
CONTENTS

1	Motivation	1
2	Introduction	5
2.1	Splitting methods	5
2.2	Composition	8
2.2.1	Construction of higher order integrators by composition	8
2.2.2	Lie derivative and integrators	10
2.3	Order conditions via BCH formula	14
2.3.1	Runge-Kutta-Nyström methods (RKN)	16
2.3.2	Near-integrable systems	17
2.4	The Magnus Expansion (ME)	18
2.4.1	Derivative of the exponential and its inverse	19
2.4.2	First few terms of the Magnus expansion	20
2.4.3	Time symmetry of the Magnus expansion	21
2.4.4	Convergence of the Magnus expansion	22
2.4.5	Numerical integrators via Magnus expansion	22
2.4.6	Commutator-Free Magnus integrators	27
2.4.7	Different time-averaging	29
3	Non-reversible systems	33
3.1	The separable non-autonomous parabolic equations	33
3.1.1	The problem	34
3.2	Splitting methods for non-autonomous problems	36
3.2.1	Splitting methods for non-autonomous perturbed systems	37
3.2.2	Order conditions	40
3.2.3	Fourth-order methods	41
3.3	Numerical examples	42
4	Exponential of perturbed matrices	49
4.1	The scaling, splitting and squaring method	49
4.2	Computational cost of matrix exponentiation	53
4.2.1	Computational cost of Taylor and Padé methods	53
4.2.2	Computational cost of splitting methods	54
4.3	The Lie algebra of a perturbed system: (p_1, p_2) methods	55
4.3.1	Error propagation by squaring	55
4.4	Splitting methods for scaling and squaring	57
4.4.1	Standard splittings	58
4.4.2	Modified splittings	61

4.5	Error analysis	62
4.6	Numerical results	65
4.6.1	Rotations	65
4.6.2	Dissipation	66
5	Matrix Hill's equation	69
5.1	Symplectic integrators for the matrix Hill's equation	69
5.2	Numerical integration for one period	72
5.2.1	Symplectic methods	73
5.3	Exponential symplectic methods for the Hill's equation	77
5.3.1	Sixth-order methods	77
5.3.2	Eighth-order methods	80
5.4	Numerical examples	81
5.4.1	The Mathieu equation	82
5.4.2	Matrix Hill's equation	84
5.4.3	The damped Mathieu equation	85
5.4.4	The non-linear Mathieu equation	86
6	Conclusions	89
6.1	Non-reversible systems	89
6.2	Exponential of perturbed matrices	90
6.3	Symplectic integrators for the matrix Hill's equation	90
A	Algebraic Tools	93
A.1	Further approaches	93
A.1.1	On processing	93
A.1.2	More exponentials	94
A.1.3	Splitting for low-order Padé	94
A.2	Efficient symplectic approximation of E_2	95
	References	95