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Elasto-plastic stability of single-layer cylindrical reticulated shells

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Abstract

Until now, researches about elasto-plastic stability of reticulated domes have gotten fruitful results, but few on the elasto-plastic stability behaviors of cylindrical reticulated shells are still unknown. Therefore, based on commercial finite element software ANSYS and self-compiled pre-post-processing programs, more than 600 examples of cylindrical reticulated shells are analyzed, using accurate complete-process method with both geometric and material nonlinearity.Some characteristic responses such as critical loads, buckling modes and distribution maps of plastic level are collected. The effects of various geometrical and structural parameters, such as initial imperfection, unsymmetrical distribution of loads, on the behaviors of elasto-plastic stability are investigated. Plastic reduced coefficients are defined to indicate the decrease of stability critical loads of reticulated shells under consideration of material nonlinearity. The plastic reduced coefficients of cylindrical reticulated shells are summarized to be 0.362～0.578.

Keywords: single-layer cylindrical reticulated shells, elasto-plasticity, buckling mode, critical load, inital imperfection

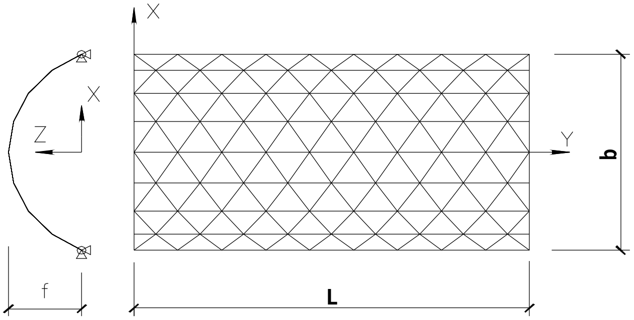
1. Introduction

The elasto-plastic stability analysis of reticulated shells has been carried out since 1990s last century in China(Hu *et al.*[1], Shen [2],[3]), and the work were mostly focused on the experimental tests and the program composition in which the material and geometrical non-linearity were taken into account. These works provides a good theoretical basis for the elasto-plastic stability study of reticulated shells (Luo [4], Wang [5], Liu *et al.* [6]). These years, with the development of computer technology and large finite element software, the comprehensive elasto-plastic whole-course analysis of reticulated shells becomes accessible. Many research productions on elasto-plastic stability of reticulated shells have been obtained by scholars (Cao[7], Cao *et al.*[8], Kato *et al.*[9]). But, most works done before have some limitations, for their main subject were spherical reticulated domes. As a matter of fact, another commonly used shell, single-layer cylinder reticulated shell, were studied with emphasis in this paper hoping to find out the complicated stability behaviors, the real process of losing stability of a cylinder reticulated shell structure and the mutual influence of various factors.

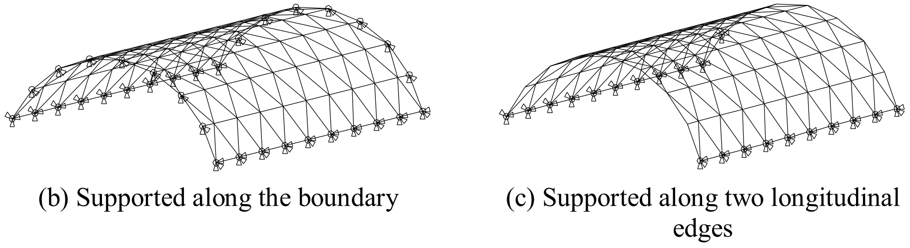
2. Analysis method and parametric scheme

The standardized technological process for complete load-displacement analysis of reticulated shells was established using the FEM software ANSYS and the self-developed programs for pre-and post-processing. During analysis, the connections of shells are assumed to be rigid, and the pipes are simulated by Beam 189 element. The plastic development situation of sections can be outputted timely, and the plastic development of the structure subjecting to load can be tracked visually during the complete progress.

Basic net system of single-layer cylindrical reticulated shells is a three-way system, and three types of supports were considered in analysis, as shown in figure 1. All the shells have a span of 15m. Five rise-width ratios (f/b=1/2, 1/3, 1/4, 1/5) and six length-width ratios (L/b=1.0, 1.4, 1.8, 2.2, 2.6, 3.0) were taken into account during analysis. For the analysis of the shells with initial imperfection, the first eigenvalue buckling mode is used to simulate the distribution of initial imperfection. Two initial imperfection values, b/750 and b/500, were used during the basic analysis of shells, and what is more, more initial imperfection values were used in some shells to investigate their influences on the stability behavior of single-layer cylindrical reticulated shells. The asymmetric distribution forms of loads are shown in figure2. Four different proportion factors of live load to dead load are p/g=0, 0.25, 0.5 and 1.0. Different section group was used for reticulated shells with different support system and different physical dimension (Shen *et al*. [10]).

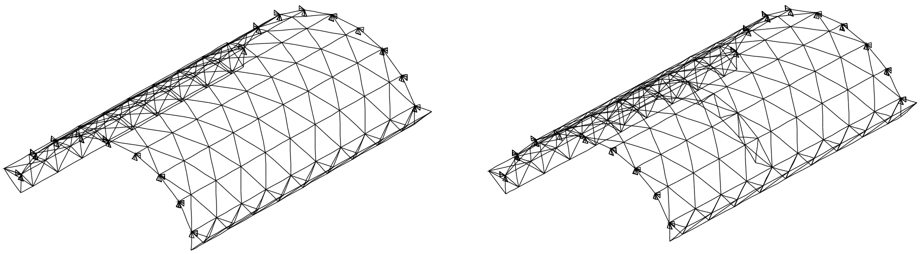


(a) Structure of cylindrical reticulated shells

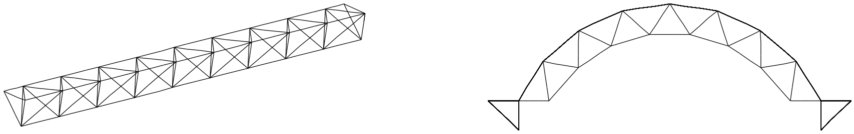


(b) Supported along the boundary

(c) Supported along two longitudinal edges



(d) Supported at two end diaphragms (e) Supported at two end with diaphragms



(f) Vertical stiffening rib (g) Diaphragms

Figure 1: Categories of single-layer cylindrical reticulated shells

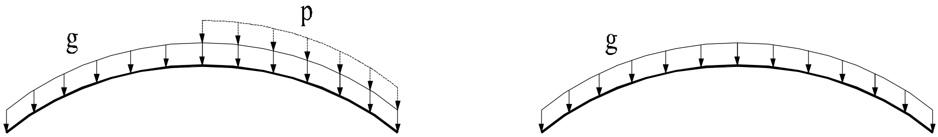


Figure 2: Unsymmetrical distribution of loads

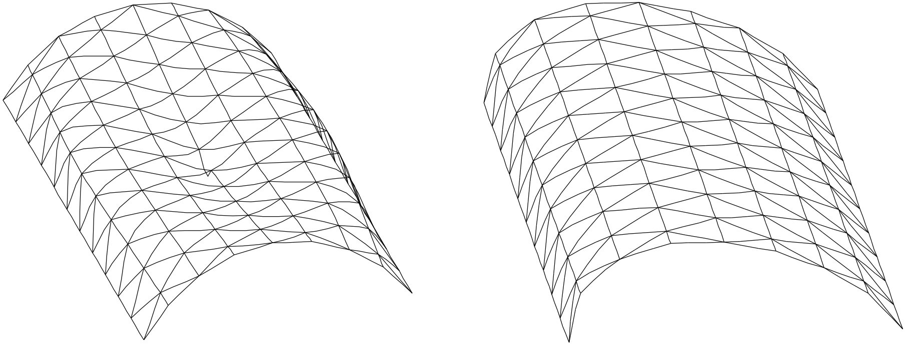
3. Results of parametric analysis

3.1. Buckling mode

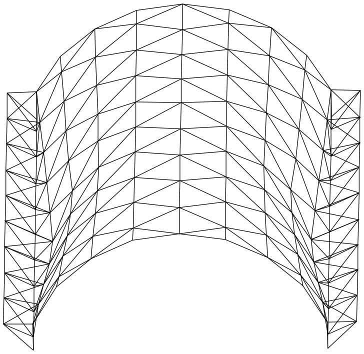
Bucking mode is regarded as the displacement tendency of structures at the critical point. Through analyzing bucking modes of reticulated shells, the weakness area of the structures can be find out and the region firstly losing stability can be predicted accurately. Figure 3 presents the buckling modes of shells with three different support systems. It can be seen from the figure that the structural performance of a single-layer cylindrical reticulated shell supported along two longitudinal edges is similar to that of two-hinged arch. Bifurcate buckling happens easily for an integrated shell with symmetric distribution of loads. During loading, the deformation is symmetric at the initial moment and becomes unsymmetrical at the buckling moment.

Buckling mode of a single-layer cylindrical reticulated shell supported along the boundary presents a concave surface with three half-waves in most cases. A big concave in the middle of the surface of a shell is shaped, and the areas at both sides are pressed outward.The longitudinal elements in middle of the surface of the shell bear large force as a press-bending element, and so, enter plastic phrase early. With the decrease of effective section modulus, the elements tend to buckle easily.

Buckling mode of a single-layer cylindrical reticulated shell supported at two end diaphragms is symmetrical under symmetrical load. The cross section of a shell has a deformation shaped as three half-waves. When the shell is buckling, the middle surface is pressed outward and the areas at both sides are concave, which are quite the opposite with the situation of the shells supported along the boundary, for the longitudinal boundary beam is too weak to restrain the edge of the shell. So, it is very important to assure the rigidity of the longitudinal ribbed stiffer to increase the overall stability behavior of a shell supported at two end diaphragms.



(a) Supported along the boundary (b) Supported along two longitudinal edges

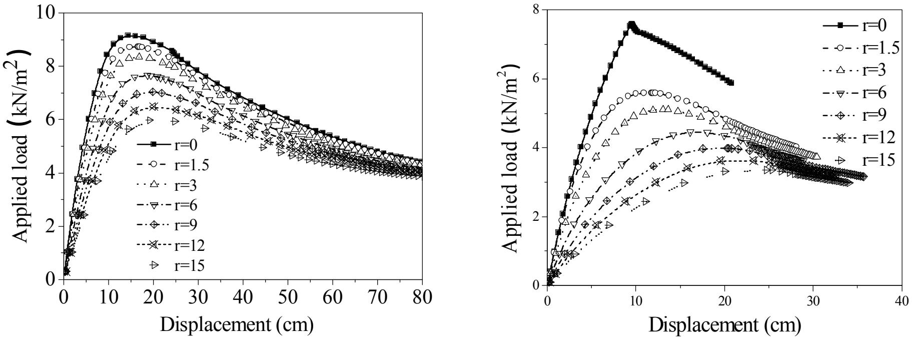


(c) Supported at two end diaphragms

Figure 3: Buckling modes of cylindrical reticulated shells

3.2 Effect of initial geometric imperfection on critical load

In practical projects, buckling model of a shell can be changed due to the existence of the initial geometric imperfection, and also, the critical load of a shell can be decreased obviously. In order to find out the effect of initial geometric imperfection on the structural performance, all the shells with initial geometric imperfection were analyzed to obtain the statistical regularity. It can be seen from the statistic analysis of the results that, contrasted to a integrated shell, the critical load decreases by 10% (15% at most) for a shell supported along the boundary, 20% at most for a shell supported at two end diaphragms, and about 10% for a shell supported along two longitudinal edges. Figure 4 presents the load-displacement curves of single-layer cylindrical reticulated shells with two supported systems and seven values of initial geometric imperfections. It can be seen from the figure that the critical load of a shell decrease gradually with the increase of the initial geometric imperfection. Based on the values of critical loads presented in figure 4 and the initial imperfection coefficients (results of critical loads of shells with initial geometric imperfections divided by that of shells without initial geometric imperfection) presented in figure 5, it can be concluded that the proper initial imperfection coefficient should be 0.8 when the value of initial geometric imperfection is b/500, and that a single-layer cylindrical reticulated shell is sensitive to initial geometric imperfection.



Supported along the boundary
(f/b=1/4,L/b=2.2)

Supported along two longitudinal edges
(f/b=1/4,L/b=2.2)

Figure 4: The load-deflection curves with different initial imperfection

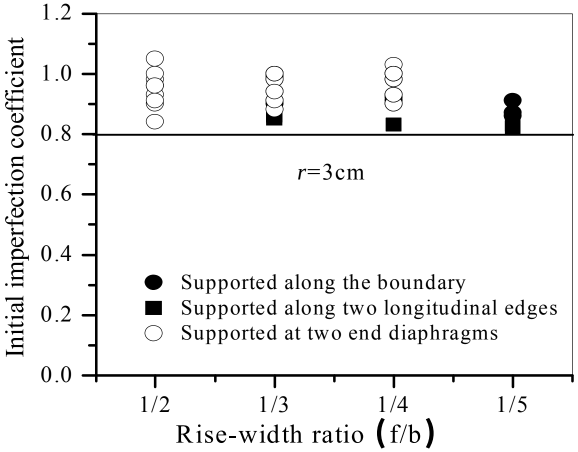
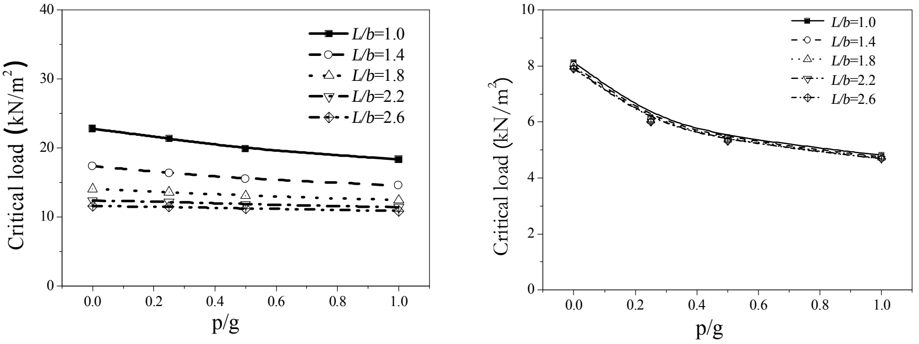


Figure 5: Initial imperfection Coefficient of different style cylindrical shells

3.3 Effect of asymmetric load distribution on critical load

Just as initial geometric imperfection, asymmetric load distribution also has great effect on the critical load of a single-layer cylindrical reticulated shell. Figure 6 presents the results for the shells with different length-width ratio and different proportion factors of live load to dead load.



(a) Supported along the boundary (f/b=1/5)

(b) Supported along two longitudinal edges (f/b=1/5)

Figure 6: Curves of critical loads under unsymmetrical distribution of loads

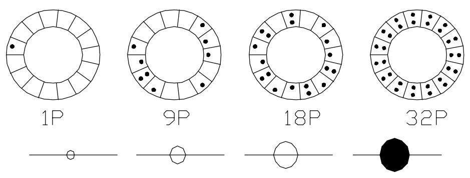
As shown in figure 6a, when the length-width ratio is small (L/b=1.0, 1.4), the critical loads of shells decrease due to the asymmetric load distribution, and it decreases by 31% at most when the p/g is 1.0. With the increase of length-width ratio, the decrease tendency of the critical load becomes weak, and when L/b > 1.4, it is concluded that the asymmetric load distribution has little effect on the critical load of shells.

The special structural performance of a single-layer cylindrical reticulated shell supported along the boundary is similar to that of two-hinged arch. The result curves of shells with different length-width ratio are almost overlapping. The critical loads of shells decrease with the increase of proportion factors of live load to dead load. The critical load of shells can decrease by 40% because of the asymmetry load distribution.

With a single-layer cylindrical reticulated shell supported at two end diaphragms, the asymmetric load distribution has little influence on the critical load.

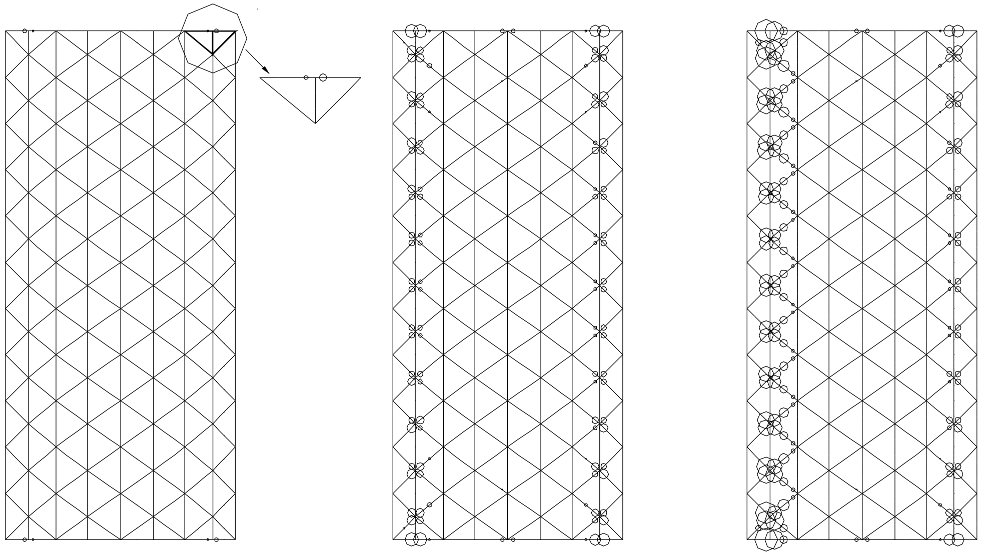
3.4 Tracking analysis of plastic development process

The buckling mode and critical load of a shell represents the macroscopic instability characters of a shell, and the inner plastic development of a shell represents the microscopic instability of a shell. Usually, some elements will enter the plastic stage before the buckling of a shell, and the plastic degree affected by geometric non-linearity will develop gradually with the increase of the load. The buckling mode and variation of the critical load can reflect the plasticity stage.



(a) Definition for different degree of plastic development of element section

(b) Displacement shape at different time (structure section)



(c) Plastic development of structures (plane)

Figure 7: Plasticity distribution of reticulated vaults at different time

The instability mechanism and instability character can be explained macroscopically and microscopically by tracking the plastic development of a shell during the whole loading progress. Figure 7 presents the plastic development status and the corresponding deformation of a shell. Circles in the figure represent the plastic position of elements, and the bigger the circle is the deeper the plastic is. At the time the shell entering the plastic stage, the elements that have entered the plastic stage centralize at the corners near the support system. At the point of critical load, all the elements near the two longitudinal boundaries enter the plastic stage, and the distribution of the plastic elements and the deformation of the structure are full symmetric. After the point of critical load, the plastic development of the structure becomes unsymmetrical and the deformation of the structure is lateral deviation.

Thus it can be concluded that the inner plastic development and the macroscopic displacement of a structure are correlated, and in fact, it is their interaction that make the structure lose the stability. But it is difficult to determine what is the key factor between them, for it varies with different structure.

4. Plastic reduction coefficient

For the stability analysis of shell structure, both elato-plastic and elastic whole-process analysis were done, and the ratio of the critical load obtained by ealto-plastic analysis to that obtained by elastic analysis was defined as 'plastic reduction coefficient' represented by c_p . The plastic reduction coefficient represents the disadvantage effect caused by material nonlinearity. Based on the large numbers of results, it is feasible to make a statistic treatment for c_p according to the general theory of statistics to find out the proper value of c_p .

The plastic reduction coefficients should be grouped into three kinds according to three types of single-layer cylindrical reticulated shells due to different support system.

For example, the average value and the mean square deviation of c_p are $\overline{c_p}$ =0.734 and d_{cp}=0.094 according to the statistic analysis of c_p of the 66 shells supported along the boundary with L/300 initial geometric imperfection. In order to assure that the probability of safety is bigger than 95%, the value of c_p is defined as:

$$c_p = \overline{c_p} - 1.645 d_{cp} = 0.578 \tag{1}$$

The plastic reduction coefficient for other single-layer cylindrical reticulated shells can be obtained in the same way, as shown in table 1, and what is more, a more detailed statistic result can be obtained according to different rise-width ratios.

Table 1 Statistics of ratio c_p for reticulated vaults

Support system	Rise-width ratio （f/L）				
	1/2	1/3	1/4	1/5	1/5~1/3
Supported along two longitudinal edges	—	0.563	0.709	0.766	0.578
Supported along the boundary	0.360	0.429	0.495	0.585	0.453
Supported at two end diaphragms	0.360	0.360	0.378	—	0.362

5. Conclusions

The decrease of critical load of a single-layer cylindrical reticulated shell with b/500 initial geometric imperfection at most is not more than 20%. The critical load decreases continuously with the increase of initial geometric imperfection. So, it is very important to restrict the installation error.

When the p/g is 1.0 and the live load distributes across half a surface of a shell, The critical load of a single-layer cylindrical reticulated shell decreases by 40%.

The geometric deformation of a shell and the plastic development of its elements are correlated and interact, for the geometric deformation can accelerate the speed of plastic development of elements, and the plastic development of elements can also weaken the rigidity of structure and speed up the instability.

Based on the plastic reduction coefficients obtained in the paper, it can be concluded that the critical load of a single-layer cylindrical reticulated shell decreases a lot considering the elasto-plastic analysis and the differences are between 0.362-0.578, which indicates that it is unsafe to estimate the critical load obtained by elasto-plastic analysis as 50% (China code for design of reitulater shells)of that obtained by elastic analysis.

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