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A mathematical modeling applied to the study of two forms of artistic representation

Un modelo matemático aplicado al estudio de dos formas artísticas de representación

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Abstract

The cultural manifestation of the Arican indigenous people from Chile, through the designs found in their garments was analyzed. Comparing their techniques of mosaic formation, using geometric transformations (bijection plans in itself), it was investigated whether, mathematically, its evolution could be explained. The mosaics, as well as the known works of Escher, are constructed from the application of translations, rotations, reflections or slip reflections of an initial motif (a rosette). With similar purpose —understanding the relationship between music and the evolution and complexity of a possible mathematical representation— were analyzed the geometric transformations and excerpts from three works of Johann Sebastian Bach. It is concluded, then, the existence of a possible line between artistic evolution (the artistic culture of a people or the work of a musician) and mathematical representation/geometry of such manifestations.

La manifestación cultural de los pueblos indígenas Arican de Chile es analizada a través de los diseños que se encuentran en sus prendas. Comparando sus técnicas de formación de mosaicos (usando transformaciones geométricas), se investigó si, matemáticamente, era posible explicar su evolución. Los mosaicos, así como las obras más conocidas de Escher, se construyen a partir de la aplicación de las traslaciones, rotaciones, reflexiones o traslaciones/reflexiones ("slip reflection") de un motivo inicial (una roseta). Con fines similares —entender la relación entre la música y la evolución y la complejidad de una posible representación matemática— se analizaron las transformaciones geométricas y extractos de tres obras de Johann Sebastian Bach. De esta manera se concluye la existencia de una posible línea entre la evolución artística (la cultura artística de un pueblo o de la obra de un músico) y la representación matemática/geometría de tales manifestaciones.

Keywords: Ornaments, music, geometric transformations

Palabras clave: Ornamentos, música, transformaciones geométricas

1 Introduction

This study explores, from a mathematical perspective, the characteristics of the manufacturing of ornamental pieces of tissue by the people of Arica (northern region of Chile), as well as musical compositions by Johann Sebastian Bach.

Analyzing such works of unquestionable aesthetic quality and translating its codes into a scientific language, there is the evolution of artistic quality, trying to find a counterpart in the evolution of reasoning and intellectual development needed for such creations. Mathematics was chosen as a verification tool for investigating such advancement, because as modern definitions (Vale, 2005), mathematics may be defined as the science of patterns. Means for patterns and regularities common properties that make some elements may be grouped under broader classification of meaning.

Thus, the ornament's details on existing clothing of the people from Arica as well as the techniques used in the canons and counterpoint in Bach's compositions links when arranged graphically, representing patterns that, through various transformations are new arrangements. These transformations are analyzed here as functions that cause translations, reflections and symmetry points in order to generate more complexes ornaments or new melodies that harmonize.

To contextualize the works analyzed, a brief biographical and historical account describes the training periods of Arica culture and the composition techniques developed and used by musicians of the Baroque period, especially Bach. Finally, from the definition and the use of geometric transformations are presented as mathematical procedure and chosen conventions enabling, thus, the analysis of such parts from a mathematical perspective.

2 Importance of analogy in the process of knowledge formation

It is understood the analogy to the relationship between elements that under certain analysis, possess qualities or common identities, fostering understanding of phenomena and enabling your best symbolic representation (Abdounur, 2003).

The process of formation of concepts presupposes a conflict between existing cognitive structure in the individual and the knowledge that you are being presented. Passing by encoding imposed by language, in any form it is presented (oral, written, graphic or musical), scientific knowledge is being constructed through comparisons with primitive concepts already assimilated and endowed with value. From this initial substrate, it is necessary to establish standards for categorizing a universe of human beings by ordinary identifications (Cruz, 2005).

The evolution in the classification made possible by such standards is noted for greater complexity in the structures and logic that are developed for a better understanding of information presented in symbolic form, dispensing with a level of abstraction much more pronounced. Bach, while teaching music made his mission to teach using these concepts intuitively. As Martino, Bach believed that students would have a better development in musical theory and composition techniques when they were more meaningful to them, treasuring an overall knowledge of music before analyzing its components (Martino, 1998). Thus, Bach already anticipated in his practice as an educator theses that would be defended centuries later by psychologists as Freinet, Wolfgang Kohler and Kurt Koffka, creators of the Gestalt theory.

In this study, we note that the establishment of classifications methods of contrapuntal composition determines classes that can be analyzed using mathematical concepts already established. Thus, the increasing complexity observed in the creative development of Bach is analogous to the more refined mathematical characteristics that are observed in the analysis of his works.

3 Educational implications

Guidelines of the NCTM (National Council of Teachers of Mathematics) argue that the identification of patterns, regularities and manipulations, that are possible from this classification of elements with common qualities, are essential to the cognitive logical-mathematical reasoning. It also covers skills such as understanding patterns, relations and functions, represent and analyze situations, use mathematical models to represent and understand quantitative relationships and analyze change in various contexts (NCTM, 2000).

The study of algebra, geometry and resolutions of problems as well as all the other areas of learning are favored if students are accustomed from an early age to work with pattern recognition and turning mathematical concepts into meaningful attributes to everyday life situations. As stated by Vale, the standards:

- Can contribute to building a more positive image of mathematics;
- Allow the establishment of mathematical connections;
- Attract and motivate students, because it appeals strongly to their aesthetic sense and creativity;
- Allow the promotion and development of capabilities and skills of the students;
- Help to develop the ability to classify and organize information;
- Give opportunity for understanding the connection between mathematics and the world in which we live. (Vale, 2005).

Therefore, through the development of a more algebraic and geometric reasoning conscious and structured, the student will be able to reach a more advanced achievements in mathematics and other sciences, and thus can perceive structures and rules as well as proceed with abstractions, generalizations and determinations of symbols.

4 Biographical and historical account

4.1 Arica

Arica is a region located on the extreme north of Chile making boundary with Peru, Bolivia and the Pacific Ocean. We can divide it into three ecological zones: the coast, the mountains and the puna.

The coast is a range of only 60 km, generally very dry and hot, despite the icy ocean. The "mountains" is the central part of the territory, occupied by a mountain range of the Andes cordillera. It is very cold in the mountains, but as rains occur, there is some vegetation adapted to the altitude. Advancing inland, one comes to the puna, with an average altitude of 4,000 m and nights so cold that prevent the growth of any type of plant.

For about 10,000 years, humans have occupied this unique region called Arica, despite its inhospitable climate and possible earthquakes. This people harmoniously adapted to scarce resources, managing to take sustenance from the sea and the difficult farming. In Arica, people seem to love the land and live intensely every day.

Human groups that lived on the coast, mountains and the ', through a permanent and secular exchange, produced a very expressive culture. The earliest manifestations of this culture precede the construction of the pyramids, in Egypt. Even the arrival of the Spaniards in 1535, has not stopped, even today, finding traces of Arican culture in inhabitants' handcrafts and in the customs of the place.

Within Arican culture, it stood out the art of weaving. The tissue function went well beyond protection against unfavorable weather. It was intimately related to economic, social, political and especially religious status of the people in that society. This was reflected in the garments and adornments as well as in the quality of the fabrics, its designs and colors.

In the museum of San Miguel de Azapa, of the University of Tarapacá, we can find different objects from all periods of Arican culture. There are blankets, hats, bags and ornaments all of them in vivid colors and varied designs. There are indications that this variety reflects the diversity of the ethnic groups that inhabits the region.

The different types of designs found in those objects are characteristics of certain historical epochs, like happening in any other form of art. Therefore, the tissue of Arica has also undergone through a historical evolution. In other words, elements of a social structure and ethnic particularities from the historical times can be seen on a single piece of object.

However, there is still a little more. Analyzing the evolution of a culture through the evolution of its mathematical/geometrical ornaments, which is found on some pieces of clothing, we can establish the evolution of the Arica's reasoning peoples.

The historic periods of Arica

The Archaic period (8000 BC-1000 BC)

The region was home to a society of fishermen (the coast) and hunters (the mountains). A rudimentary agriculture emerged in the hills, just at the end of the period, when there was some increase in population.

For about 4000 years remained the practice of mummification indicating that the people of Arica mastered this technique before the Egyptians. At this stage, the weaving had not yet born. The adornments, thongs, robes, blankets etc were made of plant fibers and feathers.

The Formative period (1000 BC-300 AD)

During this period the agriculture develops and villages begin to appear. It appears the tissues and their ornamental motifs. Ceramics also appear, but not yet with ornaments.

The Tiwanaku period (300 AD-1100 AD)

Arica society continues to progress. The agriculture expands and intensifies the commerce among the inhabitants of the coast, the mountains and the puna.

In arts, it is observed that the ceramic receives ornaments and colors. In tissues, usually made from llamas and alpacas' hair, they became more vivid, with more elaborate decorative motifs and more complexes symmetries.

The regional development period (1100 AD-1470 AD)

Historians point out this period as being the time of the largest population increase in the coast and the start of the conflicts between the inhabitants of the coast and the highlands. The art becomes richer, as observed in the ceramics ornaments and other made objects.

The Inca period (1470 AD-1535 AD)

The Inca Empire dominates the region and brings its well-structured social, political and economic systems. Many Incas' peasants settle in Arica, bringing with them new techniques of pottery and weaving.

This profound transformation can be observed also in the manufacturing arts. For some historians, the techniques of weaving introduced by the Incas are more advanced, on the other hand, in terms of its symmetric complexities there was a retrocession in the ornaments.

4.2 Bach

Bach was born on March 21, 1685, in Eisenach, a small town in Thuringia, central Germany. He was son of the violinist Johann Ambrosius Bach, who brought his eight children in music, as a family tradition. Bach became father and motherless at the age of nine and went on to adolescence in Ohrdruf, with his older brother the violinist Johann Christoph.

At the end of the seventeenth century, Germany was divided into independent states and opened to other cultures such as the Italian and the French. It was in this scenario that Bach acquired a sophisticated cultural background, absorbing the art of ancient composers and of his contemporaries Baroque.

In the year of 1707, Bach was appointed Kapellmeister (chapel master) in Cöthen. After several years working in German courts, in 1723, Bach moved to Leipzig, where he assumed the post of organist and professor at St. Thomas Church. He went on to write exclusively religious concerts.

The musical genius of science did not accumulated wealth. Surrounded by his family, Bach died on July 28, 1750. Bach lived in the time known, in the music history, as the Baroque period. Etymologically, baroque means bizarre, what demonstrate the emergence of a strong detachment of composition rules that prevailed.

Regarding the composition techniques prior to the baroque period, and therefore prior to Bach, music was limited to ancient Greek modes, adapted during medieval times for sacred songs. The sounds were mostly unison, with rare and mild variations. The Baroque emerged as an opposition, introducing the idea of polyphony, harmony, chords, themes that associate, turn and complete. This technique became known as counterpoint.

Counterpoint is a term that has its origin in the expression "punctus contra punctus", which means, "note against note". It is a combination of melodic lines with differences in contrasting melody, rhythm or both. Having appeared in the ninth century, was the main form of composition, but obtained its peak in the Renaissance, mainly thanks to the work of Bach.

Next, it is presented the concepts of the types of musical pieces that used the baroque counterpoint.

Cânones

Cânones (or Canon) is the most rigorous contrapuntal imitation, in which the polyphony is derived from a single melodic line, through strict imitation at fixed intervals or, less frequently, variable height and time. The term has been used from the sixteenth century to describe works composed in the genre (Sadie, 1994).

In general, the Canon is composed by performing similar melodic lines (or with few variations), but each of them has a lag compared to the previous, causing a feeling of harmony.

Fugues

The trail, in its essence, is a form of composition that uses the technique of counterpoint to the presentation of topics. It is usually written in several voices, each playing a role in the musical exposure. The first voice always has a short melodic passage, called theme or subject. The second voice imitates this passage, generating the response or counter-subject. If there are more than two voices, the subject and the answer is again presented sequentially, until all voices have been executed. The answer may be a melody very close to the theme, but can also be a series of notes that simply accompany the melodic idea, which is presented by the first voice.

The trails have become, in the Baroque period, the apex of musical development and it is common for composer to write leakage or fugues from topics created by other musicians.

5 Geometric Transformations

5.1 The ornaments

It is common to find ornaments planes constructed from a repeated figure (subject) in decorate walls, floor tiles, in printed made on objects, the grids of windows and several other places. In mathematics, we consider three types of ornaments:

• The decorative bars, in which the motif is repeated indefinitely, within a limited range inside of the two parallel lines

Figura 1: Decorative bar taken from a Greek vase from BC

• The tiles, in which the motif cover the whole plane

• Rosettes, in which the repetition occurs within a limited region of the plane

In rosettes, the motifs are repeated as congruent plane figures, which mean that they have equal measures, and occupy different positions in the plane. This is equivalent to draw pictures from the same mold in different positions. In other words, the figure passes from one position to another by means of a movement that does not deform it.

Apparently, these movements can occur in infinite ways. However, there are only four mathematically movements and any other basic movement can be decomposed in the four movements.

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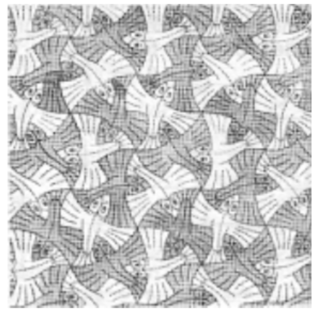

Figura 2: M. C. Escher mosaic (picture, 1954)

Figura 3: Rosetta extracted from a Russian blouse

These four movements are described below.

5.2 Movements and symmetries

A first basic motion is called translation. In this case, the figure slides on a straight line, all points of the figure run parallel and the segments have equal length.

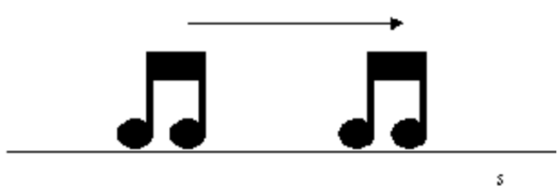

Figura 4: Translation

The second basic movement is known as rotation. The whole figure rotates a certain angle over a point O (which may or may not belong to the figure). All points of the figure move through arcs of circles with center O and all of them describe arcs correspondent to the same angle measure.

Figura 5: Rotation

The movement illustrated by the next figure 6 is a little hard to explain. In this case, we can consider a line, which acts as a mirror, reflecting the figure. The figure obtained on arrival corresponds to the mirrored image of the initial figure. This movement is called reflection.

Figura 6: Reflection

Finally, consider the motion that consists of a reflection and a translation on the same axis. This movement is called translational-reflected (or slipped reflection).

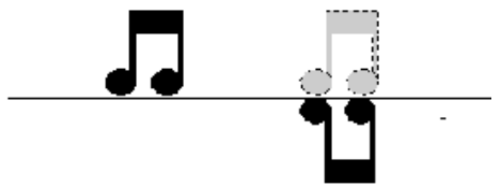

Figura 7: Slip reflection

Having defined the movements that produce repetition of motifs bars, tiles and rosettes, we can spend the final step of mathematical analysis of an ornament.

Given an ornament, we must determine which of these movements are presented in it, that is, we will be determining its symmetries. In mathematics, we say that an ornament has symmetry when it's applied some of the basics movements on it and the ornament remains the same as before.

It has also defined the complexity of an ornament by the number of symmetries that it presents. In other words, we defined that the larger the amount of symmetries of an ornament, the greater is its complexity.

5.3 Arica

Formative period

In the formative period of Arica's culture, the predominant ornaments were the tissue rosettes and the bars of bonnets and blankets. These ornaments were quite simple, both in terms of reasons and in relation to symmetries. They were produced in bar by translations and reflections perpendiculars to the direction of translation. In rosettes by reflections shown below, we can see a typical example.

On this rosette, the only symmetry is the reflection of a vertical axis. The right side of the rosette is reflected in the left and vice versa.

Tiwanaku period

From the Tiwanaku period of the Arica's culture, it is found rosettes with symmetries produced by rotations and in bars, in addition to translations and reflections of the previous period. In

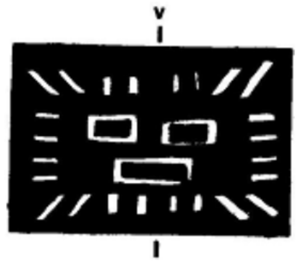

Figura 8: Rosetta from the formative period. Drawing from a blanket fragment

terms of symmetry, we now have the axis in the direction of translation. Observe, for instance, the bar of the figure 9 below. We have three types of symmetry: one produced by the translation of the arrow, another produced by the reflection of a horizontal axis and a third produced by the reflection of a vertical axis.

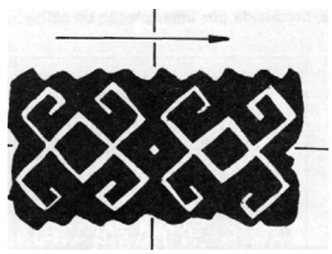

Figura 9: Bar Tiwanaku period. Drawing detail of a cap

During the Tiwanaku period appear even mosaics, which show an example to follow.

Figura 10: Mosaic of the Tiwanaku Period. Fragment of a woolen

In the mosaic above, one of the symmetries is produced by rotations of 90º around its center.

Period of regional development

In the period of regional development of the Arica's culture, we can see that the mosaic illustration is extremely elaborate. We point out the mosaic model and the symmetry produced by a rotation of 180º.

During this period, the use of stylized figures of animals, especially the condor (bird symbol of the Andes), becomes frequent. From the mathematical point of view, the appearance of symmetry translations produced by reflected or slipped reflections, increases the complexity of the ornaments.

Figura 11: Mosaic detail

Figura 12: Mosaic of the Regional Development Period. Decoration of a bag of wool

Inca Period

In the Inca period appears reflected translations and reflections. The translations reflected may indicate a high degree of development in the art of ornaments. However, the mosaic presented is less rich than the samples found during anterior period.

Figura 13: Mosaic of the Inca period. Decoration of a bag of wool

In fact, an examination of several other examples of the Inca period reveals a great similarity with the ornaments of the Tiwanaku period: their similar models and symmetries prevalent are the same. In summary, from the artistic and mathematical points of view the Inca period seems to represent an involution.

Why would this have happened? Is it because new techniques have been introduced? Or, maybe because the art from this period is the art of a dominated people? These are questions for historians.

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5.4 Bach

In order to establish a method of representing a particular melody encoding a coherent mathematical analysis, we use a plane with a Cartesian coordinate system-xy a fairly trivial representation of the scientific point of view.

Each note of the musical scale used in the compositions has properties that identify it. Two of these properties are the pitch or “the oscillation frequency of the sound”, and its duration or “the time period in which the sound is run”. These properties were “translated” into a mathematical representation matching the pitch with the y-axis and the duration with the length of each horizontal segment.

We can then translate a musical score into a mathematical way through a discrete function $f(t)$ that relates to each note an integer value; for example:

$$\text{C (do)} = 0 \text{ , C\# (do\#)} = 1 \text{ , D (re)} = 2 \text{ , D\# (re\#)} = 3, \ldots$$

as shown on the following figure:

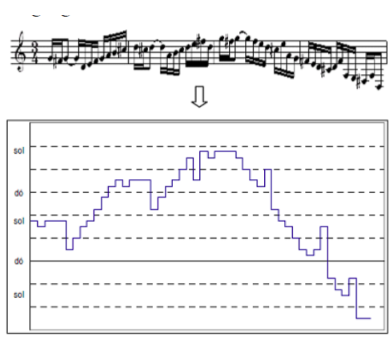

Figura 14: Graphical representation of a musical score

In the previous graph, the y-axis shows the information related to pitch, setting as reference the middle C (do), with frequency of 261 Hz. In other words, the higher a note is in relation to middle do, its representation in the graph will be displaced superiorly when compared with the middle C representation. Likewise, the lower a given note is in relation to middle do, its representation in the graph is displaced inferiorly in relation to a representation of the middle C.

The duration of the note, which indicates the time at which it should continue is associated with the length of each horizontal segment in the graphic convention adopted. Therefore, if we have a mathematical (geometrical) version of a musical score, it is possible to rewrite this song with the usual symbols just by identifying the y value and the distance between two endpoints of a segment. These numerical data will describe the pitch and duration of the specific note.

The figure 14 shows a short example of a musical score that uses only quavers and semiquavers, thinking only about the duration of the notes. The numbers 3 and 4 at the very beginning of the staff tell us that three crotchet notes (also called quarter notes) will complete each measure or by any set of notes, that fills the same amount of time. The quavers have the duration of a half crotchet, which means that two quavers are equal (in duration) a crotchet.

With this extremely extract of music lessons, one could see that a measure in a 3/4 time signature can be filled by six quavers or twelve semiquavers or other combination. In the mathematical representation, we fixed a segment length to a semiquaver note, which implies the double length to a quaver and so on.

In the works analyzed, the most common geometric transformations found were the horizontal or oblique (horizontal and vertical) translations. Less frequently, was identified horizontal reflections associated or not with translations and rarely, was recognized vertical reflections. Before checking the occurrence of these changes in more complex sections, which came from the Bach's compositions, we present in the next five figures, a piece of music and simple applications of the results of each of these changes in order to familiarize the reader with these interpretations:

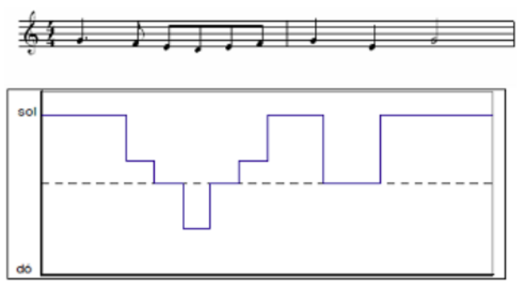

Figura 15: Excerpt musical and graphical representation agreed

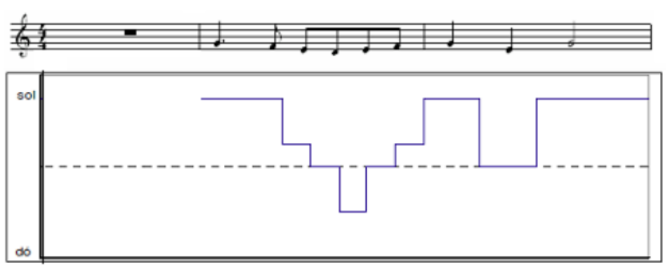

Figura 16: Horizontal translation

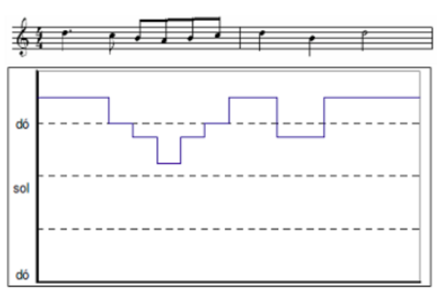

Figura 17: Vertical translation

In the analysis of the three selected sections of the Bach's compositions, we observe transformations that can be checked in each one of them.

The works chosen for this study were excerpts from "Goldberg Variations" (BWV 990), "The Art of Fugue - Die Kunst der Fugue" (BWV 1080) and "Musical Offering" (BWV 1079). Although they were published at the end of Bach's career (between the years 1740 and 1747), these works are a collection of well-organized composition techniques. These examples also gather all the musical experimentation of the artist in your life, from simpler structures intuitive, undergoing a constant evolution and reaching the most complex polyphony and execution.

Mentioning the odd musical composition by Bach is redundancy. However, it is worth men-

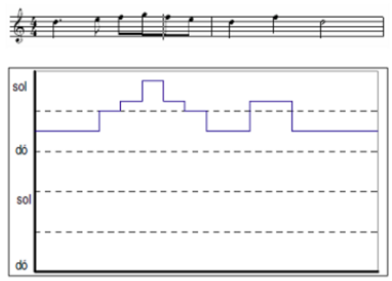

Figura 18: Horizontal reflection

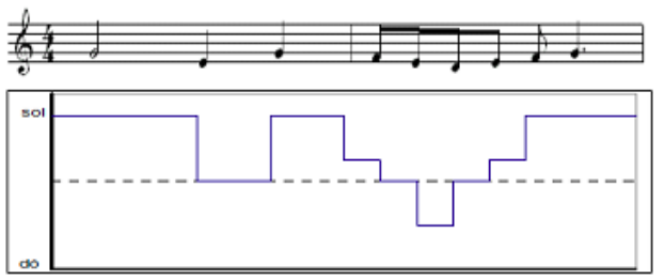

Figura 19: Vertical reflection

tioning here the didactic quality of the works mentioned above. The gradual increase of the technical level is associated with the verification of more complex geometric transformations. In other words, as much the technical level increases we are able to find more and more translations and reflections in his compositions.

Cânones

At the beginning of the Baroque period, the canons were the most common compositions and distinguished themselves very little from the unison plainchant of the Middle Ages. They were based on a simple melody with initial lag of each instrument or voice performer. Thus, under the proposed graphical analysis such melodies are represented merely by horizontal translations (x-axis).

The fact that this translation make to coincide, at an appropriate time of the melody, the second voice with the first, when the simultaneous executions of melodies occur, this causes a pleasant feeling to the listener. The determination of the time lag between the voices will not be presented here, since it would require a deeper theory of harmony that is not the focus of this article.

Subsequently, the canons, especially those that appear in the Bach's works cited, will be analyzed. It will be seen that, in such works, the lag time between the voices is not a simple melodic delay, but also the result of a vertical translation (y-axis) of the notes. That is, in the Bach's works is observed that beyond the second voice in the melody run in a time of lagged first voice, every note executed by it are higher or more severe than the first voice. The calculation of this lag factor also requires a deeper knowledge of harmony and, therefore, it will not be presented here.

In excerpts of Bach presented below, you can see a simple case of horizontal translation found in the second canon of "Art of Fugue".

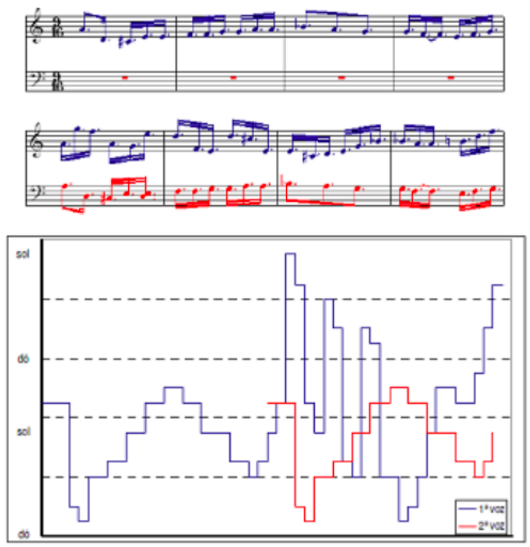

Figura 20: Second Canon of "Art of Fugue"-Horizontal translation of the second voice

You also can observe an oblique translation (vertical and horizontal) bellow in which is shown the 18th variation of the "Goldberg Variations":

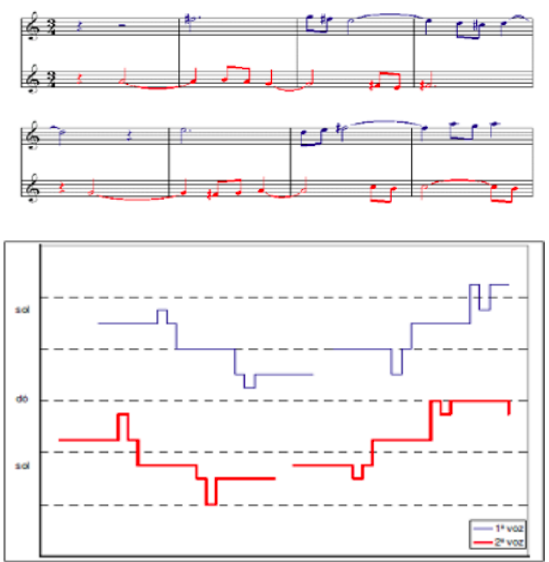

Figura 21: 18th variation of the "Goldberg Variations"-Horizontal and Vertical translations

Fugues

Due to technical and cultural Age and experiments conducted by Bach, compositions known as "Fugue" are called flight and are characterized by a theme that appears and then is modified in various ways.

One of these ways is the reversal of contrast that can be observed in our geometric representation by composing a horizontal reflection with a horizontal translation. Here, it is analyzed a snippet of the 15th variation of "Goldberg Variations".

It can also be seen in some compositions vertical reflections. This happens when performing

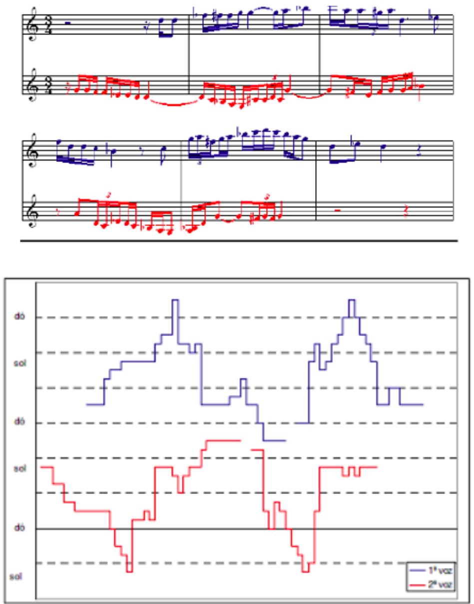

Figura 22: 15th variation of the "Goldberg Variations"-Horizontal reflection composed with a Horizontal translation

the principal melodic line of complementary shape and inverted relative to a line parallel to the y-axis. This transformation ensures the feeling of musical repetition of the initial reason but, over time, a reverse manner, i.e., from the end to the beginning.

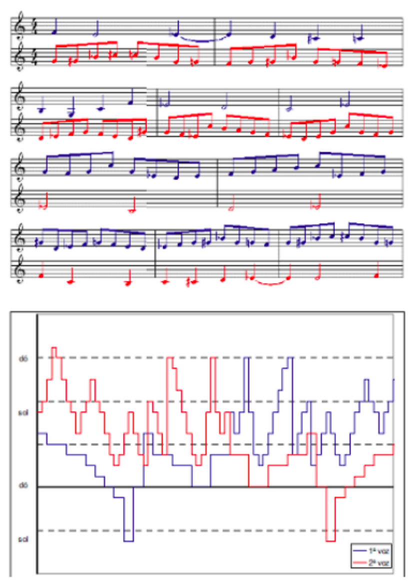

Figura 23: Canon in two voices of the "Musical Offering"-Vertical reflection

6 Conclusion and Proposals

The analogy made in this article between the evolution of Arica's Ornaments, excerpts from the works of Bach and the observed geometric standards, far from intending to minimize the necessary and required talent in the construction or development of such works, aims to show that a central science, considered by many as "cold" and without contextualization as mathematics, can serve as a stimulating factor for those who use it as a way of analyzing and understanding the development of the artistic-cultural thinking as shown here.

More than that, this study can be seen as an alternative form of analysis and understanding of the creative development, found in different forms of artistic works, from the perspective of geometric transformations.

Seeking to meet the mathematical perspective of students, of different levels of education, this paper sought to show that applied mathematics, when seeking to better understand the artwork and/or the conceptions of exponents of humanity, contributes substantially to the analysis that goes beyond the individual's perceptions and aggregate non-trivial aspects such as the geometric language here emphasized.

As stated by Vygotsky and cited by Cruz (Cruz, 1987):

> *Only in mathematics, we find a complete elimination of inconsistencies in the use of correct expressions, common and unquestionable. We can say only one thing on his swing and incongruity, our language is in a state of dynamic equilibrium between the ideals of harmony and mathematical imagination. She is in a state of continuous motion that we call evolution.*

The artistic language used in making ornaments or in music production walks on a fine line that divides the formal rigidity of mathematics and shapes, colors and inspirations of the processes of artistic creation.

As proposed application of this study in the educational environment, after the presentation of the necessary theory (geometric transformations) and the results obtained in the analysis of the works of Bach and of the Arica's Ornaments, further research can be developed, taking as object of study the artistic compositions from other artists/musicians in other crops, for instance.

Some suggestions for further work and questions to be answered:

The decoration of ceramic pieces from some indigenous peoples of Brazil presents the same evolution, under the mathematical perspective, found in the Arica's tissues? In popular or folk songs, are perceived the same geometric patterns like those seen in the works of Bach? It is mathematically possible to interpret other forms of artistic representation such as poetry, sculpture, dance, etc.? If the answer is affirmative, are the geometric transformations the best tool to be used for such studies? Some initial works have already been started in the theme "indigenous ceramics", but these questions remain unanswered and will be the goals in future works.

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