

Vibration behaviour of statically determinate arches, under pedestrian loading and using morphological indicators

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Abstract

Within the framework of sustainable development we strive for constructions with a minimum volume of material. Only considering criteria on resistance and buckling, at the stage of conceptual design a clear hierarchy among the different structural topologies can be established with Morphological Indicators (MI). Up to now, dynamic effects were not considered when designing constructions with MI. However, an optimum obtained by minimising the volume only considering strength, often results in solutions with problematic dynamic behaviour. To avoid these problems the stiffness, mass and/or damping characteristics must be modified, implying a volume increase, additional apparatus and as a consequence a cost rise. With an optimisation process that considers/predicts dynamic behaviour at the stage of conceptual design, an optimum can be obtained without the necessity to alter the (strength-optimised) construction drastically afterwards. This paper focuses on the prediction of the eigenfrequencies and peak accelerations of statically determined arches (3 hinges) under pedestrian excitation within the theory of morphological indicators. Hence, it provides a user friendly tool during the first steps of the design process for the architect/structural designer.

Keywords: conceptual design, resonance, pedestrian loading, morphological indicators, fully stressed design, parabolic arches

1. Introduction

Conceptual design, along with problem identification and analysis, make up the initial stage of the structural design process. Problem analysis transforms the often vague statement of a design task into a set of design requirements. Conceptual design encompasses the

generation of concepts and integration into system-level solutions, leading to a relatively detailed design (Kroll et al. [4]). The assessment of structural performance is, mainly at present time, the result of late finite element analysis processes (FEA) which remain computationally expensive, limiting their use to the analysis of a limited number of design alternatives. But, in the conceptual design stage, the quality depends on the comprehension and on the exploration of the design space (Yannou et al. [11]).

During the conceptual design stage, an engineer or architect creates the general outlines of a structure. In this phase a few solutions are selected out of the many possible because of their adaptedness to the most important requirements. More specifically, in the context of sustainable development, one seeks structures with minimum volume. A good conceptual design will then yield a solution (topology and geometry) that will not (or slightly) change its relative superiority to other solutions when more detailed design calculations are performed. It is hence very important to assess the possible impact of detailed calculations like (global) buckling, dynamics, weight of connections, second order effects among others. If the context of the situation allows the use of morphological indicators in their simplest form, a larger topological/geometrical solution space can be browsed for. This viewpoint can be expressed as follows: always optimize structures with a minimum number of variables as long as the detailed analysis will not alter the efficiency of the structure considerably.

1.1. Morphological indicators

Morphological Indicators are design tools allowing the optimization of structures at the stage of conceptual design using a limited number of parameters (Samyn [6]). The indicator of volume W allows the comparison of the volume of material used for different structural systems.

$$W = \frac{\sigma V}{FL} \quad (1)$$

It is the volume of an isomorphic structure with unit span L , with at least one section dimensioned at its unit allowable stress σ , subjected to a system of loads with unit resultant F .

The displacement indicator Δ compares the displacement of different structural systems.

$$\Delta = \frac{E\delta}{\sigma L} \quad (2)$$

It is the maximum displacement of an isomorphic structure with unit span L in a material with unit Young's modulus E , with at least one section dimensioned at its unit allowable stress σ , subjected to a system loads with unit resultant F .

The analytical expressions of both W and Δ have been established by Samyn [6, 7] and Latteur [5] for trusses, beams, arches, cables, cable stayed structures, masts and frames subjected to a limited number of (simple) load cases and supports. For statically determinate structures those MI are only function of the geometrical slenderness L/H

(length divided by the height of a rectangle that frames the structure). Instabilities, self weight and second order effects are neglected. Efficiency curves depicting the geometrical slenderness with respect to the minimum volume material can be established. These curves allow the comparison between different structural typologies and topologies with a reduced number of variables. In a more general way, one can conclude that MI only depend on the proportion of the rectangle in which the structure is inscribed and on the shape/topology (i.e. how this rectangle is filled). Those results are based on a fully stressed design of statically determinate structures and for a dominant load case, here a uniformly distributed vertical load (For the Parabolic truss arch asymmetric load cases are discussed in Latteur [5].)

The buckling indicator Ψ [7] takes buckling in individual elements into account

$$\Psi = \frac{\mu\sigma L}{\sqrt{qEF}} \quad (3)$$

It is the image of the buckling tendency of the compression elements in a structure with span L , composed of bars with a form factor $q=I/\Omega^2$ (Figure 1) (with I the second moment of inertia and Ω the section area) in a material with Young's modulus E , with at least one section dimensioned at its allowable stress σ , subjected to a system of load with total resultant F . μ is the proportion of the buckling length of the compression bars over their geometrical length (which depends on the connection type. Eurocodes 3 [3] give 0,9 for trusses).

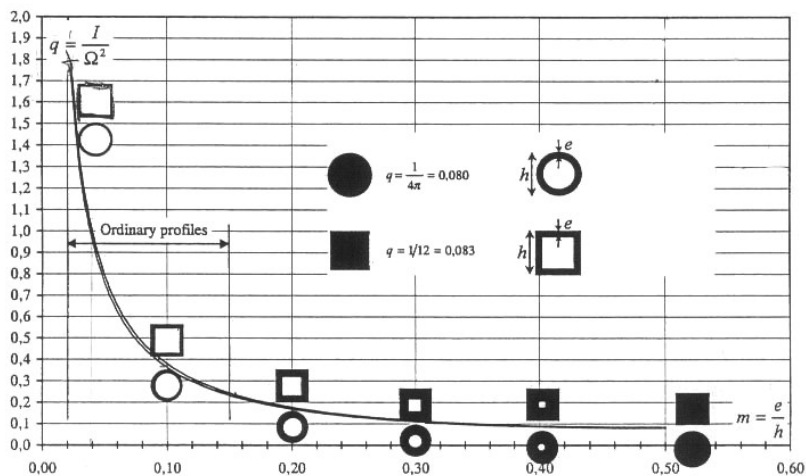


Figure 1: Form factor of double axis symmetric square and circular profile (Lateur [5])

This contribution modifies the element sizing. It implies the use of one extra parameter (Ψ), but increases the precision of the optimization process. It enables the evaluation of the extra

necessary volume of material to avoid buckling. Moreover, it affects the optimal solution. For example: the higher the value of the buckling indicator, the higher the optimal slenderness for arches. The definition of the indicator of buckling clearly demonstrates that the buckling sensitivity depends on the span-load ratio (L/\sqrt{F}). In Shanley [9] this quantity is defined as the structural index. This index illustrates that morphological indicators are dimensionless but depending on scale effects.

1.2. Dynamics

Up to now, dynamic effects were not considered when designing constructions with MI. However, an optimum obtained by minimising the volume only considering strength, often results in solutions with problematic dynamic behaviour. To avoid these problems the stiffness, mass and/or damping characteristics must be modified, implying a volume increase, additional apparatus and as a consequence a cost rise. With an optimisation process that considers/predicts dynamic behaviour at the stage of conceptual design, an optimum can be obtained without the necessity to alter the (strength-optimised) construction drastically afterwards. In this paper two new indicators are presented allowing the evaluation of the dynamic behaviour of statically determined parabolic arches (3 hinged) at conceptual design stage. These two indicators should avoid the resonance or its effects due to periodic excitation by pedestrians. (Resonance is relatively important vibration amplification when a structure is excited at one of its eigenfrequencies.)

Nowadays there are two main concepts in order to obtain satisfying vibration comfort levels: the first requires calculation of the actual dynamic response and checking if it is within acceptable limits, usually expressed as an upper limit on the maximal acceleration of the structure. The second approach is based on the request to avoid structural natural frequencies within the excitation range since this can potentially yield resonance. Therefore the two presented indicators are focussing on the eigenfrequency and the maximum peak acceleration.

2. Fundamental eigenfrequency

In 2005 Van Steirteghem [11] provides a first method to predict the first natural frequency of a beams and trusses with the indicator of displacement, based on a single degree of freedom model of a statically determined beam and the similar shape between static displacement and first global vibration mode. A nondimensional indicator of first eigenfrequency is developed which is only depending on the buckling indicator, the slenderness and the number of panels for every truss topology. Recent research by Spranghers [10] has shown that the eigenfrequencies of arches cannot be predicted using the indicator of displacement. Sensitivity analysis through FEA has show that the eigenfrequencies f_i of a fully stressed parabolic truss arch can be determinate with a reduced number of variables

$$f_i = \sqrt{\frac{EI}{z^* F_{tot}.L^3}} function\left(\frac{L}{H}\right) \quad (4)$$

In equation (4) the covibrating mass is considered through a factor z^* , the ratio of covibrating load to total load in Serviceability Limit State (or SLS). The covibrating load is essentially composed of two parts

$$z^* = y^* + \phi W \quad (5)$$

y^* represents the ratio of covibrating external load to the total load in SLS and ϕW stands for the self weight. It is the multiplication of the indicator of volume W with the indicator of selfweight ϕ , as defined for the first time in Latteur [5]

$$\phi = \frac{\rho \cdot g \cdot L}{\sigma} \quad (6)$$

in which ρ symbolizes the material's specific mass. The sum in equation (5) implies that an equal distribution for the external load and the self weight is assumed in a fully stressed arch with uniformly distributed external load. This hypothesis is not exactly correct since self weight is not uniformly distributed along a horizontal line but along a parabola. The introduced error is very low (not exceeding some percentages) and thus acceptable at conceptual design stage.

The slenderness function in equation (4) differs whether the arch is prone to buckling ($\Psi > 0$) or not ($\Psi = 0$). For buckling sensible arches the arch section (taken constant over the entire arch in this paper) is linear proportional to the indicator of buckling Ψ (Lateur [5]) or considering the form factor from q the introduction

$$I = q \cdot \Omega^2 = q \cdot \left(\frac{F \Psi}{\sigma} \right)^2 \cdot function' \left(\frac{L}{H} \right) \quad (7)$$

Substitution in equation (4) yields

$$f_i = \frac{1}{\sqrt{L \cdot z^*}} function'''' \left(\frac{L}{H} \right) \quad (8)$$

This information allows predicting the eigenfrequencies with a single design graphic. Figure 2 (upper) depicts the dimensionless eigenfrequencies as a function of the slenderness. If the arch is not subjected to buckling, the arch's second moment of inertia does not depend on the value of the buckling indicator (Lateur [5]), hence

$$I = q \cdot \Omega^2 = q \cdot \left(\frac{F}{\sigma} \right)^2 \cdot function' \left(\frac{L}{H} \right) \quad (9)$$

Substitution in equation (4) yields

$$f_i = \sqrt{\frac{qEF}{z^* \sigma L^3}} function'''' \left(\frac{L}{H} \right) \quad (10)$$

The corresponding sixteen first eigenfrequencies can be plotted as a function of the slenderness, see Figure 2 (lower).

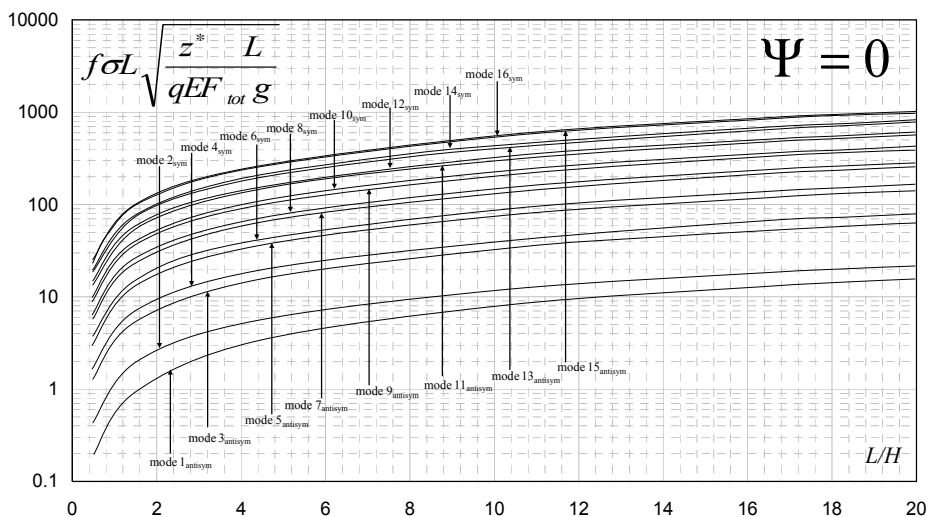
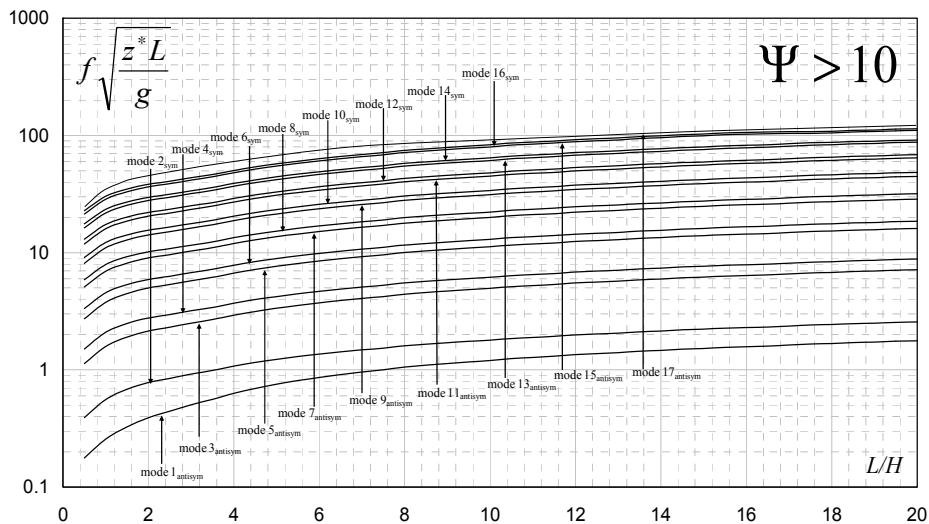


Figure 2: Dimensionless eigenfrequencies of three hinged fully stressed parabolic arch with (upper) and without (lower) buckling sensitivity

Two types of vibration modes can be distinguished: an asymmetric (or lateral) and symmetric (or vertical) mode. The first two vibration modes are show in Figure 3. Higher

modes respect the same logic, only showing successively more vibration nodes and antinodes.

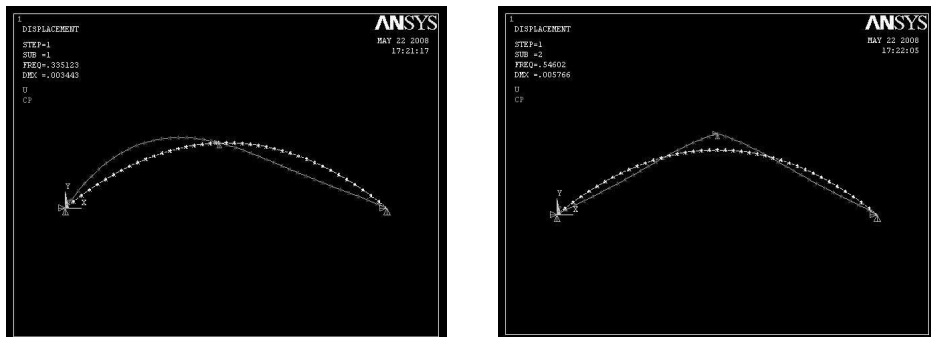


Figure 3: Two first vibration modes of 3 hinged arch: lateral (left) and vertical mode (right)

3. Peak acceleration

In a lot of practical cases the resonance region is unavoidable. The designer would have to apply very important, sometimes unacceptable, modification to the structure. Therefore some guidelines focus more on limiting the physical effect of resonance, leading to a certain level of (un)comfort: the acceleration. As opposed to the eigenfrequency, the acceleration does not only depend on structural characteristics (damping, mass,...) but also on the applied load (frequency, magnitude, distribution,...). Hence this part introduces first the applied pedestrian's load model.

3.1. French load model, Sétra-AFGC [8]

The methodology consists of different consecutive steps. Firstly the category of the bridge should be defined in accordance with the level of traffic it is intended to support and the comfort level to be obtained. It goes from Class I bridges, standing for urban footbridge connecting areas with high concentrations of pedestrians, to Class IV for bridge with very little use. With every desired comfort level an acceleration range is associated. For the vertical direction: maximal (practically imperceptible; $0-0,5\text{m/s}^2$), mean (merely perceptible; $0,5-1\text{m/s}^2$) and minimum comfort (perceptible but not unbearable; $1-2,5\text{m/s}^2$).

For footbridges of category I to III the vibration frequencies of the structure must be determined for two mass assumptions: bridge empty and bridge loaded over the entire surface with 700N/m^2 . The ranges within these frequencies lie are used to assess the risk of resonance being set up by pedestrian traffic, and, depending on the risk, to specify the dynamic load cases to be studied to verify the comfort criteria. If the risk of resonance is judged to be negligible, the level of comfort is automatically considered to be sufficient.

Finally the peak acceleration under the specified load case should be determined and compared to the allowed range. The load is always uniformly distributed over the entire bridge and pointing in the same sense as the studied vibration mode. Only the amplitude (here in N/m^2) of the load varies with the different load cases

$$F(t) = d \cdot F_1 \cdot \cos(2\pi f_v t) \cdot \Psi \cdot N_{eff} \quad (11)$$

where d is the crowd density, F_1 the amplitude of the first walking harmonic (vertically 280N), f_v the excited eigenfrequency of the bridge, Ψ correlation factor between the frequency of loading by the crowd and the eigenfrequency of the structure and N_{eff} the random footfall correlation or the equivalent percent of synchronized pedestrians.

Through modal superposition the peak accelerations for excitation on the sixteen first eigenfrequencies is calculated. From sensitivity analysis it appears

$$a_{peak,i} = \frac{F_{ex} \cdot g}{z^* \cdot F_{tot} \cdot \xi} \text{function}\left(\frac{L}{H}\right) \quad (12)$$

- with
- $a_{peak,i}$: peak acceleration for excitation on eigenfrequency i [m/s²]
 - F_{ex} : amplitude of exciting load (N)
 - ξ : damping ratio (considered equal for all vibration modes)
 - g : gravitational constant or 9,81m/s²

Hence a unique design plot, depicting the dimensionless peak accelerations as a function of the arch slenderness can be plotted (Figure 4). Note that these accelerations do not depend on the value of the buckling indicator as opposed to the eigenfrequencies.

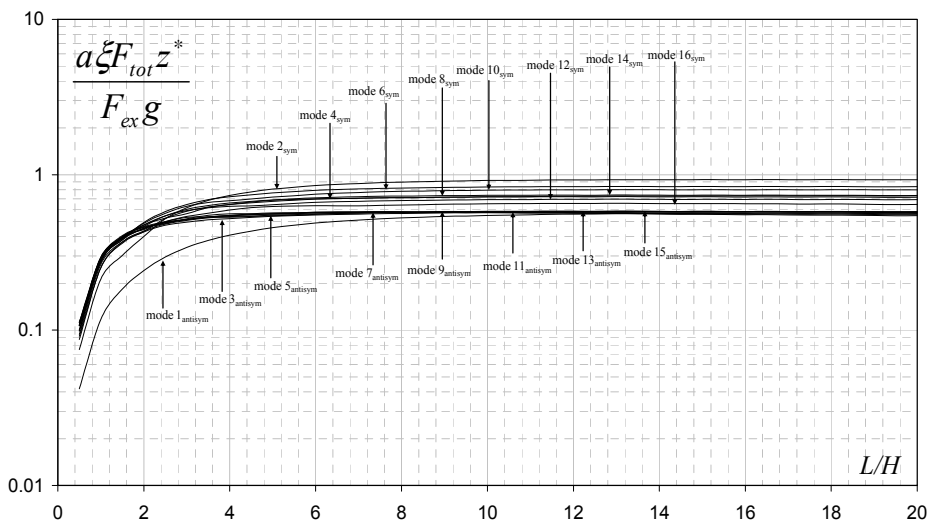


Figure 4: Dimensionless peak accelerations of three hinged fully stressed parabolic truss arch with constant section

4. Case study

The discussed case study is based on a real design problem located at the city of Rio de Janeiro, Brazil. A composite (steel/concrete) footbridge with main span of 22,5m, simply supported by columns at its ends, has been build using a T-steel beam cross section and a reinforced concrete deck. A vibration analysis performed in da Silva et al. [1] clearly indicates that the maximum accelerations violate the human comfort serviceability conditions. In this part the possibilities offered by the use of an alternative design. A parabolic arch with constant section and three hinges, is evaluated by means of morphological indicators, avoiding intensive calculations. The vibration behaviour is assessed by the above presented methodologies. The following design parameters fix the problem:

- 2 parallel vertical parabolic arches are used. One at each side of the bridge deck.
- The span is 22,5m
- The bridge deck width is 2,30m (dimensions of existing deck)
- Use of S235 steel ($\sigma_{yield} = 235\text{MPa}$, $E = 210\text{GPa}$, $\rho = 7850\text{kg/m}^3$)
- Form factor $q = 1$ or tubular profiles with thickness to diameter ratio of 4% (Figure 2)
- Estimated damping ratio $\xi = 1\%$
- Total external vertical load is composed of the pedestrian's live load $F = 7,5\text{kN/m}^2$. b.L = 388kN (according to Eurocode 1 [2]) or 194kN per arch. Additionally the load of the concrete deck can be summed to this load or an extra 85kN per arch.

Note: a three hinged arch is chosen in order to avoid additional stresses by thermal effects or differential setting of the supports.

4.1. Volume

Using Morphological Indicators, the design can be optimized with a reduced number of variables: the slenderness L/H and the buckling indicator Ψ . In order to use the appropriate design graphic Ψ must be known. With Equation (3) we find

$$\Psi = \frac{\mu\sigma L}{\sqrt{qEF}} = \frac{235\text{MPa} \cdot 22,5\text{m}}{\sqrt{1 \cdot 210\text{GPa} \cdot 279\text{kN}}} = 21,84 \quad (13)$$

In Latteur [5] the optimal slenderness and the corresponding indicator of volume can be found as a function of the buckling indicator: an optimum $W = 4,25$ and a slenderness of 4, which corresponds in this problem to an arch height of 5,625m.

4.2 First vertical eigenfrequencies

In order to calculate the eigenfrequencies, one should estimate the ratio of covibrating load, composed of self weight and external covibrating load. The self weight can be calculated with Equation (6)

$$\phi = \frac{\rho \cdot g \cdot L}{\sigma} = \frac{7850\text{kg} / \text{m}^3 \cdot 9,81\text{m} / \text{s}^2 \cdot 22,5\text{m}}{235\text{MPa}} = 7,373 \cdot 10^{-3} \quad (14)$$

For the external covibrating load two extreme load cases have to be considered. A totally unloaded bridge or ‘empty’ bridge and a bridge fully load by pedestrians. For the empty bridge the external covibrating load is the concrete deck or 85kN. Substitution in Equation (5) gives

$$z^* = y^* + \phi W = \frac{85kN}{279kN} + 7,37 \cdot 10^{-3} = 31,1\% \quad (15)$$

For the fully loaded bridge we take 700N/m² pedestrian’s load [1] or

$$z^* = y^* + \phi W = \frac{700N / m^2 \cdot 2,3m / 2,22,5m + 85kN}{279kN} + 7,37 \cdot 10^{-3} = 37,6\% \quad (16)$$

The corresponding eigenfrequencies can be estimated with Figure 2 and are found in Figure 5. Only the four first vibration modes are within the range potentially excited by pedestrians (0-5Hz) and requiring further dynamic analysis.

Vibration mode	Empty deck – Eigenfrequency (Hz)	Full deck – Eigenfrequency (Hz)
1	0,75	0,68
2	1,28	1,16
3	3,47	3,16
4	4,26	3,88

Figure 5: Eigenfrequencies with vibration modes potentially excited by pedestrians for full and empty bridge deck

4.3 Peak accelerations

The bridge category is chosen to be I since it is situated in the city of Rio de Janeiro. For every vibration mode the appropriate frequency range and load case is selected, following the Sétra guideline [8] and equation (11). The calculated vertical exciting loads are used to predict the peak vertical accelerations with Figure 4. An overview of the results is given in Figure 6. Note that for the estimation of the peak acceleration the average value for the eigenfrequencies of the empty and full deck are considered. The covibrating mass includes the weight of the participants as defined in Sétra’s guidelines [8].

Vibration Mode	Eigenfrequency (Hz)	Exciting load (N)	Peak acceleration (m/s ²)
1	0,72	NRR	NRR
2	1,22	745	5,07
3	3,32	419	1,99
4	4,07	466	2,73

NRR: No Risk of Resonance according to Sétra [8]

Figure 6: Exciting vertical loads and vertical peak accelerations under pedestrian crowd load

Typical acceptance range for peak accelerations are 0 to 1 m/s^2 . The predicted accelerations for mode 2,3 and 4 are all above this range. Hence, using a fully stressed three hinged parabolic arch would not solve the vibration problem of the pedestrian bridge. In general three main options can be taken to decrease the vibration level:

- Stiffen the structure by external components (eg. cables) or internal design modification. The designer could opt for an arch without central hinge. The impact of a modification of the bridge slenderness can quickly be evaluated using the above presented method.
- To increase the covibrating load. This option is to be avoided within the frame of volume optimization.
- Increase the damping characteristics by adding external dampers (tuned mass damper,...) since the acceleration is linear proportional with the inverse of the acceleration.

5. Conclusions and further work

The developed method combines the advantages of morphological indicators (reduced number of variables) and different existing pedestrian load models to estimate the vibration behaviour of parabolic arches at conceptual design stage, without the need of detailed calculations. Fundamental eigenfrequencies and vertical peak accelerations can be found in design graphics. The maximal peak accelerations are given for crowd loading based on the French design guideline, Sétra [8].

Moreover, the developed method gives insight into the design space (shape and topology here translated in slenderness), enabling the designer to ameliorate the dynamic response without need for detailed calculations. This method can be applied to other structural archetypes as trusses, beams, truss arches,...in order to provide a more complete overview of the design space. Another important topic is the study of lateral vibrations, including synchronization and lock in effects.

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