Application of the Game Theory with Perfect Information to an agricultural company

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Abstract: This paper deals with the application of Game Theory with Perfect Information to an agricultural economics problem. The goal of this analysis is demonstrating the possibility of obtaining an equilibrium point, as proposed by Nash, in the case of an agricultural company that is considered together with its three sub-units in developing a game with perfect information. Production results in terms of several crops will be considered in this game, together with the necessary parameters to implement different linear programming problems. In the game with perfect information with the hierarchical structure established between the four considered players (a management center and three production units), a Nash equilibrium point is reached, since once the strategies of the rest of the players are known, if any of them would use a strategy different to the one proposed, their earnings would be less than the ones obtained by using the proposed strategies. When the four linear programming problems are solved, a particular case of equilibrium point is reached.

Key words: agronomy, economics, linear programming methods, production efficiency

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The goal of the present paper is demonstrating the possibility of obtaining an equilibrium point, as proposed by Nash (Luce and Raiffa 1957), in the case of an agricultural company that is considered together with its three sub-units in developing a game with perfect information. Recently, other authors have treated similar problems of improving production efficiency in agricultural environments by using a stochastic parametric approach (Sojková et al. 2008), methods based neural networks (Trenz et al. 2011) or decision making theory (Beranová and Martinovičová 2010). Only production results in terms of several crops will be considered in this game. We will analyze the following situation, in which for simplicity, only three participants are considered. A management center A0 distributes the available resources among two production sub-units under its supervision. Let us denote these units B1 and B2. Let b be the resources vector of A0; u and v are vectors of resources received by B1 and B2. Vectors b, u and v have same dimensions. The sub-units B1

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and B2 use the assigned resources to produce goods (or services). Vectors X and Y represent the product quantities elaborated by B1 and B2. Incomes of A0, B1 and B2 depend on the production amount obtained by B1 and B2.

This situation can be considered a game in which there are three players. It can be represented by a tree, a game with perfect information (Mc Kinsey 1952; Owen 1958), whose vertex includes all possible alternatives of player A0. As long as this game is based on perfect information, it can be said that at least one equilibrium situation exists in the class of pure strategies (Luce and Raiffa 1957; Szep and Forgo 1995).

MATERIAL AND METHODS

Game strategies

The set of all possible alternatives of player A0 has the following form:

$$\{(u, v)/u + v \leq b;\ v \geq 0, u \geq 0, b \geq 0\} \quad (1)$$

The set of all possible alternatives of player B1 is given by all the possible function-vectors of parameter **u**, that is:

$$X(u) = \{x/xA \leq u + \alpha, u \geq 0, \alpha \geq 0, A \geq 0\} \quad (2)$$

where a is a constant vector and A is a known matrix. Vector a can be considered as some own resources of that unit.

The set of alternatives of player B2 is analogous to the previously defined and has the following form:

$$Y(v) = \{y/yB \leq v + \beta, v \geq 0, \beta \geq 0, B \geq 0\} \quad (3)$$

where b is a vector with analogous characteristics to a.

Earning functions

The earnings of A0 are determined by the function:

$$Z_0[X(u), Y(v), (u, v)] = a_1X(u) + a_2Y(v) \quad (4)$$

where $a_1 \geq 0$, $a_2 \geq 0$ and they both have the same dimensions as vectors X(u) and Y(v).

Expressions a_1X and a_2Y are scalar products. Vectors a_1 y a_2 are interpreted as the earning of A0 per unit produced in B1 and B2 respectively.

Earnings of player B1 are defined by:

$$Z_1[X(u), Y(v), (u, v)] = Z_1'[X(u), u] = c_1X(u) \quad (5)$$

where $c_1 \geq 0$ and c_1X is a scalar product, c_1 is interpreted as the earning obtained by B1 per produced unit.

Earnings of B2 are defined by:

$$Z_2[X(u), Y(v), (u, v)] = Z_2'[Y(v), v] = c_2Y(v) \quad (6)$$

where $c_2 \geq 0$ and it is interpreted as the earning obtained by B2 per produced unit.

Nash equilibrium (Luce and Raiffa 1957; Imbert Tamayo and Petrosian 1980):

Player B1 must calculate the set of optimal vectors $X^*(u)$ solutions of the linear parametric programming problem (Gould et al. 1993; Hillier and Lieberman 2001):

$$\max c_1X = c_1X^*(u) \quad (7)$$
$$\text{with } XA \leq u + a;\ X \geq 0;\ a \geq 0$$

where u is a parameter vector and the vector a represents resources that do not depend on A0.

Player B2 must calculate the set of optimal vectors $Y^*(v)$ solutions of the linear parametric programming problem:

$$\max c_2Y = c_2Y^*(v) \quad (8)$$
$$\text{with } YB \leq v + b;\ v \geq 0;\ b \geq 0$$

Here v is a parameter vector and b represents resources owned by B2 that do not depend on A0.

For player A0, the optimal alternative to choose is the solution to the following optimization problem:

$$\max_{u,v} a_1X^*(u) + a_2Y^*(v) \quad (9)$$
$$\text{with } u + v \leq b;\ u \geq 0;\ v \geq 0$$

Solving problem (9) gives a pair of vectors (u^*, v^*), that determine the corresponding optimum production plans from the viewpoint of A0.

If A0, B1 and B2 make their selection in an optim al manner the following set is obtained:

$$L = \{X^*(u), Y^*(v), (u^*, v^*)\} \quad (10)$$

In Imbert Tamayo and Petrosian (1980), it is demonstrated that the solution given in L is a Nash equilibrium solution. It is interesting to recall that the Nash equilibrium can be interpreted as *a set of expectations relative to the selection of each one of the players in a way that, when the selection of the rest of the players is revealed, none of them would want to change his own selection.*

This indicates that once the successive decisions conforming L is established, if one of the players takes a decision that is non-congruent with this set, he/she will obtain earnings equal or less than those guaranteed by playing according to L.

The case of the Coffee and Cocoa Company of Bayate, El Salvador. Guantánamo Province (Betancourt Valdés 2010)

Theoretically, the previous approach is in practice applicable to any real company in which all possible decisions to be taken by its management and the management of the production units follow the approach given by Equations (1)–(10). It can be generalized to any number of participants.

As it is stated at the beginning of this work, the existence of a Nash equilibrium point is demonstrated in the following for a particular company and how it can be reached. In this case, it is the Coffee and Cocoa Company of Bayate, in the town of El Salvador, the Guantánamo province. This company manages a set of the Cooperative Production Basic Units (CPBUs).

These CPBUs, on top of producing coffee, use a part of their productive land to cultivate a set of products for sale as the auto-consumption of the employees and for market sale. The amount of products to be sold to the employees is normalized, and therefore it constitutes a minimum limit for the land to be planted of each crop. An amount of land is committed to this purpose and it can be planted twice a year: in spring and winter, except for yucca that has a development period of more than six months. In the following, only these crops will be considered, not the coffee crop.

Essentially, the present work is focused on defining which one is the best policy among: (a) each unit acts on its own without considering any possible benefits to obtain collectively; or (b) that they coordinate their efforts as a whole.

If the problem of optimal distribution of the company's available resources is solved and each one of the production units uses optimally the received resources, a Nash equilibrium can be reached, since the benefits received by each one of the four players will be equal or higher to the maximum quantities if the CPBUs would have acted in a different way.

To demonstrate this, initially a Linear Programming (LP) problem will be solved covering all three CPBUs. Then, a Linear Programming problem will be solved for each CPBU separately. The results will be compared to demonstrate that in the first case, a Nash equilibrium situation will hold.

Model construction

Sub-indexes

i: indicates the product type which in this case are $i = 1$, indicates yucca; $i = 2$, sweet potato; $i = 3$, black bean; $i = 4$, red bean; $i = 5$, dry corn; $i = 6$, tender corn; $i = 7$, cucumber; $i = 8$, rice; $i = 9$, pumpkin;

j: indicates the sale destination: $j = 1$, market sale; $j = 2$, sale for auto-consumption.

k: indicates if the product is collected in spring or winter ($k = 1$, spring; $k = 2$, winter).

In summary, sub-indexes respectively indicate: *i* the product planted, *j* the production destination and *k* the production season.

Essential variables

Using a vector-based model, variables X, Y and Z can be defined with vectors representing the quantity of land devoted in each production unit to crop *i*, for destination *j* and in season *k*.

For the CPBU Gabriel Lamot (Dajao), variables have the general form: X_{ijk}. For the CPBU Camilo Cienfuegos (Melián),variables have the general form Y_{ijk}; and for the CPBU Máximo Gómez (Baltazar, they have the form Z_{ijk}, where each time $i = 1, 2, \ldots, 9$; $j = 1, 2$, and $k = 1, 2$. The example for vector X is given as Equations 11.

For example, X_{111} will represent the area (in hectares) dedicated to the yucca crop for market sale in the spring season, and equivalently X_{232} will represent the area devoted to sweet potato s for the auto-consumption in the winter season. The same scheme holds for the other essential variables and in the following for the rest of the parameters of the model.

Parameters

C: cost vector whose elements C_{ijk} represent the cost per hectare of crop *i* for sale in destination *j* planted in the *k* season, where possible index values are $i = 1, 2, \ldots, 9$; $j = 1, 2$ and $k = 1, 2$ (Equations 12).

G: earnings vector whose elements G_{ijk} represent the gross earnings per hectare for product *i* with sale destination *j* and cultivated in season *k*; $i = 1, 2, \ldots, 9$; $j = 1, 2$; $k = 1, 2$ (Equations 13).

R: yield matrix whose elements r_{ijk}: yield per hectare of crop *i* ($i = 1, 2, \ldots, 9$) to be sold externally or to employees ($j = 1, 2$) planted in spring or winter ($k = 1, 2$) (Equations 14).

$$[X] = \begin{bmatrix} X_{111} & X_{211} & X_{311} & X_{411} & X_{511} & X_{611} & X_{711} & X_{811} & X_{911} \ldots \\ X_{121} & X_{221} & X_{321} & X_{421} & X_{521} & X_{621} & X_{721} & X_{821} & X_{921} \ldots \\ X_{112} & X_{212} & X_{312} & X_{412} & X_{512} & X_{612} & X_{712} & X_{812} & X_{912} \ldots \\ X_{122} & X_{222} & X_{322} & X_{422} & X_{522} & X_{622} & X_{722} & X_{822} & X_{922} \end{bmatrix} \tag{11}$$

$$[C]=\begin{bmatrix} C_{111} & C_{211} & C_{311} & C_{411} & C_{511} & C_{611} & C_{711} & C_{811} & C_{911}\cdots \\ C_{121} & C_{221} & C_{321} & C_{421} & C_{521} & C_{621} & C_{721} & C_{821} & C_{921}\cdots \\ C_{112} & C_{212} & C_{312} & C_{412} & C_{512} & C_{612} & C_{712} & C_{812} & C_{912}\cdots \\ C_{122} & C_{222} & C_{322} & C_{422} & C_{522} & C_{622} & C_{722} & C_{822} & C_{922} \end{bmatrix} \quad (12)$$

$$[G]=\begin{bmatrix} G_{111} & G_{211} & G_{311} & G_{411} & G_{511} & G_{611} & G_{711} & G_{811} & G_{911}\cdots \\ G_{121} & G_{221} & G_{321} & G_{421} & G_{521} & G_{621} & G_{721} & G_{821} & G_{921}\cdots \\ G_{112} & G_{212} & G_{312} & G_{412} & G_{512} & G_{612} & G_{712} & G_{812} & G_{912}\cdots \\ G_{122} & G_{222} & G_{322} & G_{422} & G_{522} & G_{622} & G_{722} & G_{822} & G_{922} \end{bmatrix} \quad (13)$$

$$[R]=\begin{bmatrix} r_{111} & r_{211} & r_{311} & r_{411} & r_{511} & r_{611} & r_{711} & r_{811} & r_{911}\cdots \\ r_{121} & r_{221} & r_{321} & r_{421} & r_{521} & r_{621} & r_{721} & r_{821} & r_{921}\cdots \\ r_{112} & r_{212} & r_{312} & r_{412} & r_{512} & r_{612} & r_{712} & r_{812} & r_{912}\cdots \\ r_{122} & r_{222} & r_{322} & r_{422} & r_{522} & r_{622} & r_{722} & r_{822} & r_{922} \end{bmatrix} \quad (14)$$

$$[T]=[T_1 \quad T_2]=[T_{1,Da} \quad T_{1,Me} \quad T_{1,Ba} \quad T_{2,Da} \quad T_{2,Me} \quad T_{2,Ba}] \quad (15)$$

$$[P_{CPBU}]=[P_{Da}P_{Me}P_{Ba}]=\begin{bmatrix} P_{1Da} & P_{2Da} & P_{3Da} & P_{4Da} & P_{5Da} & P_{6Da} & P_{7Da} & P_{8Da} & P_{9Da}\cdots \\ P_{1Me} & P_{2Me} & P_{3Me} & P_{4Me} & P_{5Me} & P_{6Me} & P_{7Me} & P_{8Me} & P_{9Me}\cdots \\ P_{1Ba} & P_{2Ba} & P_{3Ba} & P_{4Ba} & P_{5Ba} & P_{6Ba} & P_{7Ba} & P_{8Ba} & P_{9Ba} \end{bmatrix} \quad (16)$$

T: available land vector whose elements T represent the available land (hectares) of each CPBU according to the season: $k = 1$ spring; $k = 2$, winter, respectively (Equations 15).

P_{CPBU}: minimum production vector (in tons) for each crop product and CPBU ($i = 1, 2, \dots, 9$ and CPBUs of Dajao (Da), Melián (Me) and Baltazar (Ba)) – (Equations 16).

B: available budget to perform planting and harvest ($).

The general model has the following matrix form:

Restriction approach:

(a) *On land availability:* where sum of all areas planted with the different crops (each of type *i*) in the different locations (CPBUs) must be equal to the total available land.

$$sum[X_k] + sum[Y_k] + sum[Z_k] = [T_k] \quad k = 1, 2 \quad (17)$$

where *sum*[.] stands for the sum of all elements of the respective vector.

(b) *On budget*: where sum of all production costs has to be lower than the actual budget B, and in terms of scalar product of the different vectors.

$$[C].[X]^T + [C].[Y]^T + [C].[Z]^T \leq B \quad (18)$$

(c) *On minimal production:* where production of each crop i at each CPBU has to comply with the minimum required production for each crop element of the minimum production vector P_{CPBU}.

$$[R].[X]^T \geq P_{Da}$$
$$[R].[Y]^T \geq P_{Me} \quad (19)$$
$$[R].[Z]^T \geq P_{Ba}$$

Target function: where cumulative earnings of all CPBUs are maximized:

$$\text{Max } Z = [G][X]^T + [G][Y]^T + [G][Z]^T \quad (20)$$

Information processing

Parameters of the cost vector C (C_{ijk}) are calculated from the cost per hectare files for each one of the crops generated by the Company (in units 10^{-3}\$/ha). This procedure integrates the fact that the employed technology and the agricultural activities performed on each crop variety are identical for all CPBUs managed by the Company (Equations 21).

Parameters of the earnings vector G (G_{ijk}, earnings per hectare) are determined, in the case of products to be sold to the employees, from prices fixed by the Minagri (the Cuban Ministry of Agriculture). In the case of products to be sold to the Acopio (National

$$[C] = \begin{bmatrix} 3.65 & 3.98 & 4.09 & 4.09 & 3.82 & 3.82 & 3.82 & 3.81 & 3.10\ldots \\ 3.10 & 3.00 & 3.00 & 3.35 & 3.35 & 2.29 & 2.29 & 2.87 & 2.87\ldots \\ 3.65 & 3.98 & 3.82 & 4.09 & 4.09 & 3.81 & 3.81 & 3.81 & 3.04\ldots \\ 3.10 & 3.00 & 3.00 & 3.35 & 3.35 & 2.29 & 2.29 & 2.87 & 2.87 \end{bmatrix} \quad (21)$$

$$[G] = \begin{bmatrix} 4.13 & 2.70 & 2.01 & 1.41 & 2.08 & 1.41 & 4.16 & 17.77 & 8.71\ldots \\ 4.10 & 2.74 & 2.01 & 1.41 & 2.09 & 1.42 & 4.17 & 20.82 & 8.72\ldots \\ 4.23 & 2.80 & 2.11 & 1.41 & 2.38 & 1.51 & 4.36 & 17.77 & 8.80\ldots \\ 4.50 & 2.84 & 2.31 & 1.41 & 2.49 & 1.62 & 4.48 & 21.86 & 8.83 \end{bmatrix} \quad (22)$$

$$[R] = \begin{bmatrix} 16.11 & 26.03 & 1.21 & 1.40 & 8.31 & 10.22 & 4.10 & 6.45 & 2.22\ldots \\ 13.22 & 19.10 & 0.90 & 0.96 & 5.19 & 7.17 & 3.22 & 3.70 & 1.82\ldots \\ 16.11 & 26.03 & 1.21 & 1.40 & 8.31 & 10.22 & 4.10 & 6.45 & 2.22\ldots \\ 13.22 & 19.10 & 0.90 & 0.96 & 5.19 & 7.17 & 3.22 & 3.70 & 1.82 \end{bmatrix} \quad (23)$$

$$[P_{CPBU}] = [P_{Da} P_{Me} P_{Ba}] = \begin{bmatrix} 0.5 & 0.7 & 0.19 & 0.19 & 0.1 & 0.1 & 0.5 & 0.5 & 0.133\ldots \\ 0.65 & 0.7 & 0.22 & 0.22 & 0.32 & 0.32 & 0.65 & 0.6 & 0.114\ldots \\ 0.7 & 0.8 & 0.32 & 0.32 & 0.17 & 0.17 & 0.7 & 0.75 & 0.2 \end{bmatrix} \quad (24)$$

Distribution Company of Agricultural and Livestock Products), it is based on the official price list fixed by the Guantánamo Province Government. From these prices, the earning per hectare is obtained in units 10^{-4}\$/ha and, subtracting the cost per hectare, the benefit is obtained (Equations 22).

Parameters of the yield vector R (r_{ijk}) are obtained from the statistical files from the last years for the considered products (Equations 23).

Parameters of the minimal areas vector P_{CPBU} devoted to each type of crop in each CPBU are given by Equations 24.

Parameter B is the budget as planned by the Company, whose total value is \$ 60 000, which is then broke down per CPBU. Hence, the budget of the CPBU of Dajao is \$ 16 235, the budget of the CPBU of Melián is \$ 19 765, and for the Baltazar, it is \$ 24 000.

Parameters of available land vector T ($T_{k,CPBU}$) are taken as:

$$[T] = [2.3 \quad 2.8 \quad 3.4 \quad 2.3 \quad 2.8 \quad 3.4] \quad (25)$$

The rest of parameters are self-explanatory.

RESULTS AND DISCUSSION

By solving the problem, the total benefit that could be obtained by the company (or A0 player) is \$ 2 394 077.0. Breaking out this global result in the values obtained for the different essential variables and crops in the different seasons, some conclusions can be extracted for each CPBU. For the CPBU Dajao, the obtained results recommend satisfying the auto-consumption needs and dedicating to the market sale of the red bean crop 0.725 the hectares in spring and 0.465 hectares in winter. For the CPBU Melián, it is recommended that, on top of satisfying the employees' needs, it should be devoting to market sale 1.15 hectares of dry corn in spring and 0.26 hectares in winter. For the CPBU Baltazar, it is obtained that 1.15 and 2.4 hectares of red bean for market sale should be planted, in addition to the necessary areas to satisfy the auto-consumption. The remaining budget of \$ 8119.57 would not be spent in the crop planting and harvesting.

For each one of the CPBUs, Linear Programming models can be easily set considering only the parameters and variables of every one of them, and having as a target the maximization of their individual benefit. By solving these problems, where only the conditions of each CPBU are considered, it can be found that:

– Maximum benefit obtained by the CPBU of Dajao is \$ 579 833.80. Also, it will not be necessary to use \$ 1836.97 of the available budget. This corresponds to the following results for the essential variable (X_{ijk}) as displayed in Table 1.
– Maximum benefit obtained by the CPBU of Melián is \$ 696 824.30. Also, it will not be necessary to

Table 1. Results for the Linear Programming model for the CPBU of Dajao in terms of the essential variable

Season (*k*)	1 – spring		2 – winter	
Destination (*j*)	1 – external	2 – employees	1 – external	2 – employees
Crop product (*i*)				
1 – yucca				0.5
2 – sweet potato		0.6		0.1
3 – black bean				0.375
4 – red bean			1.192	
5 – dry corn				
6 – tender corn				
7 – cucumber				0.133
8 – rice		0.5		
9 – pumpkin		0.2		

use $ 3558.41 of the budget. Values for the auto-consumption will be similar in this to the ones in the Table 1, and the main remarkable difference with the previous CPBU will be devoting 1.41 hectares to red bean in the winter season.

– Maximum benefit obtained by the CPBU of Baltazar is $ 1 117 419.0. Also, the remaining budget (not spent) is $ 2724.18. On top of the requirements for the auto-consumption, the external market sales would include planting 1.35 and 1.2 hectares of red bean respectively in spring and winter.

Therefore, in the game with perfect information with the hierarchical structure established between the four players, a Nash equilibrium point is reached since once the strategies of the rest of the players are known, if any of them would use a strategy different to the one proposed in Equation (10), their earnings would be less than the ones obtained by using the proposed strategies.

When all four Linear Programming problems are solved, a particular case of equilibrium point is reached. Moreover, the joint benefit obtained if the A0 strategies are mandatory for B1, B2 and B3 and they also act optimally, is equal or higher than the sum of the benefits of the CPBUs operating separately, even if they do operate optimally.

CONCLUSION

Game Theory with Perfect Information has been applied to an agricultural company that is considered together with its three sub-units. Production results in terms of several crops are considered in this game, together with the necessary parameters to implement different linear programming problems. In the game with perfect information with hierarchical structure established between the four considered players (the management center and three production units), a Nash equilibrium point is demonstrated, since once the strategies of the rest of the players are known, if any of them would use a strategy different to the one proposed, their earnings would be less than the ones obtained by using the proposed strategies. The obtained model and its application to the agricultural production company as a whole and to each of its basic units permits not only to select which crops maximize the production results, but also to investigate how the different indicators affect the final results. It is also possible to analyze hypothetic production scenarios to plan for midterm and to adopt strategic decisions.

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