Stopping of relativistic projectiles in two-component plasmas

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PACS `52.27.Gr` – Strongly-coupled plasmas
PACS `52.40.Mj` – Particle beam interactions in plasmas
PACS `73.20.Mf` – Collective excitations

Abstract – Relativistic and correlation contributions to the polarizational energy losses of heavy projectiles moving in dense two-component plasmas are analyzed within the method of moments that allows one to reconstruct the Lindhard loss function from its three independently known power frequency moments. The techniques employed result in a thorough separation of the relativistic and correlation corrections to the classical asymptotic form for the polarizational losses obtained by Bethe and Larkin. The above corrections are studied numerically at different values of plasma parameters to show that the relativistic contribution enhances only slightly the corresponding value of the stopping power.

Introduction. – Measuring energy losses of charged particles beams is an important diagnostics tool in both modern condensed matter and plasma physics. In 1930 Bethe [1] derived a simplified formula for the stopping power that neatly describes the energy losses of fast projectiles moving in a solid modeled as a system of quantum-mechanical oscillators. Later, Larkin [2] clearly demonstrated that the following analogous formula remains valid for fast ions permeating an electron gas

$$-\frac{dE}{dx} \underset{v\gg v_F}{\simeq} \left(\frac{Z_p e\omega_p}{v}\right)^2 \ln\frac{2m_e v^2}{\hbar\omega_p}, \tag{1}$$

in which the oscillator frequency was effectively replaced by the plasma frequency $\omega_p = \left(4\pi n_e e^2/m_e\right)^{1/2}$. Here $Z_p e$ and v stand for the electric charge and velocity of the projectile and the electron gas is characterized by the number density n_e with m_e and $-e$ being the electron mass and charge, respectively.

Formula (1) is usually engaged to experimentally determine the number density of electrons in a charged particles target traditionally treated as an electron fluid [3–5]. The X-ray Thomson scattering excepted, this technique remains the only suitable candidate for the diagnostics of hot and dense ($n_e \gtrsim 10^{19}$ cm^{-3}) plasmas [5], see also [6] and references therein.

Further advance was lately made in [7] where it was shown that in a two-component completely ionized hydrogen plasma with a weakly damped Langmuir mode of dispersion $\omega_L(k)$, the plasma frequency in the Coulomb logarithm of (1) should be replaced by the long-wavelength limiting value of $\omega_L(k)$, $\omega_L(0) = \omega_p\sqrt{1+H}$ with $H = h_{ei}(0)/3 = (g_{ei}(0)-1)/3$, $h_{ei}(r)$ and $g_{ei}(r)$ being the electron-ion correlation and radial distribution functions, respectively. It should be noted that the generalization of [7] to partially ionized plasmas or plasmas with complex ions and larger number of species is rather straightforward. At present the described above electron-ion correlation correction to the electron fluid stopping power might not be observable due to a relatively low accuracy of the experimental techniques available, but for dicluster heavy ions projectiles [8] that correlation correction could become more pronounced in order to be experimentally detected.

The problem of stopping power computing for relativistic projectiles has recently arisen thanks to the reported experiments with protons decelerating from velocities of up to 80% of the speed of light [9], see also [10]. The main goal of this letter is to estimate the importance of the relativistic corrections to the classical asymptotic form (1) of the stopping power as compared with the above mentioned electron-ion correlation contribution. In a partially ionized plasma the bound-electron contribution can strictly be taken into account [11] by incorporating the ionization losses, but in the sequel the plasma is considered to be completely ionized. Such an assumption is fully justified because of the high kinetic energy of projectiles and, at the same time, allows one to adopt the polarizational picture to calculate the stopping power of a simple Coulomb fluid.

In 1954 Lindhard [12] expressed the polarizational stopping power in terms of the medium longitudinal dielectric function $\epsilon(k,\omega)$. This expression can further be generalized by applying the Fermi golden rule to obtain [13–16]:

$$-\frac{dE}{dx} = \frac{2\left(Z_p e\right)^2}{\pi v^2}\int\limits_0^\infty \frac{dk}{k}\int\limits_{\alpha_-(k)}^{\alpha_+(k)} \omega n_B(\omega)\left(-\operatorname{Im}\epsilon^{-1}(k,\omega)\right)d\omega. \tag{2}$$

Here $\alpha_\pm(k) = \pm kv + \hbar k^2/2M(v)$, $n_B(\omega) = (1-\exp(-\beta\hbar\omega))^{-1}$, where $\hbar$ denotes the Planck constant, $M(v)$ is the (velocity-dependent) projectile mass and $\beta^{-1} = k_B T$ stands for the plasma temperature T in energy units with k_B being the Boltzmann constant. Two essential physical restrictions are imposed by applying formula (2). First of all, no magnetization effects are taken into account such that the plasma dielectric function depends solely on the wavevector modulus. Secondly, the interaction between the projectile and the plasma medium is treated in a linear approximation. Notice that, e.g., the Z_p^3 Barkas contribution to the stopping power [17] identically vanishes in a fully ionized plasma [18].

In the past the polarizational stopping power was quite extensively studied in the literature. The problem was thoroughly analyzed within the random-phase approximation (RPA) [13,14] and beyond by introducing an analytical formula for the local field correction (LFC) [19], derived within the T-matrix approach [20], the method of effective potentials [21], or using the Mermin or more sophisticated models for the dielectric function [22].

In spite of the fact that the coupling between the projectile and the target plasma is assumed to be treated perturbatively, no further restriction is made on the value of the coupling parameter, $\Gamma = \beta e^2/a$ with $a = (4\pi n_e/3)^{-1/3}$ being the Wigner-Seitz radius. The only limitation left is that the plasma must remain in the liquid-like phase, although, the modeling of its dielectric properties remains rather a sophisticated problem, since its characteristic lengths, i.e., the Wigner-Seitz radius and the Debye length, $\lambda_D = (4\pi n_e e^2\beta)^{-1/2}$, are estimated to be of the same order of magnitude. Note that in a non-ideal plasma of interest herein, $\Gamma = a^2/3\lambda_D^2 \gtrsim 1$, which invalidates mean field theories, such as the RPA and other analogous perturbative approaches and, at the same time, requires the electronic subsystem to be considered as partly degenerated.

The background. – All further dielectric formalism is based on the classical method of moments [23,24], which allows to express the dielectric function $\epsilon(k,\omega)$ in terms of the already known convergent frequency moments or sum rules. The sum rules to be employed are actually the power frequency moments of the positive loss function (LF)

$$\mathcal{L}(k,\omega) = -\omega^{-1}\ \mathrm{Im}\,\epsilon^{-1}(k,\omega),$$

defined as

$$C_\nu(k) = \pi^{-1}\int_{-\infty}^{\infty}\omega^\nu \mathcal{L}(k,\omega)\,d\omega, \quad \nu = 0,1,\ldots.$$

Due to the parity of the LF, all odd-order frequency moments vanish whereas the even-order frequency moments are still determined by the static characteristics of the system which, after routine but straightforward calculations, take the following form [23–25]:

$$C_0(k) = (1-\epsilon^{-1}(k,0)), \qquad C_2(k) = \omega_p^2, \tag{3}$$

$$C_4(k) = \omega_p^4(1+K(k)+U(k)+H), \tag{4}$$

with

$$K(k) = \left(\left\langle v_e^2\right\rangle k^2 + \hbar^2 k^4/\left(2m_e\right)^2\right)/\omega_p^2, \tag{5}$$

and

$$\left\langle v_e^2\right\rangle = \frac{3F_{3/2}(\eta)}{m_e\beta D^{3/2}},$$

being the average squared characteristic velocity of the plasma electrons. Here the following notations are introduced:

$$F_\nu = \int_0^\infty \frac{x^\nu dx}{\exp(x-\eta)+1},$$

$$D = \beta E_F = \beta m_e v_F^2/2 = \beta\hbar^2 k_F^2/2m_e = \beta\hbar^2\left(3\pi^2 n_e\right)^{2/3}/2m_e, \tag{6}$$

where $F_\nu(\eta)$, E_F, v_F, and k_F are the ν-th order Fermi integral, Fermi energy, velocity, and wavenumber, respectively, and the dimensionless chemical potential $\eta = \beta\mu$ is defined by the normalization equation

$$F_{1/2}(\eta) = \frac{2}{3}D^{3/2},$$

D being the degeneracy parameter.

The last two terms in the fourth moment (4) stem from the interaction contribution to the system Hamiltonian and can be expressed in terms of the partial structure factors $S_{ab}(k)$, $a,b=e,i$:

$$U(k) = \left(2\pi^2 n_e\right)^{-1}\int_0^\infty p^2\left(S_{ee}(p)-1\right)f(p,k)\,dp,$$

$$H = \left(6\pi^2 n_e\right)^{-1}\int_0^\infty p^2 S_{ei}(p)\,dp, \tag{7}$$

where

$$f(p,k) = \frac{5}{12} - \frac{p^2}{4k^2} + \frac{(k^2-p^2)^2}{8pk^3}\ln\left|\frac{p+k}{p-k}\right|.$$

It is worthwhile mentioning that the second moment in (3) is exactly the f-sum rule, i.e. $C_2(k) = \omega_p^2$. Notice that the above results for the moments are exact and these expressions are applicable in the whole range of variation of both coupling, Γ, and degeneracy, D, parameters as long as the system is in a liquid state and is not relativistic. The only simplification admitted in (**??**) is that the terms of the order of m_e/m_i are omitted in these expressions, m_i being the average mass of plasma ions.

The static structure factors $S_{ab}(k)$ of various species $a,b=e,i$ can independently be computed, e.g., from the Ornstein-Zernike equation in the hypernetted-chain approximation

[26] which enables one to calculate the power frequency moments (3) and (4). At this point it is the strength of the theory of moments that allows one to restore the dynamical behavior of the dielectric function from such scarce knowledge of the system. Namely, the Nevanlinna formula of the theory of moments provides the following form of the dielectric function to satisfy the known sum rules $\{C_{2\nu}\}_{\nu=0}^{2}$ [23,24]:

$$\epsilon^{-1}(k,z) = 1 + \frac{\omega_p^2(z+q)}{z(z^2-\omega_2^2)+q(k,z)(z^2-\omega_1^2)}, \quad \mathrm{Im}\ z > 0, \tag{8}$$

where

$$\omega_1^2 = \omega_1^2\,(k) = C_2(k)/C_0(k), \qquad \omega_2^2 = \omega_2^2\,(k) = C_4(k)/C_2(k).$$

Here the function $q(k,z)$ is introduced to be analytic, to have the positive imaginary part and to fit the limit $q(k,z)/z \to 0$ as $z \to \infty$, all uniformly in the complex upper half-plane $\mathrm{Im}\ z > 0$. Under these conditions the LF in (8) automatically satisfies the sum rules in (3), (4) by construction. By definition,

$$\epsilon^{-1}(k,\omega) = \lim_{\gamma\downarrow 0} \epsilon^{-1}(k,\omega+i\gamma).$$

Despite of its pure mathematical nature, the Nevanlinna function $q(k,z)$ plays the role of the dynamic LFC $G\,(k,\omega)$ in an electron liquid. In particular, the Ichimaru visco-elastic model [27] expression for $G\,(k,\omega)$ is equivalent to the Nevanlinna function approximated as i/τ_m with τ_m being the effective relaxation time [28].

In two-component dense plasmas the Nevanlinna parameter function stands for the species' dynamic LFCs but, in general, there is no immediate phenomenological information at hand to determine $q(k,z)$ [29]. Nevertheless, the Perel' - Eliashberg (PE) [30] exact expression for the high-frequency asymptotic form of the imaginary part of the dielectric function [23],

$$\mathrm{Im}\,\epsilon\left(k,\omega \gg (\beta\hbar)^{-1}\right) \simeq A\,(\omega_p/\omega)^{9/2}\,, \tag{9}$$

can be employed to yield [31]:

$$\begin{aligned} q\,(k,z) &= \frac{A\sqrt{\omega_p^5 z}\,(1+i)}{\omega_2^2\,(k)-\omega_1^2\,(k)} + i\frac{\omega_2^2\,(k)-\omega_1^2\,(k)}{\nu\,(0)}, \\ A &= \left(\frac{16\pi}{3}\right)^{5/4} \pi Z r_s^{3/2}, \end{aligned}$$

where $r_s = a m_e e^2/\hbar^2$ is the Brueckner parameter and $\nu\,(0)$ represents the transport collision frequency determined by the plasma static conductivity

$$\sigma_0 = \lim_{\omega\to 0} \frac{\omega}{4\pi i}\left(\frac{1}{\epsilon^{-1}\,(k=0,\omega)} - 1\right) = \frac{\omega_p^2}{4\pi\nu\,(0)}. \tag{10}$$

It should be clearly emphasized that the application of the asymptotic form (9) for a Coulombic system inevitably implies that higher even-order frequency moments $C_{2l}\,(k)\,,\ l \geq 3$ diverge.

The Nevanlinna formula which leads to (8) gives the continuous, so called non-canonical, solution of the Hamburger moment problem [32] corresponding to the set of moments $\{C_{2\nu}\}_{\nu=0}^{2}$. In what follows we will also employ the canonical solution [33] which effectively corresponds to the choice of the Nevanlinna function $q(k,\omega) = i0^+$:

$$\frac{\mathcal{L}\,(k,\omega)}{\pi C_0\,(k)} = \frac{\omega_2^2-\omega_1^2}{\omega_2^2}\delta\,(\omega) + \frac{\omega_1^2}{2\omega_2^2}\left(\delta\,(\omega-\omega_2)+\delta\,(\omega+\omega_2)\right), \tag{11}$$

where δ stands for the Dirac delta function. Physically, equation (11) describes an undamped collective excitation mode located at

$$\omega_2(k) = \omega_p^2(1 + K(k) + U(k) + H) \tag{12}$$

with an additional central peak corresponding to the hydrodynamic diffusional process [34]. Due to the presence of the $\sim k^4$ contribution in (5),

$$\omega_p^2 K(k) = \langle v_e^2 \rangle k^2 + \hbar^2 k^4 / 4m_e^2,$$

the canonical solution (11) describes not only the undamped Langmuir collective mode channel of energy transfer in collisional plasmas, but the one-particle excitations [35] and diffusional processes as well, i.e., the full range of energy loss channels.

The corrected Bethe-Larkin formula. – Our goal here is to obtain an analytic expression for the plasma stopping power asymptotic form with respect to relativistic ions. But first let us see how the Bethe-Larkin asymptotic form can be easily obtained within the above formalism. To this end, let us omit the correlation contributions, $U(k)$ and H to the fourth moment, i.e., to

$$\omega_2(k) = \left(\omega_p^2(1 + U(k) + H) + \langle v_e^2 \rangle k^2 + \frac{\hbar^2 k^4}{4m_e^2}\right)^{1/2}, \tag{13}$$

and substitute the resulting canonical solution of the moment problem (11) or

$$\omega\left(-\operatorname{Im}\epsilon^{-1}(k,\omega)\right) = \pi C_0(k)\frac{\omega_1^2}{2}\left(\delta(\omega - \omega_2) + \delta(\omega + \omega_2)\right), \tag{14}$$

into the Lindhard formula (2) and take into account that

$$n_B(\omega) + n_B(-\omega) = 1.$$

Then one gets:

$$-\frac{dE}{dx} \underset{v \to c}{\simeq} \frac{(Z_p e \omega_p)^2}{v^2} \ln\frac{k_2}{k_1}, \tag{15}$$

where the inverse wavenumbers k_1^{-1} and k_2^{-1} are not just "cut-off" characteristic lengths, but the values stemming from the inequalities

$$\alpha_-(k) \le -\omega_2(k) \le \omega_2(k) \le \alpha_+(k) \tag{16}$$

for the longest and shortest wavelengths possible, c is, certainly the speed of light. Precisely, for small k and large v we get from (16):

$$-kv \le -\omega_p \le \omega_p \le kv,$$

i.e., $k_1 = \omega_p/v$. For large k and large v we arrive to the inequality

$$\frac{\hbar k^2}{2m_e} \le kv + \frac{\hbar k^2}{2M(v)},$$

where, for heavy projectiles, we can neglect the contribution

$$\frac{\hbar k^2}{2M(v)} \ll \frac{\hbar k^2}{2m_e}$$

and thus obtain $k_2 = 2m_e v/\hbar$. Hence, the Bethe-Larkin result [1,2] is recovered in this way.

To analytically take into account Coulomb and exchange interactions in the system the following long- and short-range asymptotic forms for the electron-electron contribution $U(k)$ can be used:

$$U(k \to 0) \simeq \frac{E_{ee} k^2}{15 n_e^2 e^2}, \quad U(k \to \infty) \simeq -h_{ee}(0)/3,$$

where E_{ee} is the plasma electron-electron interaction energy density E_{ee} [16] and $h_{ee}(0) = g_{ee}(0) - 1$. Then the wavenumber k_1 is modified to become $k_1' = \omega_p'/v$ with $\omega_p' = \omega_p\sqrt{1+H}$, so that the fast projectile stopping power reads [7]:

$$-\frac{dE}{dx} \underset{v \to c}{\simeq} \left(\frac{Z_p e \omega_p}{v}\right)^2 \ln \frac{2 m_e v^2}{\hbar\omega_p \sqrt{1+H}}. \tag{17}$$

The correction H defined in (7) is due to the electron-ion correlation contribution to the moment $C_4(k)$ and is responsible for the undamped Langmuir frequency upshift in the long-wavelength limit. It is worthwhile mentioning that a precise calculation of the electron-ion correlation contribution, H, is a difficult problem [36], and, the following simplified analytic expression can be used for completely ionized hydrogen-like plasmas, obtained within a modified random phase approximation [23, 37]:

$$H = \frac{4}{3} Z r_s \sqrt{\Gamma}\left[3Z\Gamma^2 + 4r_s + 4\Gamma\sqrt{3(1+Z)\, r_s}\right]^{-1/2}. \tag{18}$$

It is obvious that in an ideal plasma, $\Gamma \ll 1$, this correction stays negligible, but in a strongly coupled Coulomb system its importance can grow significantly such that it could even turn possible to directly retrieve H (or $g_{ei}(0)$) by measuring the stopping power and, then, using (17). If $g_{ei}(0) = 10$ and $\ln\left(2mv^2/\hbar\omega_p\right) = 10$ are taken for the sake of estimate, then the stopping power obtained by the Bethe-Larkin formula is modified by $\sim 7\%$, which indicates to what extent the experimental accuracy needs to be improved in future to corroborate (17). As it has already been mentioned above, another alternative way of verification of (17) is to use diclusters as projectiles [8].

Energy loss of relativistic projectiles. – Relativistic corrections to the Lindhard formula were studied in [38]:

$$-\frac{dE}{dx} = -\frac{(Z_p e)^2}{\pi v^2} \int_0^\infty \frac{dk}{k} \int_{-kv}^{kv} \omega\, \mathrm{Im}\left(\epsilon^{-1}(k,\omega)\, \frac{\epsilon^{-1}(k,\omega) - \frac{v^2}{c^2}}{\epsilon^{-1}(k,\omega) - \frac{\omega^2}{k^2c^2}}\right) d\omega. \tag{19}$$

It can easily be shown that when the speed of light $c \to \infty$ and $M, m_i \gg m_e$, (19) actually turns into (2). As we have seen, within the present approach the quantum corrections in the frequency integral limits are negligible, and since

$$\mathrm{Im}\left(\epsilon^{-1}(k,\omega)\, \frac{\epsilon^{-1}(k,\omega) - \frac{v^2}{c^2}}{\epsilon^{-1}(k,\omega) - \frac{\omega^2}{k^2c^2}}\right) =$$

$$= \left(\mathrm{Im}\,\epsilon^{-1}(k,\omega)\right)\left(1 + \frac{\left(\frac{\omega}{kc}\right)^2\left(\left(\frac{v}{c}\right)^2 - \left(\frac{\omega}{kc}\right)^2\right)}{\left(\mathrm{Re}\,\epsilon^{-1}(k,\omega) - \left(\frac{\omega}{kc}\right)^2\right)^2 + \left(\mathrm{Im}\,\epsilon^{-1}(k,\omega)\right)^2}\right)$$

we can once employ (14) to simplify (19) into:

$$\left.\frac{-dE}{dx}\right|_{v\to c} \simeq \left(\frac{Ze\omega_p}{v}\right)^2 \ln\frac{2mv^2}{\hbar\omega_p\sqrt{1+H}} +$$

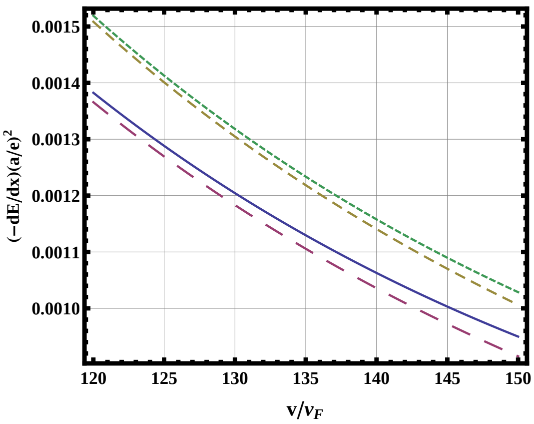

Fig. 1: The energy losses of relativistic protons in plasmas of $T = 1\ eV$, $\Gamma = 10.7725$, and $r_s = 2.5256$. The tiny dashed line shows our result in comparison with the Bethe-Larkin asymptotic expression (medium line) for the electron fluid, while the large dashed line, corresponds to the Bethe-Larkin asymptotic expression and the full line is corrected one (20) for two-component plasmas.

$$+\left(\frac{Ze\omega_p}{c^2}\right)^2 \int\limits_{\frac{\omega_p\sqrt{1+H}}{v}}^{\frac{2mv}{\hbar}} \frac{dk}{k^3} \frac{\omega_2^2(k)\left(1-\frac{\omega_2^2(k)}{k^2v^2}\right)\left(\omega_2^2(k)-\omega_1^2(k)\right)^2}{\Omega^4(k)+\left(\frac{\omega_p^2\omega_2(k)\,\mathrm{Im}\,q(k,\omega_2(k))}{|q(k,\omega_2(k))|^2}\right)^2}, \tag{20}$$

where

$$\Omega^2(k) = \omega_p^2 + \left(\omega_2^2(k)-\omega_1^2(k)\right)\left(1-\frac{\omega_2^2(k)}{k^2c^2}\right) + \frac{\omega_p^2\omega_2(k)\,\mathrm{Re}\,q(k,\omega_2(k))}{|q(k,\omega_2(k))|^2}.$$

The relative importance of (20) as compared to (17) has numerically been analyzed and the results are presented in figs. 1-5. In these figures the plasma stopping power is multiplied by the factor $(a/e)^2$, and the projectile speed is normalized to the Fermi velocity of plasma electrons. The upper pairs of curves on all figures correspond to the electron fluid under the same thermodynamic conditions, while the lower ones represent the stopping power of two-component plasmas. In each pair of curves, the lower one corresponds to the corresponding Bethe-Larkin asymptotic expression, i.e., without (in an electron fluid) or with the electron-ion correlation correction, respectively. For numerical evaluations the plasma static characteristics have been calculated in the hypernetted-chain approximation [26] using the Deutsch effective potential [39].

Conclusions. – In this Letter, two factors have been studied to influence the polarization stopping power described by the Bethe-Larkin formula. The first correction is due to the presence of the ionic component [7] whereas the second amendment is caused by relativistic effects. It has been demonstrated that for projectiles with a velocity of up to 80% of the speed of light, the relativistic correction enhances only slightly the Bethe-Larkin-type asymptotic value of the stopping power of moderately coupled plasmas. At the moment it is rather difficult to expect the experimental confirmation of the presented results, but they might become crucial for future studies of the ion-driven inertial fusion.

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This research was financially supported by the Spanish Ministerio de Educación y Ciencia Project No. ENE2010-21116-C02-02 and by the Science Committee of the Ministry of Education and Sciences of the Republic of Kazakhstan under Grants No. 1128/GF, 1129/GF and

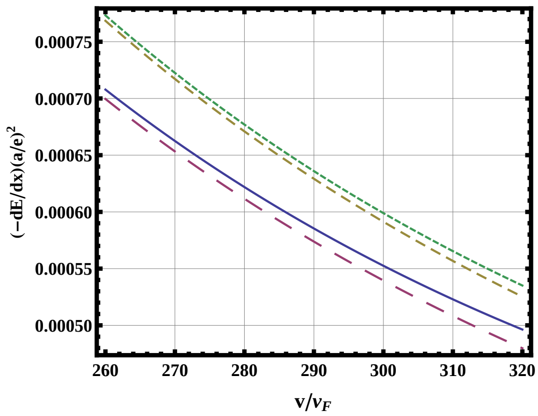

Fig. 2: Same as in fig. 1 but for $T = 10\ eV$, $\Gamma = 0.5$, and $r_s = 5.441$.

1099/GF. I.M.T. acknowledges the hospitality of the al-Farabi Kazakh National University during his leave of absence.

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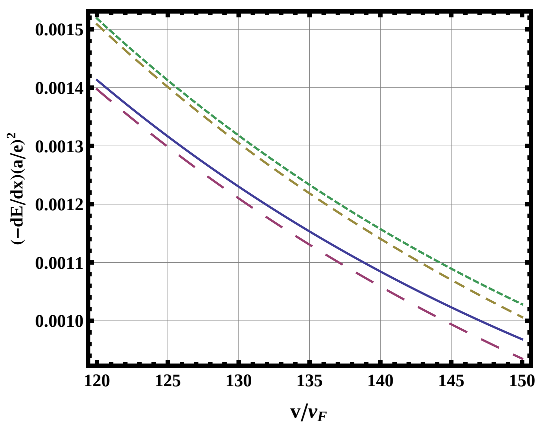

Fig. 3: Same as in fig. 1 but for $T = 10\ eV$, $\Gamma = 1.07724$, and $r_s = 2.5256$.

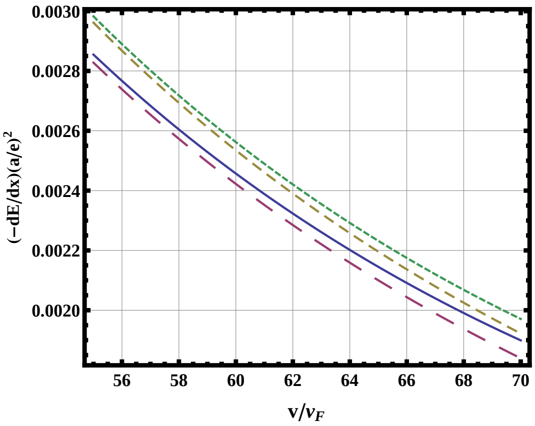

Fig. 4: Same as in fig. 1 but for $T = 10$ eV, $\Gamma = 2.321$, and $r_s = 1.172$.

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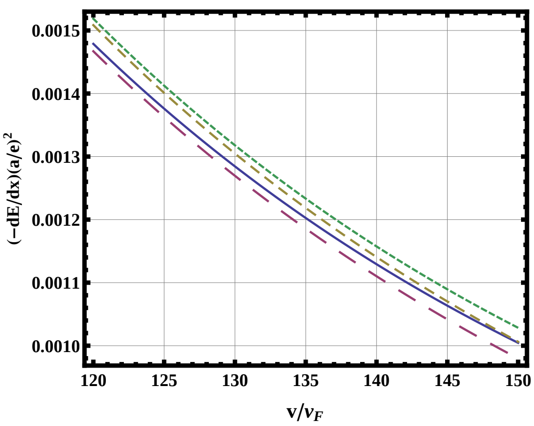

Fig. 5: Same as in fig. 1 but for$T = 100$ eV, $\Gamma = 0.107725$, and $r_s = 2.5256$.

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