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A bi-stable revolute hinge for Variable Geometry Structures

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Abstract

A Rolamite hinge is made up of two rollers held by three tensioned bands attached to either part, with ends of each band terminating on opposite sides of the hinge, ensuring a no-slip condition. This paper investigates the existence of alternative forms of the Rolamite hinge by changing the profile of the rollers or tension band surfaces. The aim is to achieve a novel bi-stable revolute hinge that can be built into Variable Geometry Structures (VGSs). This hinge consists of two prismatic blocks connected through three fiber wires wrapped around them, that prevent any motion between blocks unless relative rotation along their axis. A set of design parameters is used for introducing the concept of bi-stability. The relationships concerned in either design or mechanical behavior are established in providing a proper use of the hinge. However, these relationships represent only a preliminary analysis of a deeper one that it should be carried out to build a physical model.

Keywords: Rolamite hinge; Variable Geometry Structure; bi-stable revolute hinge; bi-stable structural system.

1. Introduction

Certain structures, having the function to respond to changing situations in their use, operation or location, offer the advantage of modifying their configuration. Such structures are commonly known as Variable Geometry Structures (VGSs) and fall into two principal categories according to their process of transformation. The first category includes those that rely on the intrinsic property of their material to change configuration, like engineering balloons that are blown up with hot air, whereas the second category consists of those that rely on the geometric inter-linking of their elements to change configuration; this latter category usually contains a number of essentially resistant bodies, which are connected by hinges employed to enable movement of one or more degrees of freedom.

This paper is focused on the second category, in which the design of hinges addresses a key role in providing the motion that will ensure the seamless from one configuration to the other.
It should be pointed out that the critical factor is for these structures to maintain stability at their working configuration, since they behave like mechanisms. An elastic structure is assumed to be stable in a particular configuration if any small change in that configuration requires an increase in the total potential energy of the system, i.e. the sum of its potential energy and strain energy. Some VGSs accomplish this requirement through external forces, supplied by both additional elements and specific devices that retain the structure in its altered position. Another way to obtain stability (Ziegler [1]) is to constrain the assemblage of the components, which characterize the structures, to withstand a state of deformation that prevents any other motion. Problems arise in the above-mentioned solutions in that they are either costly, from an energy point of view, or the state of deformation that is used to retain the structure in its altered position leads to wear and an eventual decrease in performance.
There are some other structures that, thanks to their particular geometry or their material property, spontaneously find a stable configuration. Not only do they lack strained elements, they don't even need additional elements to constrain the motion (Krishnapillai [2], Gantes [3]).
These structures are usually identified as bi-stable structural systems, because they present at least two stable configurations. Either the geometry or the material for these structural systems are chosen in order to exhibit minima in potential energy corresponding to the stable configurations. Bi-stable can be the whole structure or the hinge system built into it.
It turns out that it is important to look more closely at the nature of hinges for both transmitting motion and stabilizing a VGS at its working configuration.
We are going to present a type of revolute hinge that behaves like a bi-stable structural system, suitable for use within Variable Geometry Structures for both space and architectural uses, simple, reliable and predictable in its properties.

2. Revolute hinges for VGSs: presentation of two types

There are several types of connection that can be used within VGSs; each type is employed to restrain the relative motions between two components, depending on the relative degrees of freedom that are to be prevented. We have focused our attention on a type, called revolute joint, which forces the components to rotate around a common axis and prevents any other relative motion.
Before to get down to the introduction of the hinge, we'll give a briefly description of few works that were carried out by previous researchers.
Donald Wilkes [4] of Sandia National Labs reported on a device known as "Rolamite", it was made up of a band wrapped in one direction around two cylindrical rollers ensuring a no-slip condition. One end of the band was attached to an upper plate and the other end to the lower, the plates were arranged in such a way to guarantee that the rollers could not be removed from the system (Figure 1).

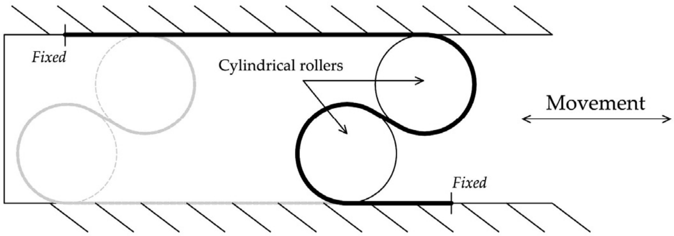


Figure 1: Model of the Donald Wilkes' Rolamite

Some years later Hilberry and Hall [5] also describe a "Rolamite" hinge (Figure 2) similar to the previous one. It was manufactured with rollers held by tensioned bands; the bands are attached to either part, with ends of each band terminating on opposite sides of the hinge. The minimal number of bands is three to prevent any motion between rollers unless relative rotation along their axis.

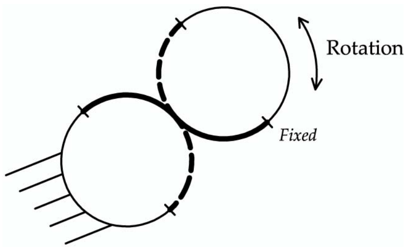


Figure 2: Plan view of the Hilberry's Rolamite

Rolamite hinges have the ability to maintain several positions without any external stabilizing force and contain minimal volume and parts. Further, due to the fact that there is no sliding, it is possible to construct hinges with very low coefficients of friction. The hinge designed by Hilberry can be used instead of a door hinge, even though the door, as all the standard doors, will require a device to be locked, or stabilized, at the close and open position. The same type of hinge has a benefit over standard revolute joints in that the door can be folded flat against the jamb in both directions, giving rotations of up to 360°; as example, it will represent an improvement of the hinge for saloon door, in which this last requirement is obtained by putting two standard hinges next to each other (Figure 3).

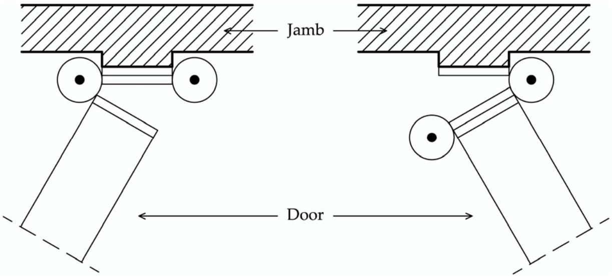


Figure 3: A model of double acting spring hinge

To summarize the Rolamite hinges present certainly some benefits compared to traditional hinges, nevertheless the Variable Geometry Structures built by them will require a power source to maintain at least one of their possible states.

3. Purpose of a novel bi-stable revolute joint

The idea of hinge we studied is an extension of the principle and benefits of the Rolamite to an hinge; we were looking for an hinge that behaves as bi-stable structural systems employed to stabilize a variable geometry structure at specific configurations.
The model consists of two prismatic blocks, with bases that are regular polygons, held by three wires wrapped around them. Each cylinder is turned into prism, corresponding to a prismatic block; the wires are still looped around cylindrical surfaces and follow the surface of three circle grooves cut into the transversal section of the prismatic blocks (Figure 4).

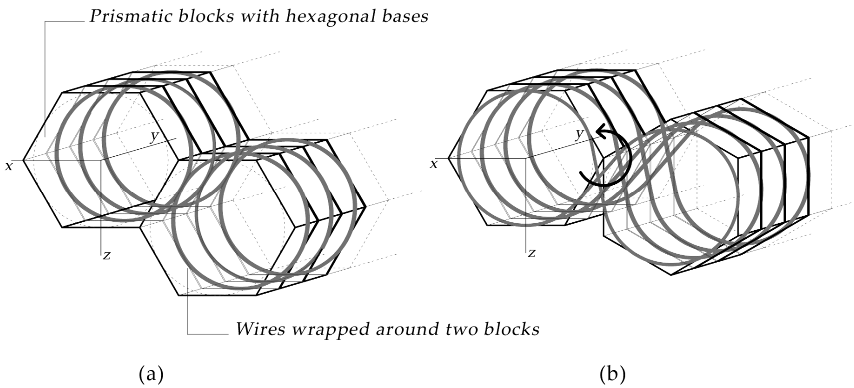


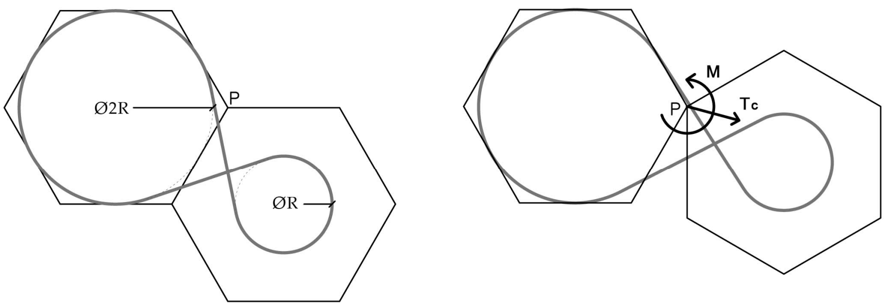
Figure 4: Prospective views of the model: stable position (a) and during the motion (b)

Assuming that the two blocks are essentially rigid and the wires are flexible, the hinge can be rotated from one configuration to the other around an axis that passes through one common vertex of the bases, which are chosen by the designer for the regular prisms. In other words, during the motion the only contact between the two prismatic blocks becomes a line that passes through one common vertex of their bases. The length of the wires becomes incompatible, shorter than it should be, with the position of the two hinges in between one configuration and the other, and for this reason each wire is deformed in providing this geometrical incompatibility. The wires are stress free in all the positions that are compatible with their unstressed length, those in which the edges of the bases are adjacent, as consequence that a force will have to be applied to move the hinge from one compatible position to the other. Each compatible position identifies a stable configuration because any small change in this requires an increase in the total potential energy of the hinge system.

Design parameters as well as kinematic behavior of the hinge can be simply observed by considering a plan view of a transversal section that includes only a single wire.

The Figure 5 shows a transversal section of a hinge made up of two regular prisms with hexagonal bases, held by wires that are wrapped along two different circle surfaces, one of diameter 2R, which is inscribed in the first section and another of diameter R, marked in the second section.

The following parameters are defined in the Figure 5: the vertex P, line projection on the plane where the prism bases lay, the two sections rotate around this point, the vector T_c that represents the contact force due to both the elongation and stiffness of the wires, and the vector M that indicates the moment, action that should be applied to go by one stable configuration to the other, and that is equal to the contact force multiplied by the distance between the point P and the line of action of the wire.



(a) (b)

Figure 5: A transversal section of the hinge: its stable position (a) and during the motion (b)

Looking at the vectors T_c and M throughout the motion, we analyzed the geometrical relationships between some parameters that characterized the hinge. These parameters are pointed out in the Figure 6. Through this study we found out the equations of both tensile force along the wire and resultant moment of this force with to respect to the point P, as function of those parameters that change during the motion. In the Figure 6(a) the following parameters are drawn: the centers of the circle surfaces, that keep in tension the wires, are marked with letters C_L and C_R, with R and L in subscripts, respectively, for the center on the left and the center on the right, the edge of the polygon is marked as s, the unstressed length of the wire is indicated as l_o, and γ is the angle between the line passes through C_LC_R and the orthogonal line to the piece of wire that doesn't adhere to either of circle surfaces. The wires are wrapped around circles of radius R_L and R_R, with letters in subscripts that refer, respectively, to the circle on the left and to the circle on the right. The choice of the regular polygon, base for the prisms, affects the value φ, angle of rotation from one stable configuration to the other; the angle is showed in the Figure 6(b). In the same figure we can see some of those parameters, already showed in the Figure 6(a), that refer to a position of the hinge impressed during the motion.

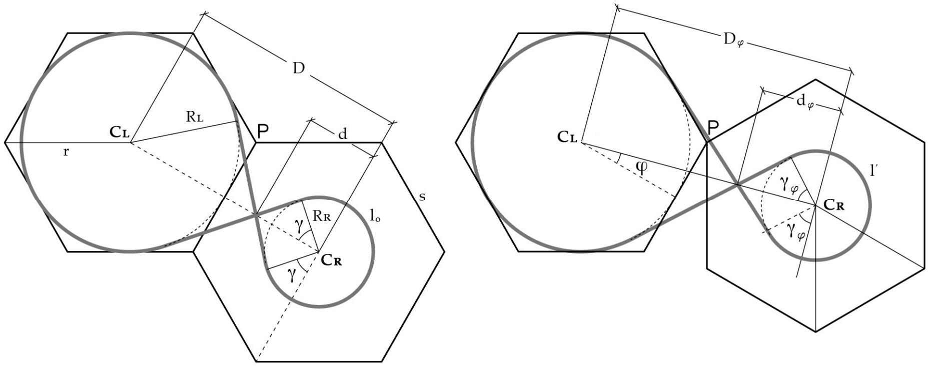


Figure 6: A set of design parameters: a stable position of the hinge (a) and a position during the motion (b)

Assuming that n is the number of edges for each regular polygon chosen for the bases, and given $R_{L,}$ as radius of a circumference inscribed into it, R_R, as radius of the circle on the right where the wire is wrapped, then we can find the value of the edges of each polygon

$$s = 2R_L \sin\frac{\pi}{n} \tag{1}$$

and the radius r of the circumference circumscribed to the regular polygon

$$r = \sqrt{(R_L)^2 + \left(\frac{s}{2}\right)^2} \tag{2}$$

From these parameters the following relationships for a stable position, showed in the figure 6(a), can be obtained:
the distance d, between the center C_R and the point where the tensioned wire comes into contact, this value is obtained as function of the shortest distance between the two center C_LC_R, equal to $2R_L$ and marked with the letter D

$$d = \frac{D}{\frac{R_L}{R_R} + 1} \tag{3}$$

similarly we can achieve the angle γ between the line that passes through $C_L C_R$ and the orthogonal line to the piece of wire that doesn't adhere to either of circle surfaces

$$\gamma = \cos^{-1} \frac{R_R}{d} \tag{4}$$

The hinge can rotate through an angle φ = 2π/n from one stable configuration to the other, the equation (2), (3) and (4) can be re-written throughout the motion for all those parameters that are introduced in the Figure 6(b). We found that $D_£$ defines the distance between the two centers due to an imposed rotation φ

$$D_\varphi = \sqrt{2r^2 \left[1 - \cos \pi(1 + 2/n)\right]} \tag{5}$$

similarly we worked out $d_£$ that defines the distance between the point, where the tensioned wire comes into contact, and C_R

$$d_\varphi = \frac{D_\varphi}{\frac{R_L}{R_R} + 1} \tag{6}$$

at last we defined the angle $\gamma_£$

$$\gamma_\varphi = \cos^{-1} \frac{R_R}{d_\varphi} \tag{7}$$

Now by using E_w and A_w for, respectively, the Young's Modulus of the material of each wire and the area chosen for their cross section, we have all parameters useful to write the relationships of both tensile force T and resultant moment M with the angle φ

$$T = \left(\frac{\pi + tg\gamma_\varphi - \gamma_\varphi}{\pi + tg\gamma - \gamma} - 1 \right) E_w A_w + T_o \tag{8}$$

$$M = 2Tr \sin\gamma_\varphi \sin\pi\left(\frac{2n-1}{n}\right) \tag{9}$$

the ratio between T, the tensile force, and A_w, the area of each wire, gives the stress into it, T_o is a possible pre-tension given to each wire. Applying a sine's law we can easily find out from T the value of the contact force transmitted across the interface of the prismatic blocks.
The equations (8) and (9) are plotted in the Figure 7. These diagrams show the main paths that both tensile force T and resultant moment M follow throughout motion from one stable

position to the other. We marked with letters A, B, C, D and E the key points to explain the advance of T and M due to an imposed rotation φ. As we see in the Figure 7(a) the value of T, the tensile force, goes up from zero to a maximum, corresponding to a rotation of φ/2, it peaks at C and afterwards it drops back to a zero value, when the hinge raised the consecutive stable position. In the Figure 7(b) the resultant M goes up from zero to a maximum value for φ/4, it peaks at B and falls at a negative value in D, going through the zero value for φ/2, point C. Finally the resultant moment, as well as the tensile force, goes back to zero in E, being this corresponding to the next stable position.

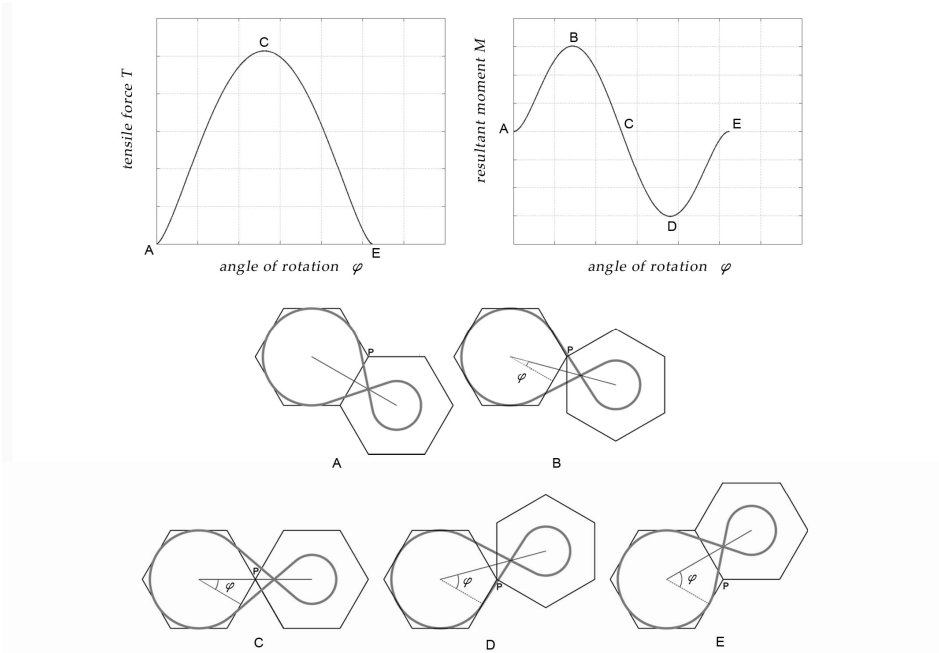


Figure 7: The main path of tensile force T and result moment M throughout the motion

The relationships between T, M and φ has been worked out for three hinges made up of regular prisms with, respectively a hexagonal, a pentagonal and an octagonal bases, held each one by three wires of *Vectran* fiber with an initial unstressed length of l_o.
The input values for the hinges were R_L and R_R equal to 6,7mm and the product between Young's Modulus and surface of the wires, E_wA_w equal to 1500N. The results are plotted in the Figure 8.

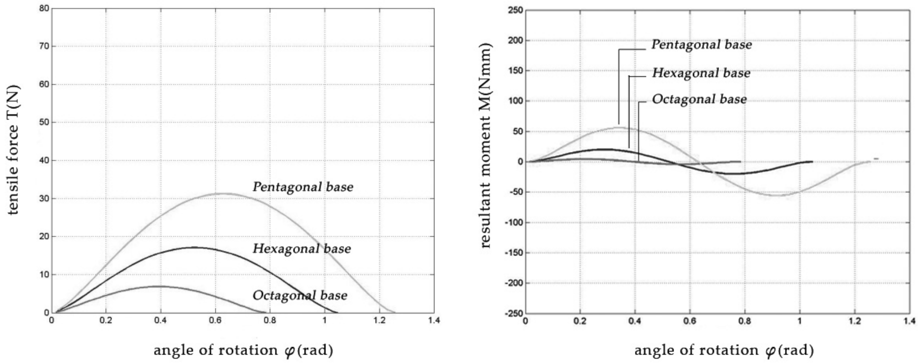


Figure 8: Tensile force and result moment versus angle φ for wires of lenth l_o

The same analysis has been carried out for three hinges, distinguished by the previous for only the initial length of the wires, in these latter case the length is shorter than l_o, and it is marked with letter l_1. The grooves are such that the length l' fits into them. The results are plotted in the Figure 9.

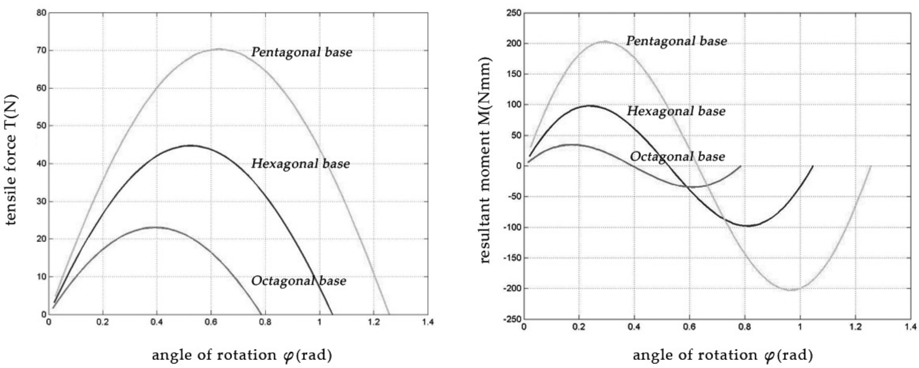


Figure 9: Tensile force and result moment versus angle φ for wires of length l_1

To summarize the diagrams plotted in the Figure 8 and Figure 9, higher values of the tensile forces and the resultant moments can be achieved either by decreasing the number n of edges that characterize a polygonal base or by using shorter wires. Obviously these options can be realized only if both the size and the mechanical properties of the wires are able to withstand the corresponding higher values of stress.

Although this basic design achieves the desired locking effect, it does have one large drawback. When the hinge is rotated from one configuration to the other, a contact force

acts through the corners in P. This induces localized stresses in the corners. The solution is to round the corners sufficiently to keep their deformation elastic.
An analysis is then required to predict the magnitude and distribution of surface tractions, normal and tangential, transmitted across the interface, with the aim to compare these values to the yield limit of the material chosen for the prisms. We used the Hertz theory of elastic contact between two-dimensional cylindrical bodies (Hertz [6]), and satisfied all the assumptions that must be imposed for the above theory. Some important assumptions are that the surface in contact should be continuous and small compared with the dimensions of the prisms themselves, and the stresses are highly concentrated in the region close to the contact zone and are not greatly influenced by the shape of the prisms at a distance from the contact area. In order to satisfy these assumptions, firstly we turned the vertex P into a smooth corner designed with a circle of radius R as shown in the Figure 10.

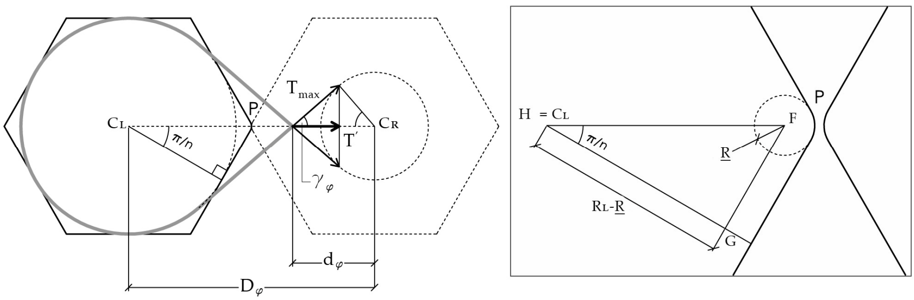


Figure 10: Set of parameters to determine the stress transmitted across the interface

In the triangle ΔFGH for the cosines' law

$$FH = \frac{R_L - \underline{R}}{\cos\frac{\pi}{n}} \tag{10}$$

the equation (5), written for a general configuration during the motion, becomes the equation (11) for the angle of rotation φ/2, corresponding to the position where the tensile force raises the maximum value

$$D_{\varphi/2} = 2\left(FH + \underline{R}\right) \tag{11}$$

substituting equation (11) into (6), (7) and (8) yields the maximum value of T. At this value the contact force can be easily found from the sine's law

$$T_c = 2T_{max} \sin\gamma_\varphi \tag{12}$$

T_c should be compared to the ultimate strength T_U of the material chosen for the prismatic blocks. T_U is given as function of τ_U that is the ultimate stress of the material

$$T_U = \left(\frac{\tau_U}{0.3}\right)^2 \frac{\pi(1-\nu)R}{2G} \tag{2}$$

where ν is the Poisson's ratio and G is the shear modulus.
The final value of the tensile force that occurs in each wire should be lower or at the most equal to the ultimate tensile value T_U. Not only does the proper geometry of the hinge contribute to this value, but also a pre-tension, given by an external power to the wire, can be add in order to compute the total force.
Although the operation of increasing the value of the tensile force leads to an effective bi-stable structural system, the corresponding resultant moment necessary to rotate the hinge, from a stable configuration to the other, will have to be too high. In other words a hinge that offers a strong resistance to change configuration will loose the ability to grant one degree of freedom between the components linked in through it.
We conclude that the analysis made can be, certainly, used to describe the relationship between design criteria of the hinge and its static and kinematic properties, but only as a preliminary study versus a deeper investigation for an efficient construction of the hinge.

4. Discussion

In this paper, a concept of utilizing a novel revolute hinge for the construction of Variable Geometry Structures has been presented. It has been proved that by modifying the geometry of a Rolamite hinge it is possible to obtain an hinge that behaves as a bi-stable structural system. Thus, this hinge could be used to construct VGSs since becomes a good device to stabilize them at some specific configuration. Detailed mathematical derivation has been given leading to the relationships between design criteria of the hinge and its static and kinematics properties.
The hinge we studied is made up of two same prismatic blocks held by three wires, wrapped around them through circle surfaces cut into the cross-sections. The length of the wires is taken to be unstressed when the lateral surfaces through one edge of the prisms are adjacent. Assuming that the two blocks are essentially rigid whereas the wires are flexible, the hinge can be rotated from one configuration to the other, as the wires are able to deform in providing that motion. In fact, the wires are stress free in all the positions that are compatible with their unstressed length, those in which the edges of the bases are adjacent, but they are deformed during the seamless rotation. During the motion, the only contact between the prisms is a line through a vertex of the bases, a contact force occurs in that point that is proportional to the elongation of the wires. Further, a resultant moment occurs, that is equal to the contact force multiplied by the distance between the contact point and the line of action of the wire. Both the tensile force into the wires and the resultant moment

throughout the motion has been found in order to compute the strength that will have to be applied to move the hinge.
Higher values of the tensile forces and the resultant moments can be achieved either by decreasing the number n of edges that characterize a polygonal base of the prisms or by using shorter wires that held them.
The maximum material stresses occur when the contact force is highest, this happens when the angle of rotation is $\varphi/2$. The stresses that occur in the interface of the prisms due to this contact force are best calculated using Hertzian contacts.
Although the operation of increasing the value of the tensile force leads to an effective bi-stable structural system, the corresponding resultant moment necessary to rotate the hinge, becomes too high, as to say that a hinge offers a strong resistance to change configuration for significant lack of ability to enable one degree of freedom between the components linked through it.
We conclude that the analysis we made represents a preliminary study versus a deeper investigation for an efficient construction of the hinge. The hinge should easily excute its motion, but behaves as a bi-satble hinge system.

Acknowledgement

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