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Alberto DOMINGO and Carlos LAZARO (eds.)

Universal equivalent static wind loads of one single-layer reticular dome

Qingshan YANG*, Bo CHEN[a]

* School of Civil Engineering, Beijing Jiaotong University, Beijing 100044 China
qshyang@bjtu.edu.cn

[a] School of Civil Engineering, Beijing Jiaotong University, Beijing 100044 China
chenbohrb@163.com

Abstract

A new method to analyze universal equivalent static winds (ESWL) simultaneously reproducing multiple largest load effects (names as multiple equivalent targets) is proposed, on basis of analysis method of ESWL for one certain largest load effect and the method proposed by A. Kasumura and Y. Tamura [5]. Furthermore, this method is employed to study universal ESWL of one single-layer reticular dome. The principles of this method are: according to characteristics of wind-induced response, dominant eigen-modes of wind loads and dominant vibration modes are chosen as fundamental vectors to express universal ESWL for multiple targets; and a least square approximation method is employed to calculate the weighting factors of these fundamental vectors, thus, universal ESWL for multiple targets are obtained. The analysis results of the single-layer reticular dome show that: structural response of all nodes or all supports under the same ESWL agrees well with peak values under actual dynamic wind loads at the same time.

Keywords: equivalent static wind loads; wind-induced response; eigen-mode; vibration mode.

1. Introduction

Equivalent static wind load (ESWL) produces the same maximum dynamic response under actual wind loading. Thus, wind-induced response is expressed in a static form through ESWL, which overcomes complex stochastic dynamic analysis and makes wind-induced response analysis much easier for structure designer. So, ESWL is used for design and relates wind engineer to structure engineer.

Foremost, most studies focused on ESWL of high-rise buildings from 1960s. The original concept of the GLF method was proposed by Davenport [1]. M.kaserski [2] advised load-response correlation method (LRC), which was an important milestone to analyze ESWL and is efficient to calculate background component of ESWL. ESWL gained with above methods is the unfavorable loading distribution for one certain largest load effect (maximum nodal displacement is usually chosen). In most cases, the error is small between load effects under this ESWL with actual dynamic peak response of all load effects, for high-rise buildings whose fluctuating wind induced response is dominated by the first vibration mode.

Recently, ESWL of large span space structures for one certain load effect is studied with similar methods to high-rise buildings by Gu Ming [3] and Chen Bo [4]. However, such ESWL is obtained for a specific largest load effect of a structural member or a nodal displacement, not for all largest load effects of all structural members and nodes. For large span space structures, many largest load effects are important for structural design, including stress of all members and nodal displacements. And for this type of structures, many vibration modes have important contribution to fluctuating wind-induced response, and it means that all load effects can't simultaneously reach their maximum values under the same ESWL. Thus, many ESWL distributions are needed for those important maximum load effects, if traditional methods for ESWL are adopted.

A. Kasumura and Y. Tamura [5] proposed universal equivalent static wind load (universal ESWL) that simultaneously reproduce multiple largest load effects, and the universal ESWL distribution was shown by combination of eigenmodes calculated by POD analysis of fluctuating wind pressure. On basis of this method, Hong Xionghong [6] expressed universal ESWL as combination of several ESWL distributions for some typical load effects, each of which was gained with those traditional methods of ESWL for one certain load effect.

Referring to A. Kasumura's method, this paper will discuss the method to analyze universal ESWL of large span space structures, on basis the method for one certain load effect. Moreover, this method is employed to analyze universal ESWL of one single-layer reticular dome.

2. Analysis method of ESWL for one certain largest load effect

Reference to Chen Bo [4] and M.kaserski [7] and，ESWL for one certain largest load effect is expressed as combination of three components, including mean component, background component and resonant component, by the following equation.

$$\left\{F_e\right\} = \left\{\overline{F}\right\} \pm \left[\gamma_b \left\{F_e^b\right\} + \gamma_r \left\{F_e^r\right\}\right] \tag{1}$$

where $\left\{\overline{F}\right\}$, $\left\{F_e^b\right\}$, $\left\{F_e^r\right\}$ are mean, background resonant component respectively; γ_b , γ_r are weighting factor of background and resonant component respectively.

$$\gamma_b = \frac{g_B \sigma_b}{\sqrt{(g_B \sigma_b)^2 + (g_R \sigma_r)^2}} ; \gamma_r = \frac{g_R \sigma_r}{\sqrt{(g_B \sigma_b)^2 + (g_R \sigma_r)^2}} \tag{2}$$

When proper orthogonal decomposition (POD) technique as in Y.Tamura [8] is incorporated into load-response correlation method (LRC) as in M.kaserski [2], background component of ESWL can be written as

$$F_{e,k}^{b} = g_B \rho_{ri,Fk} \sigma_{Fkk} = \sum_{m=1}^{M} \alpha_m G_m(x_k, y_k, z_k) \tag{3}$$

where g_B is peak factor of background component, $\rho_{ri,Fk}$ is the correlation coefficient between fluctuating wind load of point k and structural response of point *i*, σ_{Fkk} is the standard deviation of the pressure, α_m is the weighting factor for the contribution of the *m*th POD eigenmode to background component of ESWL, and

$G_m(x, y, z)$, λ_m are eigenmode and eigenvalue of the covariance matrix of fluctuating wind pressures respectively, which are the solution of following eigenvalue equation.

$$\mathrm{C}_r \mathrm{G}_k = \lambda_k \mathrm{G}_k \tag{4}$$

$$\mathrm{C}_r = \overline{\{F(x,y,z,t)\}\{F(x,y,z,t)\}^T} \tag{5}$$

where $F(x,y,z,t)$ is fluctuating wind pressure.

Derived from stochastic vibration theory and eigenvalue equation of structure, resonant component of ESWL can be expressed as combination of inertial loads of several dominant vibration modes.

$$\{F_e^r\} = g_R \sum_{j=1}^{n} \{P_{e0}\}_j \, w_j \tag{6}$$

where g_R is peak factor of resonant response, $\{P_{e0}\}_j = M\{\varphi\}_j \omega_j^2$ is inertial force vector of the *j*th vibration mode, and w_j is the weighting factor of the *j*th mode to resonant compo nent of ESWL. Dominant vibration modes can be chosen by the method proposed by Chen Bo [9]

Above method relates ESWL for one certain largest load effect to dominant eigenmodes of fluctuating wind loads and dominant vibration modes. This ESWL is the instantaneous distributions coinciding with the certain peak load effect.

3. Analysis method of universal ESWL for multiple largest load effects

In most cases, most load effects under the same ESWL, which is gained with the method in above section will deviate much from their peak values under actual wind loads. Thus, many different ESWL distributions are necessary for all important load effects if above method is adopted, therefore, it will be very inconvenient for structural design.

From Eq.(1), Eq.(3) and Eq.(6), it’ seen that ESWL for any certain largest load effect can be expressed as the combination of eigenmodes of fluctuating wind loads and structural vibration modes. The weighting factors of these eigenmodes and vibration modes depend on each largest load effect.

Dynamic behaviors of large span space structures show that there are several dominant eigenmodes in background response and several dominant vibration modes in resonant response under wind actions. In other words, correlations exist among some load effects in the wind-induced response, and some load effects will simultaneously reach their maximum values if the corresponding ESWL is expressed as the combination of these dominant eigenmodes and vibration modes of dynamic response. Based on this concept, this paper proposes that dominant eigenmodes and vibration modes are regarded as fundamental vectors to express universal ESWL for multiple largest load effects. The combination factors of these fundamental vectors should meet that all static load effects under the universal ESWL will be simultaneously equal to peak responses under actual dynamic wind loads, or the errors between them are least. The strongpoint includes that this universal ESWL reflects dynamic behavior of wind-induced response in some degree, thus these fundamental vectors are more efficient than other vectors, and least number of vectors are enough to express universal ESWL for the same calculation precision requirement.

Above concept is derived from the method of ESWL for one certain largest load effect and dynamic behaviors of wind-induced response. Of course, this concept is similar to A. Kasumura and Y. Tamura [5] in some degree, but some difference exists. In Ref. [5], universal ESWL is described as a combination of dominant eigenmodes of fluctuating wind pressure in any case, but dominant eigenmodes and vibration modes are chosen in this paper.

From Eq.(1), Eq.(3) and Eq.(6), ESWL excluding mean component can be rewritten in matrix form as follows.

$$\begin{aligned}\left\{\widetilde{F_e}\right\} &= \left[c_1\{G\}_1 + c_2\{G\}_2 + \cdots + c_{n1}\{G\}_{n1}\right] + \\ &\left[c_{n1+1}\{P_{e0}\}_1 + c_{n1+2}\{P_{e0}\}_2 + \cdots + c_{n1+n2}\{P_{e0}\}_{n2}\right] = \left[F_0\right]\{c\}\end{aligned} \tag{7}$$

where the vector {C} is weighing factors and represents the contribution of each vector in $\left[F_0\right]$ to the ESWL, $\{G\}_i$ and $\{P_{e0}\}_j$ are given in Eq.(3) and Eq.(6) respectively, and $\left[F_0\right]$ is called fundamental load distribution matrix.

$$\left[F_0\right] = \left[\{G\}_1, \{G\}_2, \cdots, \{G\}_{n1}, \{P_{e0}\}_1, \{P_{e0}\}_2, \cdots, \{P_{e0}\}_{n2}\right] \tag{8}$$

The ESWL of one certain largest load effect $\hat{y}_i$ should satisfy flowing equation.

$$\{\beta\}^T \left\{\widetilde{F_e}\right\} = \{\beta\}^T \left[F_0\right]\{c\} = \hat{y}_i \tag{9}$$

where $\{\beta\}^T$ is structural influence function.

If ESWL is the instantaneous distributions coinciding with the certain peak load effect in statistics sense, Eq.(9) has only unique solution. If this ESWL is suitable to multiple largest load effects (called universal ESWL), following equations should be met at the same time.

$$\begin{cases} \{\beta\}_1^T \{\widetilde{F_e}\} = \{\beta\}_1^T [F_0]\{c\} = \hat{y}_1 \\ \{\beta\}_2^T \{\widetilde{F_e}\} = \{\beta\}_2^T [F_0]\{c\} = \hat{y}_2 \\ \qquad\qquad \cdots \\ \{\beta\}_m^T \{\widetilde{F_e}\} = \{\beta\}_m^T [F_0]\{c\} = \hat{y}_m \end{cases} \tag{10}$$

where $\{\beta\}_i^T$ is structural influence function for one certain load effect *i*, and $\hat{y}_i$ is peak load effect under wind action for load effect *i*.

Eq.(10) can be rewritten in matrix form.

$$[\beta]\{\widetilde{F_e}\} = [\beta][F_0]\{c\} = [R_0]\{c\} = \{\hat{y}\} \tag{11}$$

where $[\beta]$ is influence function matrix, and $[R_0] = [\beta][F_0]$ represents structural response under the fundamental load distribution $[F_0]$。

The solution of {C} in Eq. (11) depends on the number (marked as *m*) of largest load effects and rank (marked as *n*) of matrix $[R_0]$. If *m* is equal to *n*, {C} has unique solution. If *m* is less than *n*, {C} has many feasible solutions. If *m* is greater than *n*, {C} has no accurate solution.

For large span space structures, so many important largest load effects should be concerned. In general, the number *m* of largest load effects is much greater than the number *n* of fundamental load distribution, thus, no accurate solution exists for Eq.(11). In this case, the weighting factor {C} in Eq.(11) can be solved by a least-square method. In addition, it is easy to obtain least-square method with software, for example, Matlab7.5.

From Eq.(11), universal ESWL (excluding mean component) for multiple largest load effects is expressed as.

$$\{\widetilde{F_e}\} = [F_0]\{c_0\} \tag{12}$$

During structural design stage, both positive peak load effects $g\sigma_y$ and negative peak load effects $-g\sigma_y$ are usually very important, therefore, responding universal ESWLs $\{\widetilde{F_e}\} = \pm[F_0]\{c_0\}$ are necessary. If mean component is included, total universal ESWL can be expressed as.

$$\{F_e\} = \{\overline{F}\} + \{\widetilde{F_e}\} = \{\overline{F}\} + [F_0]\{c_0\} \tag{13a}$$

$$\{F_e\} = \{\overline{F}\} + \{\widetilde{F}_e\} = \{\overline{F}\} - [F_0]\{c_0\} \tag{13b}$$

To sum up, analysis method of universal ESWL for large span space structures include three main steps: （Ⅰ）dominant eigenmodes and vibration modes are chosen to construct fundamental load distribution matrix $[F_0]$; （Ⅱ）Eq. 11 is solved to obtain weighting factor $\{c_0\}$; （Ⅲ）Total universal ESWL is calculated by Eq. (13).

Fundamental load distribution matrix $[F_0]$(seen Eq. (8)) may be simplified, on basis of behaviors of structural wind-induced response. If the structure is rigid, and background response is much greater than resonant response, only dominant eigenmodes are needed to construct $[F_0]$. If the structure is flexible, and resonant response are much greater than background response, only vibration modes are needed to construct $[F_0]$. In other cases, both dominant eigenmodes and vibration modes are needed.

4. Universal ESWL of one single-layer reticular dome

The model is a rigidly jointed single-layer reticular shell of Kiewitt system with pin supports at the surrounding, a span of 80*m* and rise-span ratio is 1/6 at a height of 20*m*. There are 127 nodes and 654 degrees of the freedom. The mean wind velocity is 25*m/s* at 10m. The fluctuating pressure on the roof is measured with simultaneous pressure measurement technique in the wind tunnel. Figure1 shows the structural model.

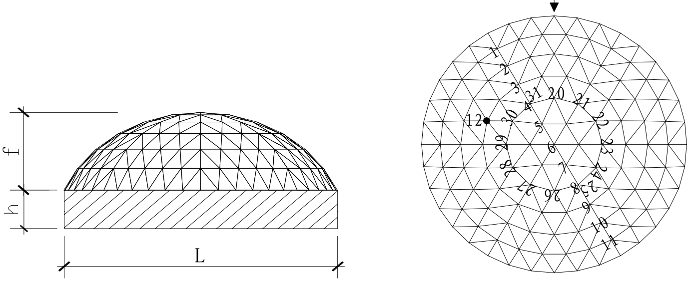


Figure 1: Structural model of the single-layer reticular dome

4.1 Wind-induced response of this single-layer reticular dome

Table 1 shows the mean response, maximum response, ratio between maximum response and mean response, and ratio between resonant response and background response of typical nodes of Figure1. It is seen that for most nodes, the ratio between maximum response and mean response is greater than 2.0, thus fluctuating response has important effect on wind-induced response and can’t been neglected; the ratio between resonant response and background response is greater than 2.0, thus background response can be neglected. Therefore, background response is neglected when universal ESWL is calculated, and only inertial loads of dominating modes are chosen to express universal ESWL.

Table 1 All response components of wind-induced response

Node number	Mean response(mm)	Maximum response(mm)	$\frac{\text{maximum response}}{\text{mean response}}$	$\frac{\text{resonant response}}{\text{background response}}$
1	-8	-46.5	5.81	2.64
2	10.9	51.9	4.76	2.53
3	16.2	57.7	3.56	2.64
4	21.2	58.1	2.74	3.37
5	28.4	68.3	2.40	2.92
6	30.9	62.2	2.01	2.00
7	26.7	61.7	2.31	2.36
8	17.9	47.9	2.68	2.50
9	16.1	47.9	2.98	2.16
10	11.4	42.8	3.75	2.34
11	5.5	32.9	5.98	1.70
20	24.8	64.9	2.62	3.03
21	19.7	53.7	2.73	2.60
22	28.1	57.3	2.04	2.17
23	21.9	51.9	2.37	2.17
24	29.2	66.2	2.27	2.26
25	17.9	47.9	2.68	2.50
26	21.1	59.6	2.82	2.64
27	19.5	52.1	2.67	2.02
28	21.5	57.1	2.66	2.09
29	21.5	56.4	2.62	2.22
30	25.1	65.3	2.60	2.77
31	21.2	58.1	2.74	3.37

Strain energy contribution of each mode to total system strain energy is shown in Figure2. From Figure2, it is seen that the contribution of the 91st mode, 1st mode, and 80th mode … is the most. Total contribution of the first most important 75 modes amounts to 90%. Thus, modal inertial forces of the first most important 75 modes are chosen as fundamental vectors to express universal ESWL in Eq.(12).

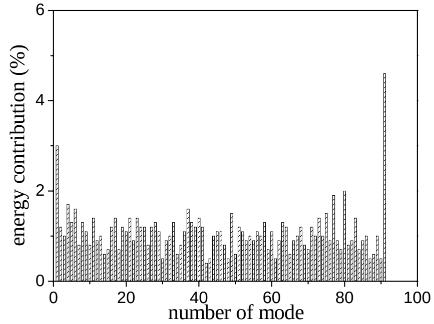


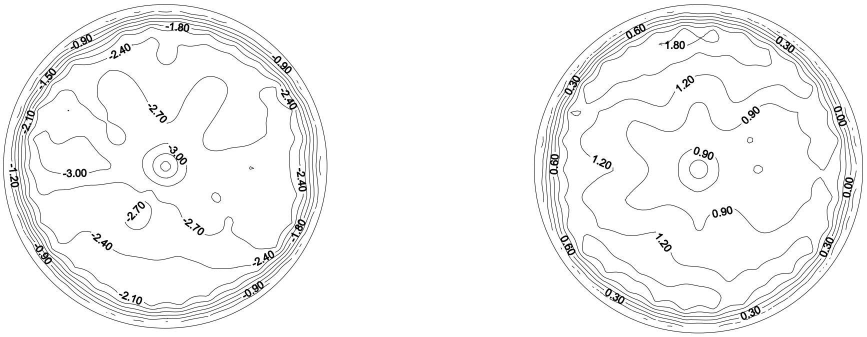
Figure 2: Strain energy of each structural mode to fluctuating response

4.2 Universal ESWL of this single-layer reticular dome

In view of strucutural design, displacements of 91 nodes, stresses of 342 element members and reactions of 36 supports must be concerned. One universal ESWL or several universal ESWLs are necessary to reproduce all these peak structural responses.

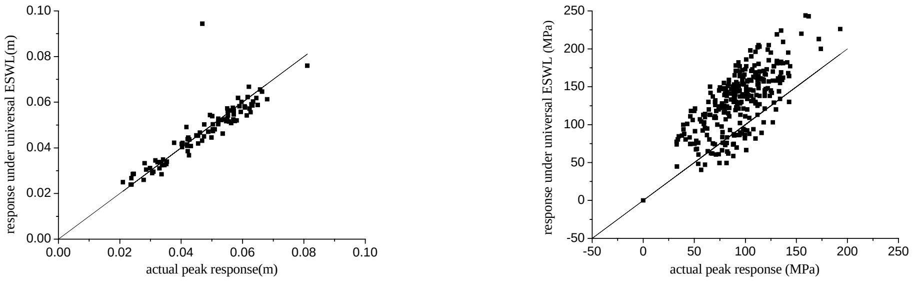
Universal ESWLs of five cases are analyzed, which include case 1: universal ESWL for peak displacements of 91 nodes; case 2: universal ESWL for peak axial stresses of 342 element members; case 3: universal ESWL for peak bending stresses of 342 element members; case 4: universal ESWL for peak reactions of 36 supports; case 5: universal ESWL for all peak diplacements, peak member stresses and supporting reactions.

Figure3 shows universal ESWL of case1 aims to reproduce peak displacements of 91 nodes, which is expressed by pressure coefficients in Figure3. Figure4 shows strucural response under this universal ESWL versus actual peak response. It’s seen that all nodal displacements agree well with actual peak response except one node, whose actual peak response is small, but static response of axial stresses and supporting reactions are obviously different from actual peak values. Thus, this universal ESWL is only suitable to calculate structural displacement response.

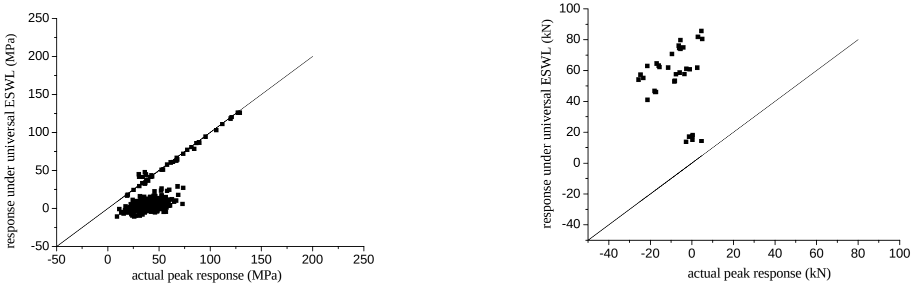


(a) ESWL（from Eq. 13a） (b) ESWL（from Eq.13b）

Figure 3: Total universal equivalent static wind loads for all nodal displacements



(a) largest displacement (b) largest axial stress



(c) largest bending stress (d) largest supporting reaction

Figure 4: Response under universal equivalent static wind loads for all displacement

All these 5 cases are analyzed, and Figure5 shows universal ESWLs for case 2, case 3, case 4, case 5. The results show that each universal ESWL is only suitable to calculate one kind of structural responses, but it is not suitable for other kinds of structural response, for example, Figure5(a) is suitable for axial stresses, Figure5(b) is suitable for bending stresses, and Figure5(c) is suitable for supporing reactions. The error of case 5 is very big between static responses under ESWL in Figure5(d) and peak dynamic response under actual wind loads, when this universal ESWL aims to reproduce all peak diplacements, peak member stresses and supporting reactions at the same time. Thus, each kind of structural response should corespond its own universal ESWL.

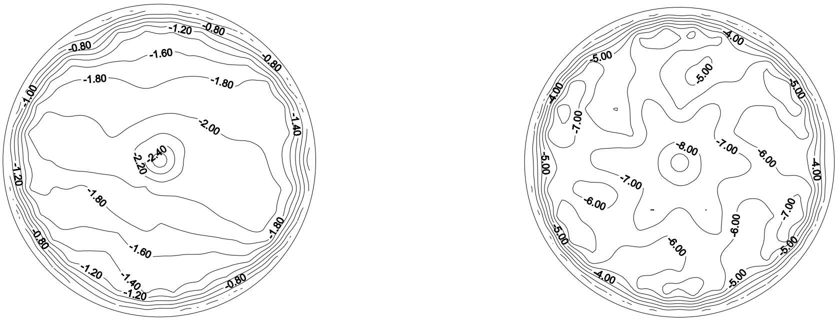
5. Conclusions

Analysis method of universal ESWL simultaneously reproducing multiple largest load effects is discussed, on basis of the analysis method of ESWL for one certain largest load effect. Moreover, universal ESWL of one single-layer reticular dome is studied. The following conclusions are reached:

(1) A method to analyze universal ESWL of large span space structures is proposed. Dominant eigenmodes and vibration modes of wind-induced are chosen to express universal ESWL.

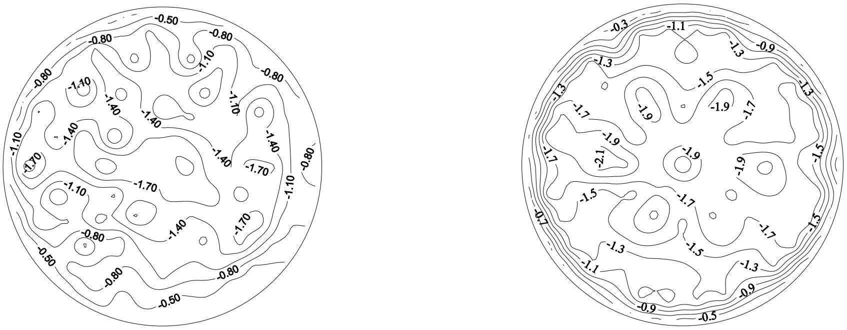
(2) Resonant response are much greater than background response of this single-layer reticular dome, and the latter can be neglected.

(3) For this single-layer reticular dome, each universal ESWL can reproduce peak load effects simultaniously for one kind of strucural response, but it is not suitable for other kinds of structural response, thus each kind of structural response coresponds one universal ESWL.



(a) ESWL for case 2 (from Eq. 13a)

(b) ESWL for case 3 (from Eq. 13a)



(c) ESWL for case 4 (from Eq. 13a)

(d) ESWL for case 5 (from Eq. 13a)

Figure 5: Total universal equivalent static wind loads for different cases

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