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Finite element analysis of tensioned fabric cone structures using a modified assumption on the meridianal stress

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Abstract
In this latest paper concerning the shaping of tensioned fabric cone structures without meridianal cables, a modification of the assumption concerning the circumferential distribution of the meridianal stress is incorporated. In the previous study an assumption was made that the axial force in the structure was uniformly distributed about a perimeter of a section perpendicular to the axis of the structure. In this study, it will be assumed that the meridianal stress itself is uniformly distributed about a perimeter of a section perpendicular to the axis of the structure. This assumption would agree with results obtained when using uniform bi-axial pre-stress in the shaping analysis. Results between the two studies are compared for structures with bi-planar symmetry.

Keywords: Tensioned fabric structures, cone structures, unequal bi-axial stresses, membrane finite elements, form finding analysis.

1. Introduction
A series of papers presented at past IASS conferences (Gellin [1 – 4]) explored the behaviour of the shaping of tensioned fabric cone structures without radial (now called meridianal) cables. These papers revealed the following characteristics about these structures: (1) that only certain ranges of the geometric parameters of the fixed boundaries lead to stable equilibrium configurations; (2) that the range of these geometric parameters could be expanded by increasing the ratio of the meridianal stress to that of the circumferential stress; (3) that uniform bi-axial stress solutions only existed if this stress ratio was 1:1; and (4) membrane finite elements formulated theoretically using these principles and incorporated into special and existing software yielded promising results.

One of the theoretical principles derived was the necessity to specify the total axial force transmitted by the structure. In the most recent of the papers, an assumption was made that for any section of the structure at a particular axial coordinate that the vertical component of the force was independent of the circumferential position. This did lead to equilibrium shapes which had variable stress distribution both meridianally and circumferentially, even when the ratio at any point between the stresses was 1:1. However, an equilibrium shape was derived using a uniform, equal bi-axial pre-stress distribution with conventional software.

In this paper, in order to maintain the desired principle that the equal, uniform bi-axial stress shaping formulation is a special case of the more general formulation explored herein, an assumption is made that the meridianal stress will be independent of the circumferential position at a particular axial coordinate. This assumption was tested for bi-planar symmetric cone structures and compared with previous results.

2. Test Cases

The test cases will be analyzed using the same procedure as outlined in Gellin [4]. Summarizing, an equilibrium shape is derived using a finite element model using constant stress triangular membrane elements. Each element in the model is prescribed a ratio of its meridianal stress resultant to its circumferential stress resultant, denoted as α. In addition, the total axial force F_z is prescribed. The meridianal stress resultant, S_r^0, in the structure must satisfy the relation:

$$F_z = \oint S_r^0 s_z dt \tag{1}$$

where the integral is taken around the circumference at a particular value of z, s_z is the direction cosine of the local element meridianal direction in the z direction, and t is the coordinate in the circumferential direction. For axisymmetric structures, it was reasonable to assume for each element that the meridianal stress resultant, as well as s_z, was constant. In Gellin [4], it was assumed that for bi-planar symmetric cone structures that the axial force was uniformly distributed around the circumference at a particular value of z; thus the product of the meridianal stress and the axial direction cosine was held constant. In this paper, in order to match results found with conventional software, it is assumed that the meridianal stress is constant around the circumference at a particular value of z; thus:

$$S_r^0 = \frac{F_z}{\oint s_z dt} \tag{2}$$

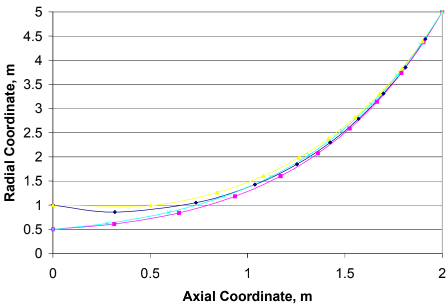
A special version of existing proprietary software was used with these features incorporated.

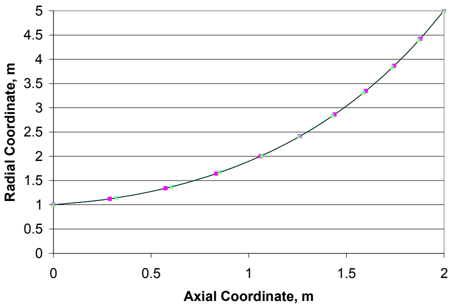
The test case structures are the same as in Gellin [4]. The structures consist of two rings, the planes of which are perpendicular to the z axis, separated by a fixed distance. One ring is elliptical. The elliptical ring has a radius denoted as a along the x axis of the ring at $z = 0$ m. The value of a in this study is fixed at 1 m. The radius along the y axis is denoted as b, which will take on values between 0.50 m and 2.00 m in 0.25 m increments. The outer ring is at $z = 2$ m, and is circular with a radius of 5 m. Two values of α will be investigated, specifically, 1 and 2. The value of 1 is chosen because it can be compared easily with results obtained using conventional methods and software. The value of 2 is chosen because many of the results with this value can be obtained in closed form (Gellin [1]).

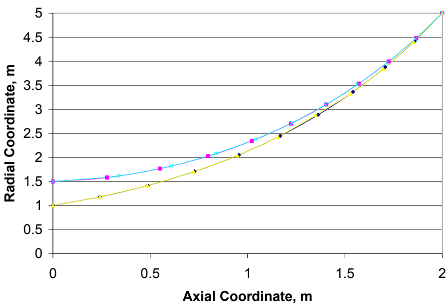
For each geometric case, a conventional analysis using constant and equal bi-axial stresses was performed. The value chosen for the stress was 4.4 kN/m. Material properties associated with PTFE fabric were employed. Each of these analyses resulted in a converged equilibrium shape. The total axial force at each ring was used as a basis for the axial load in the corresponding case using the formulation described above.

3. Results

Figures 1 – 4 display the results of the study. Each plot shown is for a different value of *b*. The upper left plot is for *b* = 0.50 m; the upper right plot is for *b* = 1.00 m; the lower left plot is for *b* = 1.50 m; and the lower right plot is for *b* = 2.00 m. The results for those values of *b* analyzed but not displayed are qualitatively similar to those included herein. In each plot, the dark blue curve refers to the results along the meridian in the *x*-*z* plane for this study; the magenta curve refers to the results along the meridian in the *y*-*z* plane for this study; the yellow and light blue curves refer to the same set of results as the dark blue and magenta curves, respectively, but for the results obtained in Gellin [4]. When *b* = 1.00 m, the structure is axisymmetric. A theoretical curve, based on the results of Gellin [1], is added in black for these plots.







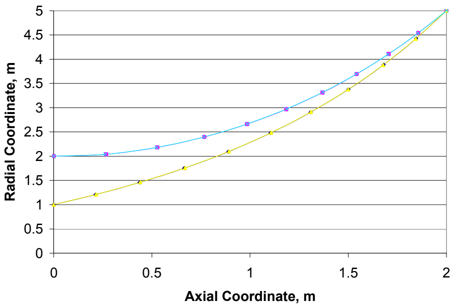


Figure 1: Equilibrium shape along a meridian ($\alpha = 1$)

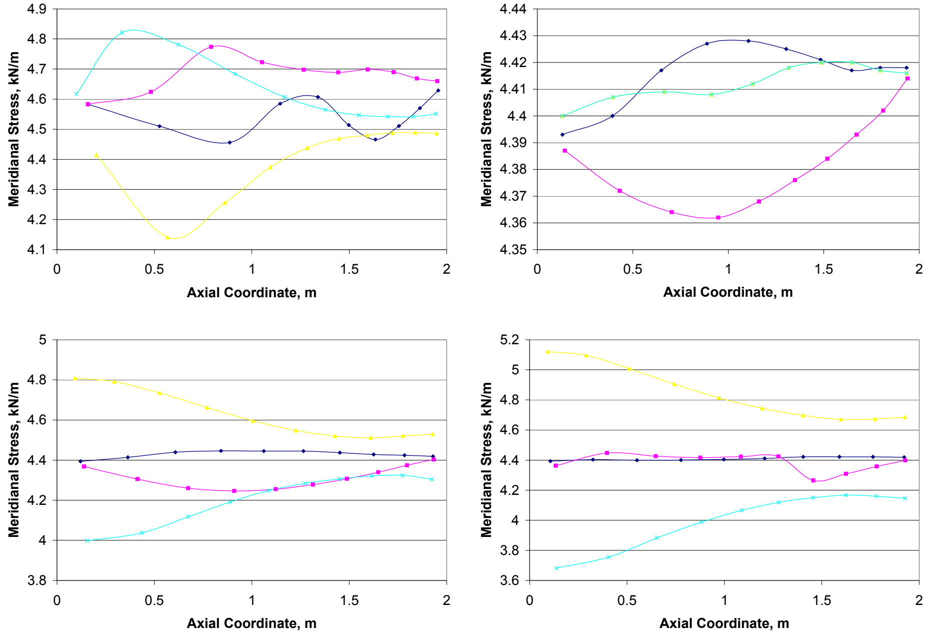
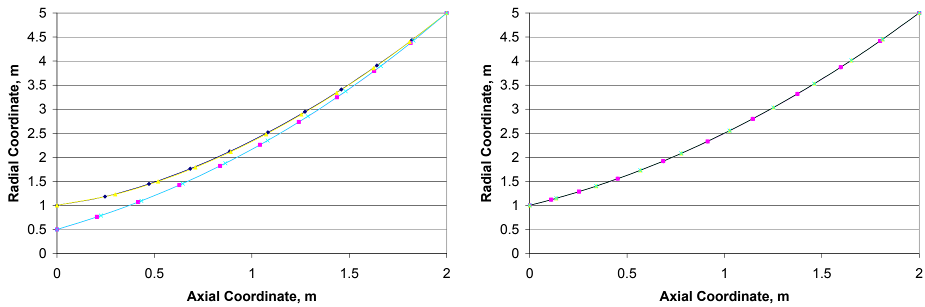


Figure 2: Meridianal stress as a function of the axial coordinate ($\alpha = 1$)



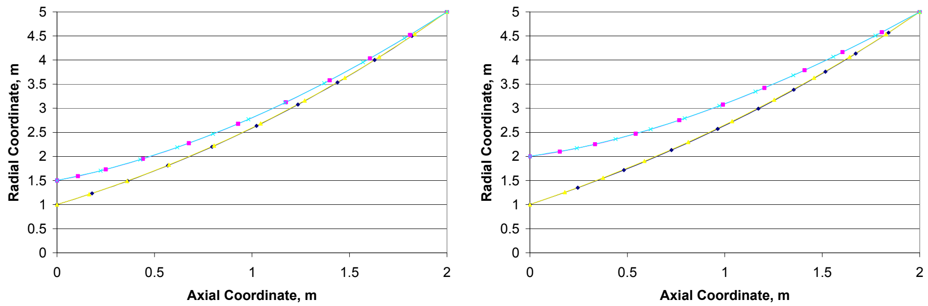


Figure 3: Equilibrium shape along a meridian ($\alpha = 2$)

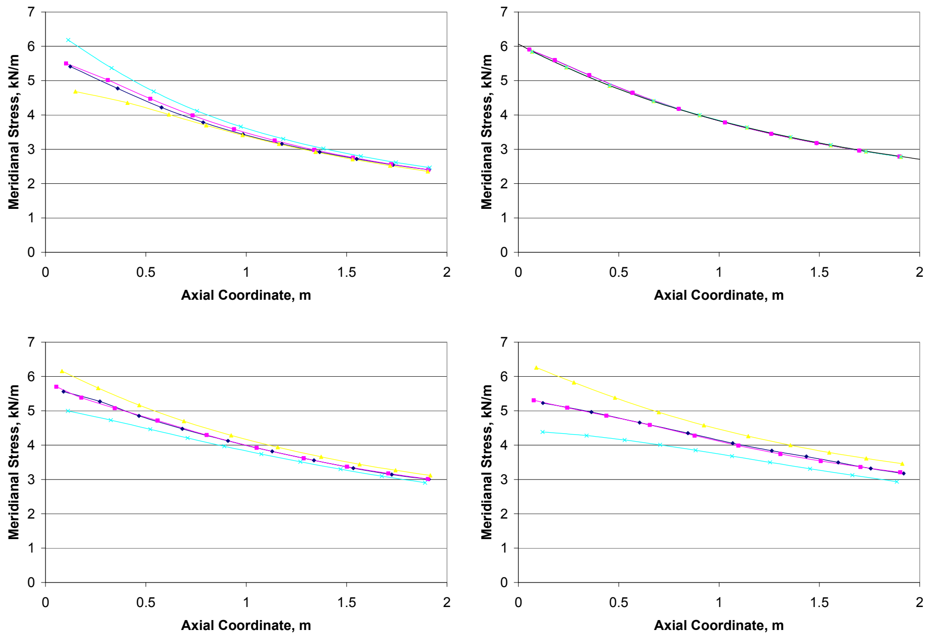


Figure 4: Meridianal stress as a function of the axial coordinate ($\alpha = 2$)

The primary differences in the two studies are demonstrated in Figures 2 and 4. The stress field in this study is designed to be independent of the circumferential coordinate, while in Gellin [4] the stress was found to be greater on the meridian with tighter circumferential curvature at the top. Figure 4, which displays the results when $\alpha = 2$, clearly demonstrates the success of the method in obtaining the desired outcome. Note that the stress at each axial coordinate appears to be approximately equal to the average of the stresses on the two primary meridians. For the axisymmetric structure, both case results are in agreement with the theoretical results.

The resolution on the vertical axis of Figure 2 is much greater; ideally, the stress should be 4.4 kN/m for all cases in the present study. The results for b = 1.50 m and b = 2.00 m appear to be more stable than the results for b = 0.50 m particularly.

Figures 1 and 3 indicate a trend found by many researchers: that small (by engineering standards) differences in equilibrium shape can lead to significant differences in stress distribution. The shape results for this study and that of Gellin [4] are nearly indistinguishable. Again, for the axisymmetric structure, the results agree with theory (Gellin [1]).

4. Conclusions and Future Research

The results of this study indicate that for the purposes of developing a shape for bi-planar symmetric cone structures without meridianal cables of varying ratios of meridianal to circumferential stress that the form of implementation of Eq. (1) appears to have minimal effect, at least for those cases studied; however, the form of implementation of Eq. (1) on the stress distribution has a profound effect. The assumption used in this study is considered preferable to that used in Gellin [4] only because this assumption is satisfied for the results obtained by existing proprietary and commercially available software.

One of the long-term objectives of this study was to incorporate the procedures used to obtain these results into existing proprietary software. It is believed that this study may have reached the limitations of relatively simple modifications for that software. Any other theoretical approach for these structures and the interpretation and implementation of Eq. (1) will require major modifications of this software.

Acknowledgement

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References

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