Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2009, Valencia
Evolution and Trends in Design, Analysis and Construction of Shell and Spatial Structures
28 September – 2 October 2009, Universidad Politecnica de Valencia, Spain
Alberto DOMINGO and Carlos LAZARO (eds.)

Optimization of the roundness of the soccer ball

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Abstract
Most soccer balls are made of stitched leather or synthetic panels with a bladder inside. The initial configuration of the balls can be represented by polyhedra in the three dimensional space. For the numerical evaluation of the sphericity of the ball, a method previously proposed by the first author is used in this paper. Following previous studies on the optimizaton of the initial uninflated configuration of the soccer ball, this paper aims to calculate the shape and the sphericity of the inflated soccerball under given internal pressure and material properties. For this purpose two large displacement mechanical models are applied. The most popular configuration, the truncated icosahedron is considered as the initial shape of the ball, and a parametric study is conducted to determine the optimal shape. Our results show that there is significant difference between the optimal shape of the inflated and uninflated shapes.

Keywords: soccer ball, sphericity, truncated icosahedron, optimal shape, dynamic relaxation method

1. Introduction
Soccer is one of the most influential sport games in the world. The soccer ball itself has so far received little scientific attention apart from the economic interest of sports accessories manufacturers. Most balls are made of flat panels sewn together and inflated by means of an inner balloon. FIFA, the top governing body of football imposes strict standards for ball designs in competition games, elaborated in the 2nd law of the game [3]. Designing balls that meet the requirements poses scientific challenge. The basic aim is the construction of

the initial flat configuration which requires the study of polyhedra, bodies formed by planar polygons. The geometric model provided with material properties forms the membrane structure intended to model the inflated ball. It is expected to provide the highest possible game experience, i.e. to be as spherical as possible. Last but not least, the manufacturing of the ball should be feasible at a relatively low cost. The optimal ball design hence has an agreement with various requirements and restrictions. Following previous studies (Geiger and Lengyel [4]) on the morphological and metric analysis of various polyhedra this paper deals with the mechanical modeling of the ball. For this purpose we investigate the truncated icosahedron, the configuration of the most popular ball design and we introduce mechanical models to determine the shape of the inflated balls under given internal pressure. A parametric study is performed to calculate the optimal metric properties of the configuration in terms of various parameters in order to find the best approximation of the sphere.

Chapter 2 gives an overview of the measuring of the sphericity and the optimization of the ball, Chapter 3 describes the methods presented by the authors to calculate the inflated ball shapes, and Chapter 4 presents the numerical results of the parametric study on the truncated icosahedron.

2. Measure of sphericity

The design of early soccer balls in the 19th century was based on a heuristic construction of different panel configurations aimed to get a spherical shape. Since then a large variety of different panel topologies were created. A comprehensive historical overview was given by Tarnai [9], who provided numerous examples and analysed their symmetry properties. Recently a new ball design was introduced by one of the major sports accessories manufacturers, which applied thermal bonding between the synthetic panels instead of stitches.

Polyhedra belonging to different symmetry groups have fascinated mathematicians since antiquity. The five convex regular polyhedra are associated with Plato, the well-known mathematician and philosopher in ancient Greece. The truncation of regular polyhedra by cutting planes can yield so-called semi-regular polyhedra that are composed of regular polygons of two or more kinds. One of these Archimedean solids, the truncated icosahedron gives the shape of the most popular soccer ball.

Sphericity is a measure of the roundness of a body. Wadell [10] defined the sphericity of a particle as the ratio of the surface area of a sphere (with the same volume as the particle) to the surface area of the particle. His definition yields a unit value for the sphere and less for other forms. The sphericity of the five Platonic solids increases with the number of their faces. Mathematicians in recent times usually define a dimensionless quantity, the so-called isoperimetric quotient to describe the sphericity of a convex body as $IQ = A^3/V^2$ where A and V denote the surface area and the volume of the body, respectively. The problem is formulated as follows: find the body with the smallest surface area of all bodies having the same given volume, i.e. the one with the smallest isoperimetric quotient. For polyhedra, L. Lindelöf formulated his famous theorem [7], which states that among all convex polyhedra composed of faces with given orientation the smallest isoperimetric quotient is obtained if all faces are tangent to the inscribed sphere. An inverse formulation for the isoperimetric quotient is also possible (Pólya [8]). For the approximation of the sphere Goethals and Seidel [5] studied the distribution of discrete point sets on a unit sphere and they defined the strength of a point set as an integer number. A detailed geometric analysis on various polyhedra was published by Huybers [6]. He investigated two isodistant truncations of the icosahedron and an isodistant configuration of the snub dodecahedron and the octahedron. The term isodistant refers to equal distances of the faces from the centre. He presented a number of geometric properties focusing on the difference between areas of different panels, on the inscribed circles, and on the total seam length. Similar panel sizes are expected to provide uniform deformations and stresses in the inflated ball, while smaller seam lengths serve manufacturing purposes.

Geiger and Lengyel [4] generalized Goethals and Seidel's formula for arbitrary surfaces in three dimensional space and hence enabling it to deal with continua such as the surface of a soccer ball. Let (ξ,η,ζ) and (x,y,z) denote the Cartesian coordinates of a unit sphere ($\mathbf{\Omega}$) and an arbitrary set (**S**) in 3-space, respectively, so that the coordinate system is centred at the centre of the sphere. Let the moment of order *k* of the set **S** be defined by

$$M^{(a,b,c)}=\frac{1}{|\mathbf{S}|}\int_{\mathbf{S}} x^a y^b z^c ds \tag{1}$$

where $k=a+b+c$ and *a*, *b*, and *c* are non-negative integers. The integration in Eqn (1) is performed on the entire surface. Thus by definition a continuous set **S** has the strength *t* if the moments of the set and those of the unit sphere are equal for all positive integers *k* up to *t*:

$$\frac{1}{|\mathbf{S}|}\int_{\mathbf{S}} x^a y^b z^c ds=\frac{1}{|\mathbf{\Omega}|}\int_{\mathbf{\Omega}} \xi^a \eta^b \zeta^c d\omega \tag{2}$$

Higher *t*-values indicate better approximation of the sphere. Surfaces with equal strengths can be ranked by the deviation of their moment from that of the unit sphere. This approach provides a more detailed comparison of different surfaces. Geiger and Lengyel [4] analysed six different flat polyhedral configurations for soccer ball design and set up a ranking. They calculated the optimal truncations in order to obtain the best approximation of the sphere.

3. Mechanical models of the ball

In this study we investigate the inflated shape of the polyhedral ball configurations and qualify them by their strengths according to the method above. The ball has a complex structure where the main loadbearing component is a synthetic fabric. The contribution of the bladder and the outer layer to the strength and stiffness is small. The synthetic fabric is usually orthotropic with perpendicular fibre directions. The exact modeling of the ball would require the consideration of all layers with their material properties, interaction between layers, and fibre directions. We do not aim to develop the most realistic model of the ball but instead our aim is to investigate the effects of a number of key parameters on the sphericity, such as modulus of elasticity, internal pressure, discretization density and geometric parameters that define the initial configurations. The sphericity of the inflated balls should be compared to that of the initial configuration. For this purpose we apply two iterative numerical procedures for the calculation of large displacements: one is a quasi-static method seeking the minimum of the total potential energy of the elastic structure, and the other one is the dynamic relaxation method often used for tensile structures. For both methods a homogeneous and linearly elastic material model is used. Both methods use linear displacement-strain relationship.

The quasi-static method generates a triangular mesh of the initial shape. The surface is then approximated by the set of flat triangular faces with in-plane forces. The actual position of each node is defined by a vector in a Cartesian coordinate system centred at the geometric centre of the initial uninflated configuration. The deformation of the elements are expressed in terms of the position vectors. The total potential energy of the structure is composed as the sum of the total deformation energy of the elements and the sum of the potential of load on the nodes derived from the distributed pressure on the elements. The equilibrium shape of the structure is associated with the zero value of the first variation of the total potential energy function. No direct exact solution can be obtained for the non-linear equation system, therefore a numerical iterative procedure is required. An initial value is chosen for the variable vector and then each consequitive value is obtained by means of the Newton-Raphson iteration. The iteration goes on until an acceptable small error threshold is reached.

The other method we apply is the implementation of the dynamic relaxation method (Day [2]). The dynamic relaxation method (DRM) is a step-by-step numerical method, which is especially suitable for the solution of nonlinear problems. The idea is to trace the fictitious motion of the structure in time steps Δt, from an initial shape to the equilibrium shape. In this case the initial shape is the polyhedron and the equilibrium shape corresponding to the given pressure is to be determined. Fictitious masses are placed into the joints of the finite element network of triangular elements before the iteration. Then in every step of the iteration the member forces in the elements are calculated on the basis of the current coordinates. The resultant of the member forces and external loads (in this case the internal pressure) are calculated for every joint. These unbalanced nodal forces accelerate the fictitious masses according to Newton's second law. The accelerations, velocities and displacements of the joints are calculated in every iteration step. During the DRM, kinetic damping provides fast and stable convergence. Kinetic damping means that fictitious movement of the joints is stopped every time when a local peak of the total kinetic energy of the structure is detected Cundall [1]. The iteration is stopped and the current shape is accepted as an equilibrium shape when the maximum of the unbalanced nodal forces is smaller than a given limit that depends on the required accuracy.

4. Optimization of the truncated icosahedron

The Platonic solids are polyhedra composed of identical regular polygonal faces and identical angles. The Archimedean solids on the other hands are composed of two or three different kinds of regular polygonal faces. Some of them are obtained from a corresponding regular polyhedron by an appropriate truncation of the edges. Figure 1 shows the icosahedron (20 regular triangles and 12 vertices) and the truncated icosahedron (20 regular hexagons and 12 regular pentagons and 60 vertices). This latter one is the model for the most popular soccer ball design.

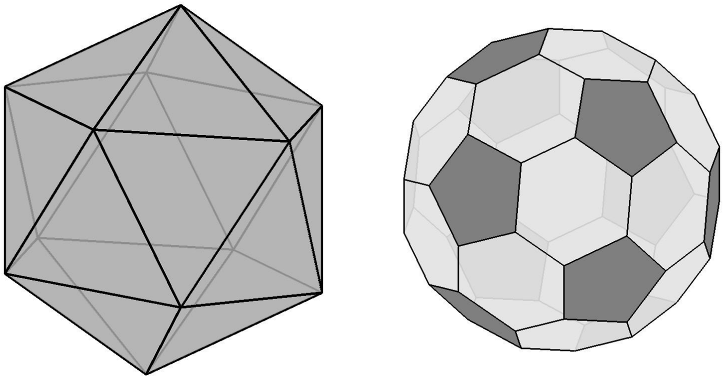

Figure 1: The icosahedron and the truncated icosahedron

Consider cut points at the thirds of all edges on the icosahedron and cut off pentagonal caps at all vertices. Thus regular pentagons are formed and the triangles are turned into regular hexagons. However, if the truncation is performed at a proportion other than one third of the edges, not an Archimedean solid will be obtained as the hexagons will not be regular. The ratio of the cut off edge section to the total edge length is introduced as parameter p that defines the geometry of the generalized truncated icosahedron.

In the vast majority of soccer balls the standard one third truncation is realized, though other truncations can also be found (see Figure 2). It is possible to define the cut points so that all faces of the polyhedron will have the same distant measured from the centre. This configuration proposed by Huybers [6] is called isodistant. It refers to truncation parameter $p = 0.37147...$ However, the first author of this paper (Geiger and Lengyel [4]) proposed to use the moments defined in Eqn (1) to describe the roundness of polyhedra. They found that the truncated icosahedron has the strength 3 and qualified the different truncations of the icosahedron according to their fourth order moments. The fourth order moment of $(a,b,c) = (2,2,0)$ and all its permutations gave identical results as well as $(a,b,c) = (4,0,0)$ and all its permutations, while all other moments were zero. Hence the moments $M^{(2,2,0)}$ and $M^{(4,0,0)}$ were calculated and compared to those of the unit sphere ($M^{(2,2,0)} = 1/15 = 0.066666...$ and $M^{(4,0,0)} = 1/5 = 0.2$). The optimum was reached for both of these moments simoultaneously at cut ratio $p = 0.36507...$ with $M^{(2,2,0)} = 0.06678267...$ and $M^{(4,0,0)} = 0.20034802...$.

The truncation parameter is thus different for the standard soccer ball ($p = 0.33333...$), the isodistant version ($p = 0.37147...$), and the optimal version by Geiger and Lengyel [4]

($p = 0.36507...$). In this study we calculate the inflated shape of truncated icosahedron under internal pressure and perform a parametric study to analyse the effect of the various parameters on the optimum.

Figure 2: Regular and non-regular truncation of the icosahedron

The parameters defining the initial configurations are the radius of the circumscribed sphere, the governing parameter of the mesh density, and the truncation parameter *p*. The parameters required to calculate the inflated shape are the tensile stiffness and the Poisson ratio of the material and the internal pressure.

For the mesh generation we chose a discretization of the initial configuration which preserves the icosahedral symmetry. Each pentagon and hexagon is divided into triangular segments around their centres respectively, and then further divided into smaller triangles by defining *n* equal sections along each edge, as shown in Figure 3 for $n = 4$. FIFA rules require the ball to have a circumference of 68-70cm and internal pressure of 0.6-1.1atm. For our analysis therefore we chose an initial radius of 0.1m.

The internal pressure is acting perpendicularly to the surface elements, and the structure is supported against rigid body motions by a few springs. The spring stiffness is small enough not to affect the results but their presence is required to avoid numeric singularity in the matrix analysis.

The most important geometric parameter is the number *n*, that defines the mesh density. Figure 4 shows moment $M^{(2,2,0)}$ against *n* for different values of the tensile stiffness *S* with Poisson's ratio 0.2 under an internal pressure of $p_{\text{int}} = 100\,\text{kPa}\ (\approx 1\,\text{atm})$ and truncation parameter $p = 0.35$. (We note that the other relevant moment $M^{(4,0,0)}$ has similar characteristics to those of the former one, therefore it is not reproduced here for brevity.)

The values fall with increasing n, then reach an approximately constant plateau at $n = 6$. Hence we use this value for the further calculations.

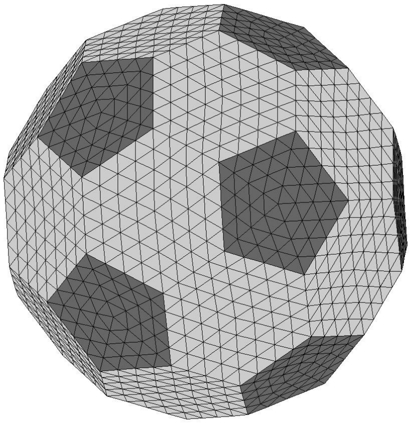

Figure 3: Triangular discretization of the initial surface

Figure 5 shows the moment $M^{(2,2,0)}$ against the internal pressure in the interval $p_{\text{int}} = 0 - 100\,\text{kPa}\ (\approx 0 - 1\,\text{atm})$ for different values of the tensile stiffness at truncation parameter $p = 0.35$. With the increase of the pressure the shape of the ball converges to the perfect sphere thus the moments decrease towards those of the sphere. In the range of the required pressure, the values show little differences. Higher tensile stiffness values produce less spherical shapes at constant pressure. If the truncation parameter is considered a variable, then repeated analysis can yield the optimal value, which is associated with the minimum values of the moments.

Figure 6 shows the optima against the pressure for different mesh density values. The optimum obtained for zero pressure (i.e. the initial configuration) falls rapidly as very small pressure makes the initially flat surface curved. In the range of $p_{\text{int}} = 60 - 100\,\text{kPa}$ the optimum can be regarded constant if the mesh density is not less than 4. Therefore for further calculations we use $p_{\text{int}} = 100\,\text{kPa}$.

The moment can be plotted in the neighbourhood of the optimum against the truncation parameter for different values of the tensile stiffness, see Figure 7. The optimal truncation parameter is indicated as the series of the minimum points. Figure 7 also displays the optimal truncation parameter for the initial uninflated ball configuration for three cases: the regular, the isodistant, and the Geiger-Lengyel truncations. The optimal truncation does not differ much with the tensile stiffness. Figure 8 shows the optimal truncation parameter against the tensile stiffness at $p_{\text{int}} = 100\,\text{kPa}$ and $n = 6$ for both methods we applied in our calculations. It is clear that the methods yield similar optima. Figure 9 shows a typical example for the principal stresses in the structure plotted over the inflated shape.

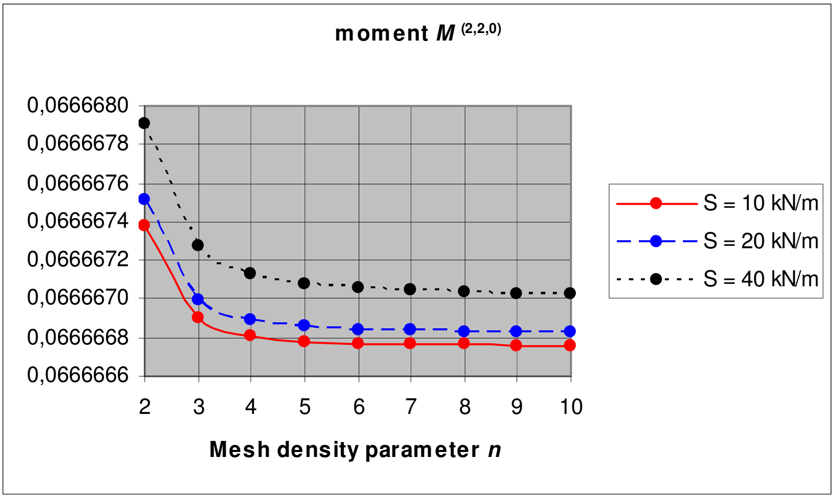

Figure 4: Moment $M^{(2,2,0)}$ against mesh density parameter n

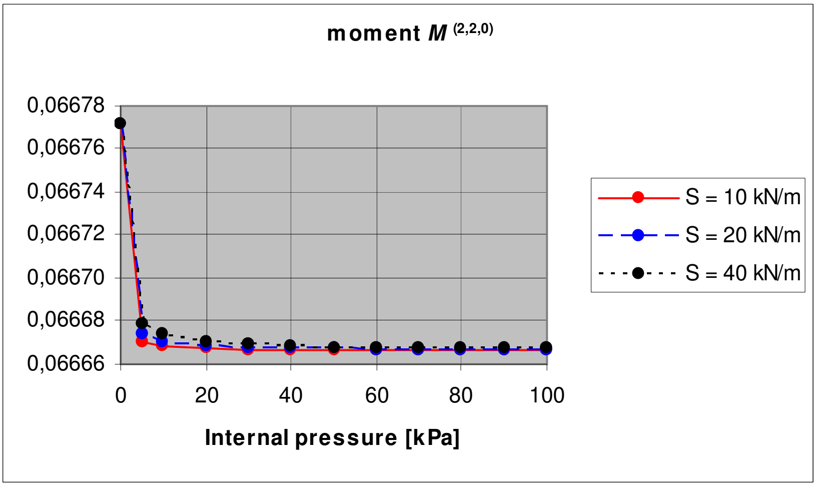

Figure 5: Moment $M^{(2,2,0)}$ against internal pressure

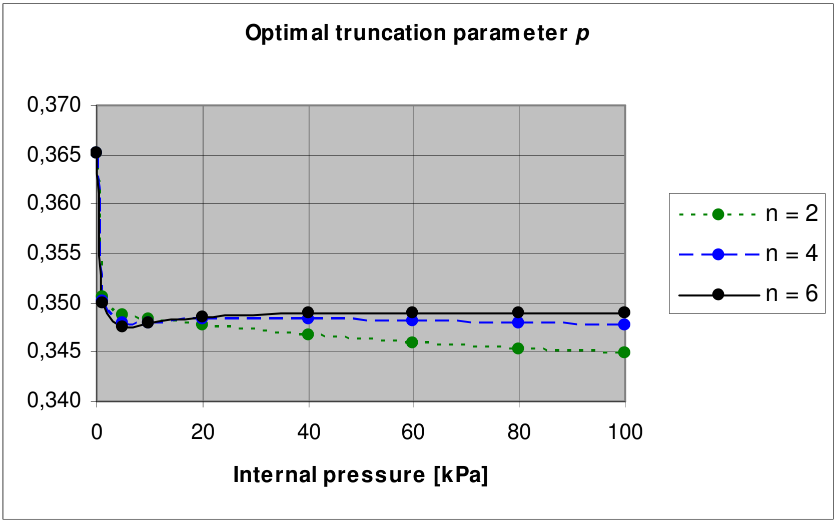

Figure 6: Optimal truncation parameter *p* against internal pressure

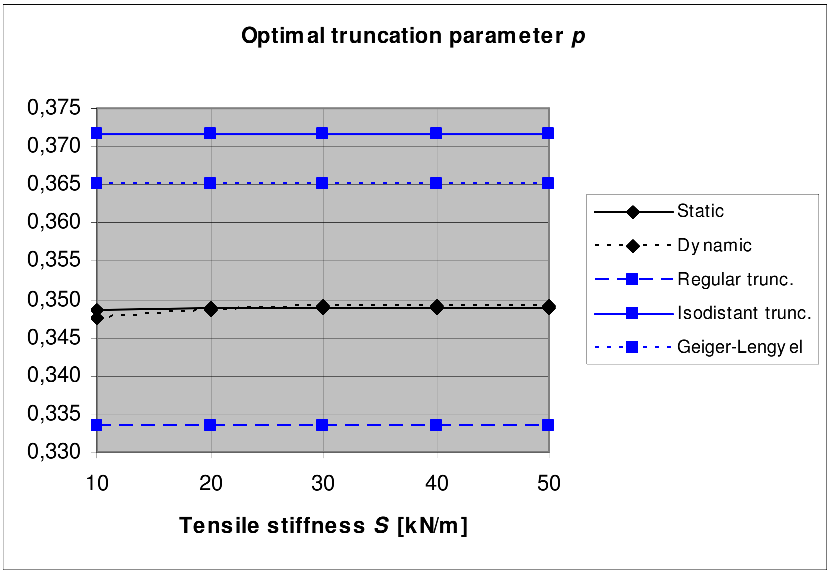

Figure 7: Optimal truncation parameter *p* against tensile stiffness

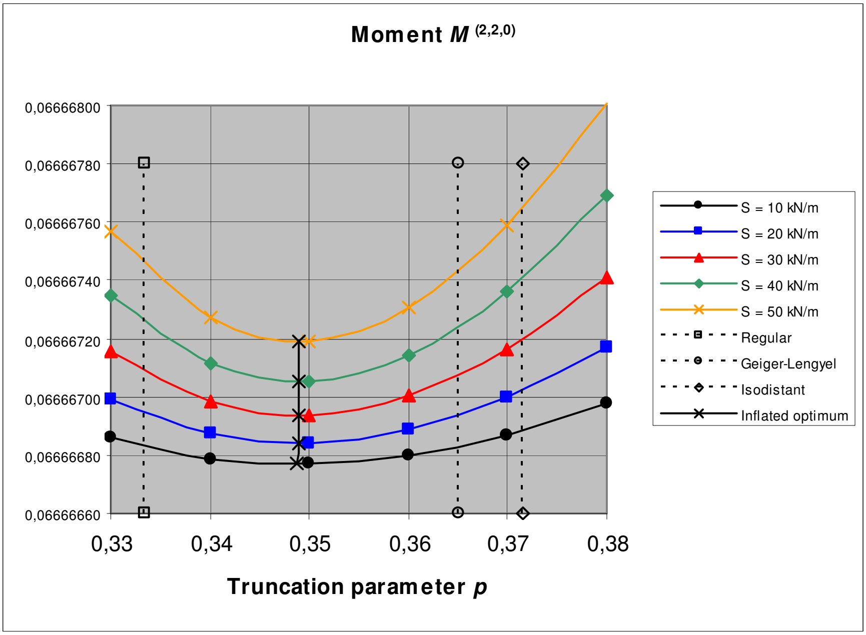

Figure 8: Moment against truncation parameter and optimal truncation parameters

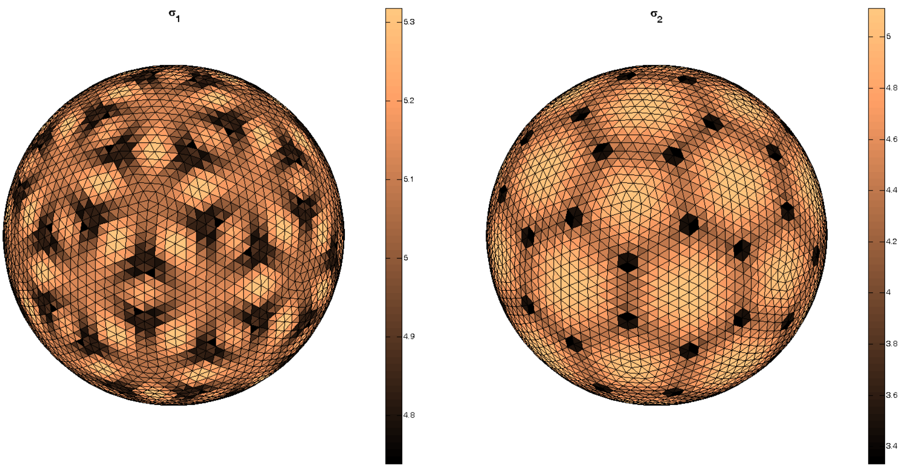

Figure 9: Principal stresses

5. Conclusions

In this study we analysed the roundness of the truncated icosahedron in order to find the optimal cutting pattern for soccer ball design. We applied mechanical models for the calculation of the shape of the inflated ball under given internal pressure and computed quantities to measure the roundness based on previous studies. It was found that the internal pressure has little effect on the optimal cutting pattern in the standard range of pressure. It is also concluded that though the roundness of the ball improves with decreasing tensile stiffness, the optimal cutting pattern is virtually independent. The optimum is significantly different from values previously obtained for the flat configuration: it is between the usual standard truncation and the optimal truncation provided by Geiger and Lengyel. Further studies along this line can be conducted to incorporate more defining factors and advanced methods.

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