike *et al.* [10]), which do not capture the experimentally observed phenomena well (Rotter [14]).

In older design standards (AS 3774 [2], DIN 1055-6 [4], ISO11697 [8]), the effect of eccentricity of filling and discharge of solids was treated as an unsymmetrical additional component to the axisymmetric solid pressures. A simple ‘patch’ of normal pressures of prescribed magnitude, distribution and location was implemented in design. This approach, though rudimentary, rightfully identified unsymmetrical normal pressures, rather than

frictional tractions, as the main catalyst for failure. Unfortunately, codified representations of patch loads differ considerably from one standard to another (Song and Teng [23]). They have been found to be very detrimental in linear bifurcation analyses, but their effect on geometrically nonlinear bifurcation loads is very small (Song [22]).

The recently published EN 1991-4 [5] defines three Action Assessment Classes which require increasing levels of sophistication in design, and a range of properties for each stored material since different properties cause different aspects of the design to become critical. In this study, the unsymmetrical pressures caused by eccentric discharge are investigated using the new rules of EN 1991-4 [5], based on a simplified version of the theory of Rotter [15]. This theory proposes a distribution for the pressures resulting from a parallel-sided circular flow channel forming against the wall, shown in Fig. 2. In this version, the solid exerts Janssen pressures outside the channel, elevated pressures at the edges and decreased pressures within the flow channel. The relationship between the pressure drop and increase is such that the horizontal equilibrium is satisfied, though it does lead to a global overturning moment on the silo. EN 1991-4 [5] requires this distribution to be used in the design of silos where eccentric discharge is expected and the silo is in Action Assessment Classes 2 or 3.

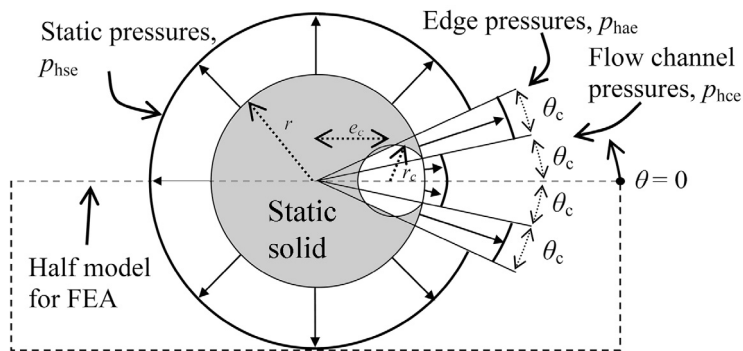


Figure 2: Circumferential cross-section of eccentric flow channel horizontal pressures, after EN 1991-4 [5]

3. Design of an example steel silo for axisymmetric loading

A simple cylindrical steel silo with a vertical wall, flat bottom and conical shell roof (inclination 30° to the horizontal) was designed for symmetrical loads only arising from the storage of 390 tonnes of wheat with friction properties for a D2 wall ('smooth'). Structural design was done according to EN 1993-1-6 [6], with properties for wheat taken from EN 1991-4 [5] using the maximum friction case since the design against buckling is dominant.

The cylinder wall height was 26 m and the radius 2.5 m, giving an aspect ratio of 5.2 (classified as 'Slender'). Action Assessment Class 2 was assumed based on storage capacity. The requirement for a small unsymmetrical patch load on the silo was ignored to keep the design as simple as possible for future interpretation. Discharge factors for normal

pressures (C_h) and frictional tractions (C_w) were taken as 1.15 and 1.1 respectively. The partial safety factor for unfavorable actions (γ_F) and for stability (γ_{M3}) were taken as 1.5 and 1.1 respectively, separating the characteristic values by a factor of $1.5 \times 1.1 = 1.65$. This value is important in the context of the outcome of later nonlinear computations against which it may be assessed.

A silo design was produced with a wall thickness varying in a stepwise manner from 3 mm at the top to 7 mm at the base to follow engineering practice. This made the wall just thick enough at the base of each strake and at the silo base. The design axial membrane stress resultants are shown in Fig. 3, while Fig 4 shows the corresponding design thicknesses as well as those required to withstand simple bursting failure.

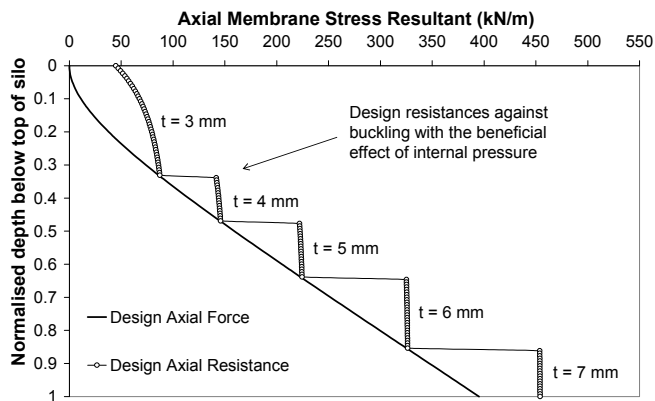


Figure 3: Distribution of design axial membrane stress resultants

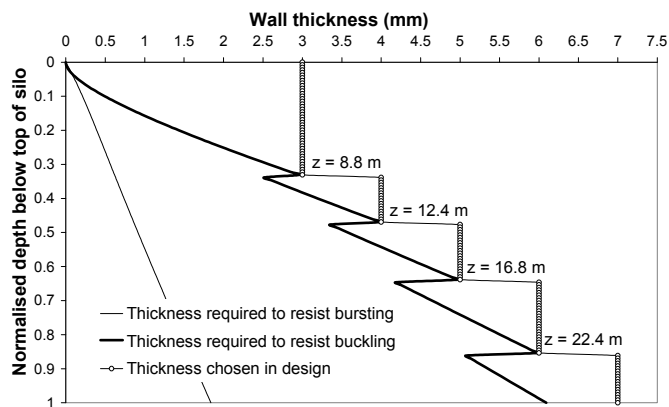


Figure 4: Distribution of design thicknesses to resist bursting and buckling

A Fabrication Tolerance Quality Class of C (i.e. ‘normal’) was adopted, requiring a thicker and more imperfect wall. The shell material was assumed to be isotropic steel with an elastic modulus of 200 GPa, a Poisson’s ratio of 0.3 and a yield stress of 250 MPa.

4. Numerical analysis of the example silo

The silo was analyzed first under the symmetrical design loads and then under eccentric discharge using the commercial finite-element package ABAQUS (HKS [7]). A pinned lower edge was assumed, with the conical roof allowing a realistic out-of-round displacement restriction to the upper edge, important under unsymmetrical loads [Calladine [3], Rotter [16]]. Using symmetry conditions only half of the silo was modeled with nine-node reduced-integration S9R5 elements (S4R5 for the roof). The mesh resolution was increased near changes of wall thickness, weld depressions, the entire flow channel and at regions where buckles are expected to form. An ideal elastic-plastic material law was assumed where applicable. The geometrically nonlinear load-deflection path was followed using the modified Riks procedure [13].

Local axisymmetric imperfections representing weld depressions in the form Type A defined by Rotter and Teng [21] were introduced at all changes of plate thickness and at selected intervals in between. The depression amplitude was chosen as identical, in each stroke, to the value adopted in the hand design process according to EN 1993-1-6 [6].

The full suite of computational shell buckling calculations were performed according to EN 1993-1-6 [6]: LA – Linear elastic Analysis to find the reference stresses; LBA – Linear Bifurcation Analysis to find the lowest linear buckling eigenvalue and eigenmode; MNA – Materially Nonlinear Analysis to find the reference plastic collapse load; GNA/GMNA – Geometrically Nonlinear Analysis without/with material plasticity to find the lowest bifurcation load and mode; and GNIA/GMNIA – Geometrically Nonlinear Analysis with Imperfections without/with material plasticity to find the lowest bifurcation load and mode.

5. Results and discussion

5.1. Behavior of the example silo under symmetrical discharge pressures

The silo was first analyzed under symmetrical loading at characteristic discharge values assumed in the design calculations. Many conservative assumptions are incorporated into the hand design process according to which the silo was designed, and the design safety margin (=1.65) is expected to be exceeded when the silo is analyzed using a GMNIA analysis with axisymmetric loading. A summary of the load proportionality factors at failure achieved for the case of concentric discharge is given in Table 1. The buckling modes for these are shown in Fig. 5 while the nonlinear load-axial displacement paths are presented in Fig. 6.

Concentric discharge load proportionality factors	LBA	MNA	GNA	GMNA	GNIA	GMNIA
	9.07	6.43	8.90	5.11	4.40	3.77

Table 1: Summary of load factors for the example silo under concentric discharge

The critical locations for buckling failure under axisymmetric loading are always at the base of a strake of any single particular thickness. In this case the critical zones are either the base of the silo or the bottom of the thinnest 3 mm strake, thus all the load factors relate to failure in the elephant's foot mode (Rotter [19]). The close proximity of the LBA and MNA load factors suggests that stability and plasticity will interact. Additionally, the GNA is almost identical to the LBA (and the load-deflection paths overlap) which shows that the pre-buckling behavior is very close to linear. With the introduction of circumferential weld imperfections, plasticity plays a smaller role, and the GMNIA load factor is not much lower than the GNIA factor with a very similar diamond pattern buckling mode.

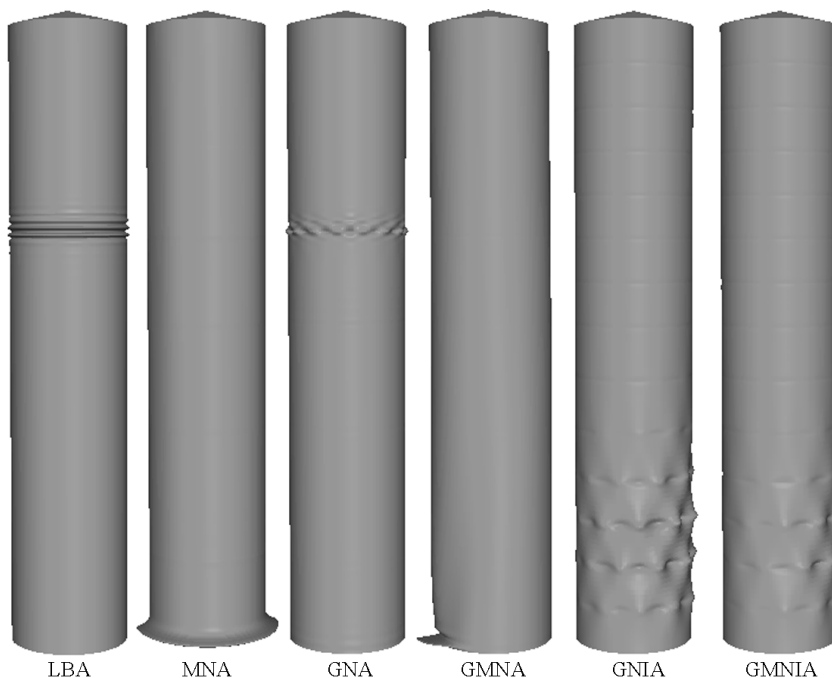


Figure 5: Incremental buckling modes (except for LBA and MNA) under concentric discharge

The lowest GMNIA factor of 3.77 is over 2.2 times the hand calculation value of 1.65, indicating that the assumptions in the hand design process are very conservative, both for the elastic stability and plastic collapse calculations.

The nonlinear load-displacement paths are very typical for shells under axisymmetric loading. The node being followed is at the top of the silo at the centre of the flow channel. The introduction of axisymmetric weld imperfections reduces the stiffness, resulting in earlier failure. The more sophisticated the analysis, the lower the load factor, so both geometric and material nonlinearity must be considered in silo design.

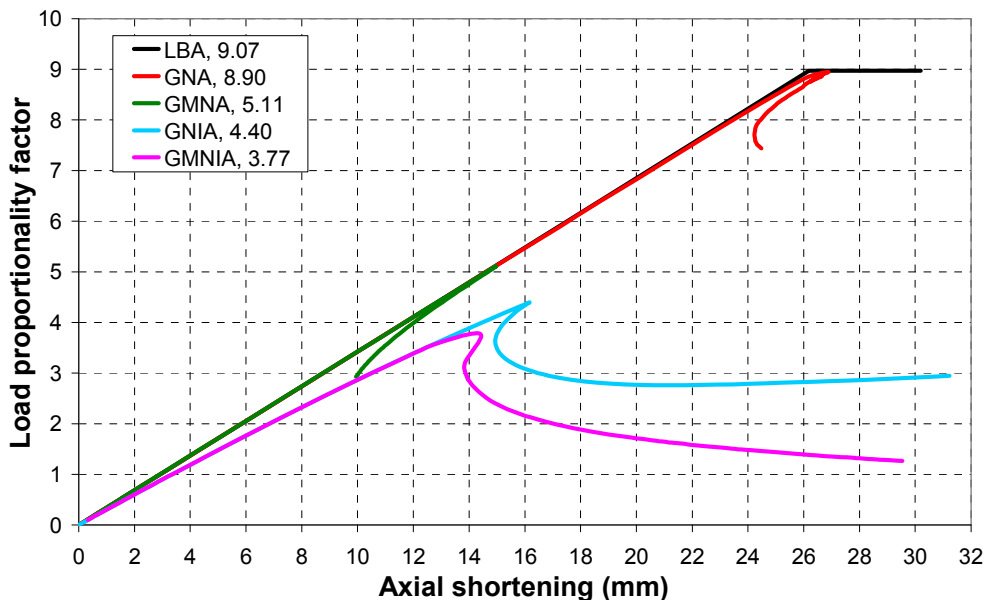


Figure 6: Nonlinear load-axial displacement paths under concentric discharge

5.2. Behavior of the example silo under eccentric discharge pressures

The goal of this paper is to explore the behavior of the silo under (accidental) eccentric discharge. Such conditions often precipitate silo failures, when either a feeder malfunctions, an outlet intended for final cleanout is opened when the silo is full, a new discharge device is fitted without proper testing and other similar conditions (see EN 1991-4 [5]). Thus the following calculations give a good insight into many silo disasters. Here, the eccentric discharge flow channel size (r_c) is taken as 0.6 times the silo radius (see Fig. 2).

Under this set of unsymmetrical pressures associated with a flowing channel of stored solid, high axial compressive membrane stresses develop close to the midheight of the silo down the centre of the flow channel. By contrast, high axial tensile stresses develop at the edges of the flow channel, with compressive values at the base. Clearly, either of the two regions of high compressive stresses may become critical for buckling failures, depending on the design of the silo and axial variation of plate thicknesses. This stress distribution, first

discovered by Rotter [15], is shown in Fig. 7 for the GNA/GMNA analysis at the instant before bifurcation.

The compressive stresses at the bottom of the silo at the edge of the flow channel are the largest, and if the wall thickness had been uniform throughout the silo, this location would be critical and susceptible to local buckling failure. However, since high compressive stresses also develop at midheight at the centre of the flow channel, this location becomes critical when the local wall thickness is smaller, as is always the case in practice. On the opposite side of the silo to the channel, the axial membrane stress resultant is unaffected, and corresponds to the axisymmetric loading case

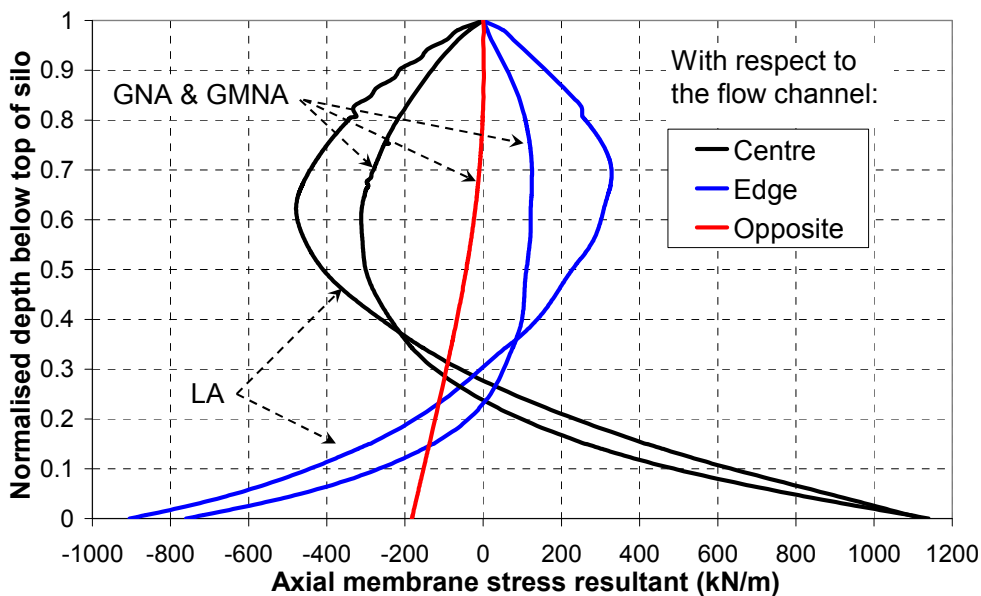


Figure 7: Axial membrane stress distribution (at instant of bifurcation) under eccentric discharge analyzed with GNA/GMNA and LA (factored with LBA)

A summary of the load factors at failure achieved for the example silo under eccentric discharge is presented in Table 2. The incremental buckling modes (where applicable) are shown in Fig. 8, while the nonlinear axial displacement paths are presented in Fig. 9.

Eccentric discharge load proportionality factors	LBA	MNA	GNA	GMNA	GNIA	GMNIA
	0.28	0.80	0.39	0.39	0.24	0.22

Table 2: Summary of load factors for the example silo under eccentric discharge

All of these load factors are very far below the concentric discharge values. This illustrates the very damaging effect of unsymmetrical pressures on cylindrical shells. The design, which was so conservative under concentric discharge, is no longer safe.

The GNA factor is, surprisingly, almost double the LBA factor, suggesting that large deformations change the geometry considerably and result in significant strength gains. Fig. 7 also shows the reference LA values (at the LBA factor), which are much greater than the GNA buckling stresses. The MNA factor, relating to a circumferential bending mechanism is quite high and does not contribute to the behavior at all.

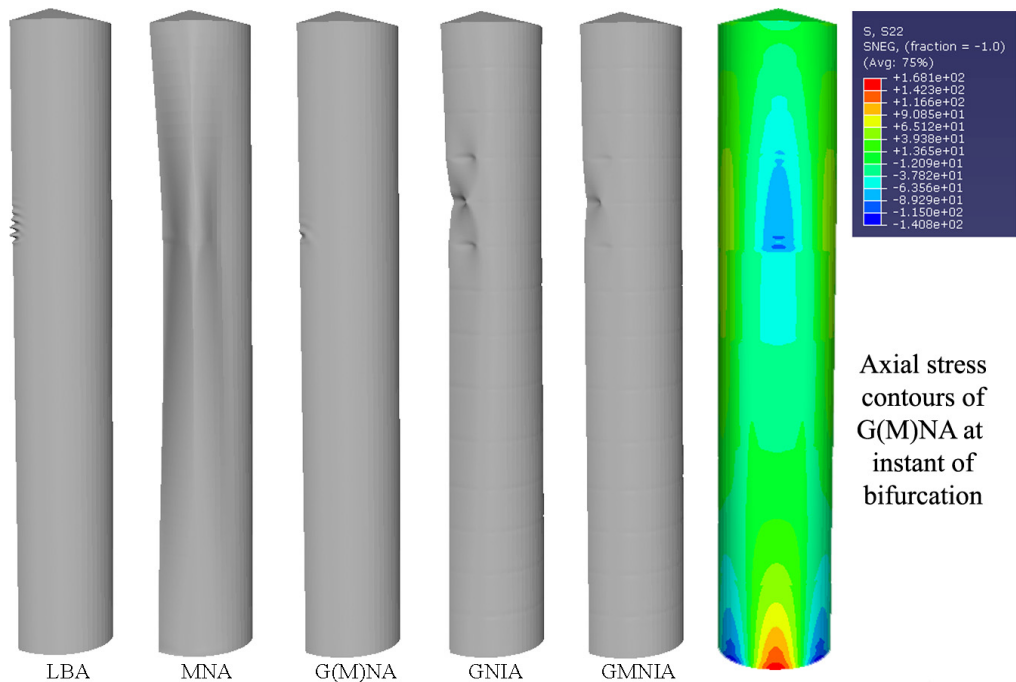


Figure 8: Incremental buckling modes (except for LBA and MNA) and 3D contour plot of G(M)NA axial stresses at instant of bifurcation under eccentric discharge

From Fig 8 it is evident that buckling occurs exclusively at midheight. Additionally, the identical values for the load factors of both the GNA and GMNA analyses, and of the GNA and GMNIA analyses, show that the buckling is entirely elastic. This failure mode relates well to known failures in service.

Considering the load-displacement paths of Fig. 9, it is clear that the structure exhibits significant stiffening behavior with geometric nonlinearity on the GNA/GMNA paths. There is a sharp peak followed by a reversal in the loading path, typical of highly imperfection-sensitive unstable post-buckling behavior. The GNA and GMNIA paths,

however, show a clear point of inflexion at a load factor of approximately 0.24 and indefinite geometric hardening with a progressive growth of the imperfection mode. With no negative eigenvalues reported at the change of slope, it is evident that an early bifurcation point has been bypassed but stable unsymmetrical buckling displacements develop strongly after this point. This phenomenon often occurs when imperfection amplitudes are large and result in a blurring of the buckling behavior (Rotter [20], Yamaki [25]).

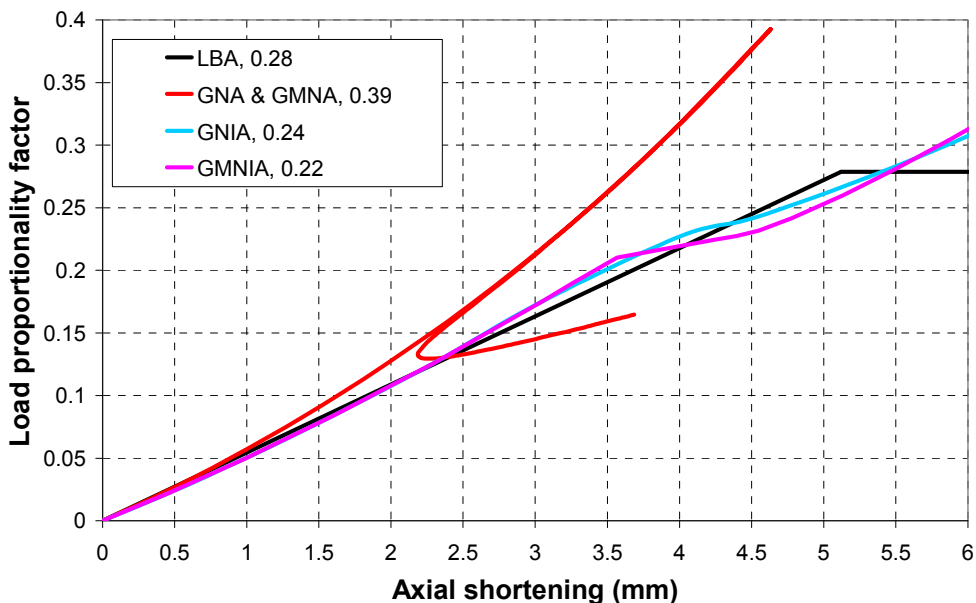


Figure 9: Nonlinear load-axial displacement paths under eccentric discharge

6. Conclusions

The following conclusions may be drawn based on the results of this study:

1. A silo designed according to the new rules of the European Standard EN 1991-4 for concentric filling, storage and discharge of contents is found by nonlinear finite element analysis to have a large reserve of strength beyond what is required in the structural assessment with EN 1993-1-6. This is due to the conservatism of the assumptions upon which the hand design is founded.
2. The European Standard EN 1991-4 limits the range of silos which must be designed explicitly for eccentric discharge to the ones with higher capacities. The example silo considered here, still a considerable structure (5×26 m), falls far short of this requirement, and would not have been designed to withstand such an accidental eccentric discharge event. This has often occurred in practice.

3. Under the eccentric discharge pressures of EN 1991-4, the regions with highest axial compressive membrane stresses are at the centre of the channel at midheight, and at the edge of the channel at the silo base. Due to the lower wall thickness higher up the silo, buckling occurs near midheight. The midheight buckling mode has often been observed in practice and is responsible for many silo failures.

References

- [1] Arnold P.C., McLean A.G. and Roberts A. W., Bulk Solids: Storage, Flow and Handling. *Tunra Bulk Solids Handling Research Associates*, University of Newcastle, Australia, 1978, 2nd edition 1980.
- [2] AS 3774., *Loads on bulk solids containers*. Sydney: Standards Australia, 1996.
- [3] Calladine C.R., *Theory of Shell Structures*. Cambridge University Press, 1983.
- [4] DIN 1055-6., *Design loads for buildings: loads in silo bins*. Deutsches Institut für Normung, Berlin, Germany, 1987.
- [5] EN 1991-4., *Eurocode 1: Basis of Design and Actions on Structures, Part 4 – Silos and Tanks*. CEN, Brussels, 2007.
- [6] EN 1993-1-6., *Eurocode 3: Design of Steel Structures, Part 1.6 – Strength and Stability of Shell Structures*. CEN, Brussels, 2007.
- [7] HKS., *ABAQUS Version 6.6 Standard user's guide and theoretical manual*, Hibbit, Karlsson and Sorensen Inc., Pawtucket, Rhode Island, USA, 2006.
- [8] ISO 11697, *Basis for design of structures – loads due to bulk materials*. International Standard, 1995.
- [9] Jenike A.W., Denting of Circular Bins with Eccentric Drawpoints. *J. of the Struct. Div.*, ASCE, 1967; 93 (ST1); 27-35.
- [10] Jenike A.W., Johanson J.R. and Carson J.W., Bin Loads – Parts 2, 3 and 4: Concepts, Mass Flow Bins, Funnel Flow Bins. *J. of Eng. for Industry*, Trans. ASME, 1973, Vol. 95, Series B, N. 1, pp 1-5, 6-12, 13-16.
- [11] Nielsen J., Pressures from flowing granular solids in silos. *Phil. Trans. Royal Society of London: Philosophical Transactions: Mathematical, Physical and Engineering Sciences*, 1998, Series A, 356, 2667-2684.
- [12] Ooi J.Y., Pham L. and Rotter J.M., *Systematic and random features of measured pressures on full-scale silos*. *Engineering Structures*, 1990, Vol. 12, N. 2, 74-87.
- [13] Riks E., An incremental approach to the solution of snapping and buckling problems. *Int. J. of Solids and Structures*, 1979, Vol. 15, 529-551.
- [14] Roberts A.W. and Ooms M., Wall Loads in Large Metal and Concrete Silos and Silos due to Eccentric Draw-Down and other Factors. *Proc. of the 2nd Int. Conf. on Design of Silos for Strength and Flow*, Stratford-upon-Avon, 1983, 151-170.
- [15] Rotter J.M., The analysis of steel bins subject to eccentric discharge. *Proc. of the 2nd International Conference on Bulk Materials Storage, Handling and Transportation*, IEAustralia, Wollongong, 264-171, 1986.

- [16] Rotter J.M., Bending Theory of Shells for Bins and Silos. *Trans. of Mech. Eng.*, IE Aust, 1987, ME12 (3), 264-271.
- [17] Rotter J.M., Pressures, Stresses and Buckling in Metal Silos containing Eccentrically Discharging Solids, in *Festschrift Richard Greiner*, Celebration volume for the 60th birthday of Prof. Richard Greiner, TU Graz, Austria, 2001, 85-104.
- [18] Rotter J.M., *Guide for the Economic Design of Circular Metal Silos*. Spon Press, 2001.
- [19] Rotter J.M., Elephant's foot buckling in pressurized cylindrical shells. *Der Stahlbau*, 2006, 75 (9), 742-747.
- [20] Rotter J.M., A framework for exploiting different computational assessments in structural design. *Proc. 6th Int. Conf. on Steel and Aluminum Structures*, 6th ICSAS 07, Oxford, June 2007.
- [21] Rotter J.M. and Teng J.G., Elastic stability of cylindrical shells with weld depressions. *J. of Struct. Eng.*, ASCE, 1989, 115(5), 1244-1263.
- [22] Song C.Y., Effects of patch loads on structural behavior of circular flat-bottom steel silos. *Thin-Walled Structures*, 2004, 42, 1519-1542.
- [23] Song C.Y. and Teng J.G., Buckling of steel silos subject to code-specified eccentric discharge pressures. *Engineering Structures*, 2003, 25, 1397-1417.
- [24] Wood J.G.M., The Analysis of Silo Structures Subject of Eccentric Discharge. *Proc. of the 2nd Int. Conf. on Design of Silos for Strength and Flow*, Stratford-upon-Avon, 1983, 132-144.
- [25] Yamaki N., *Elastic Stability of Circular Cylindrical Shells*, North Holland, Elsevier Applied Science Publishers, Amsterdam, 1984.