

Adaptive planar structures

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Abstract

Panes and shells as only in-plane loaded structural elements play an important role in lightweight structures. However their great bearing capacity will be reduced significantly when their homogeneous stress distribution is disturbed. These disturbances can be caused for example by cuttings or constraints. Present research to reduce the disturbances by form optimization of the cuttings or by adding local bracing elements can only respond to particular loading conditions.

Adaptive structures are a new innovative alternative for the optimization of panes and shells. With the use of sensors, control units and actuators they can register different external stimuli and adapt to them. This allows an optimal reaction to different loading conditions and so a reduction of the disturbances in the homogenous stress distribution.

Keywords: adaptive structures, smart structures, optimization, membrane, lightweight structures

1. Introduction

The optimization of structural systems to more efficient and lighter structures is element of multiple investigations. Most cases show that hereby the development of new structural approaches is more effective than the optimization of existing structural systems (Wiedemann [8]).

Adaptive structural systems represent such a new structural approach. They replace material which is not necessary in all loading conditions by energy which is induced in the structural system (Rogers, C.A., et al [3]).

Three different states can be distinguished in such an adaptive system (Weilandt, *et al.* [5], Sobek, *et al.*[4]). The passive state is defined as the state where the system is without manipulation and burdened only with external loads; the activated state as the condition where only the actuators are active and a third state the so called adaptive state which is defined as the superposition of the passive and the activated state.

Passive + activated = adaptive

The system usually consists of four main components already defined by Yao [9]. The structural system, which itself is equipped with sensors for monitoring on one side the external loads acting on the system and on the other side the response of the system due to adaptive manipulation. The sensors transmit their information to a controller unit i.e. a computer which calculates the necessary response in order to fulfill the requirements defined by the designer. The controller transmits this information to the actuators integrated into the structural system.

Actuators can be categorized in two main groups: the induced strain actuators and the stiffness actuators. Induced strain actuators are elements with varying lengths and are therefore able to introduce a controlled stress scenario in the system which is superposed with the stress states from the external loads. The same effect can be achieved by changeable supports as part of the induced strain actuators group. The second group, the so called stiffness actuators, can be based on materials which can change their properties and therefore their stiffness resulting in a redistribution of the load paths within the structure.

Planar structures which are submitted to membrane state loading conditions represent lightweight constructions of high efficiency. They achieve a great bearing capacity due to a homogenous stress distribution. If the homogeneous stress distribution is however disturbed by interferences as for example local charges or cuttings the load bearing capacity is reduced significantly due to high stress concentrations. Present research to reduce the stress concentrations by form optimization of the cuttings or by adding local bracing elements can only respond to particular loading conditions; but in regard to different loading conditions these optimizing methods can induce even higher stress concentrations to the structure. This is specially the case for very light structures without dominant form defining load case.

The research activities in adaptive structures have shown that adaptive structures are able to react to different loading conditions (Teuffel [5]). So it seems obvious to use the approach of adaptive structures to optimize lightweight planar structures to achieve homogenous stress distributions.

Different kind of actuators can be defined for planar structures (Weilandt [6]). Firstly the structure itself can represent the actuator, the so-called self-activated structures. If a strain induced actuator is used in this case (strain-induced structure), the structure will be activated by strains of continuous distribution which are induced in a part or the total area of the structure. Secondly the structure can be activated by integrated discrete induced strain or stiffness actuators. Such actuators can for example be represented by local elements fixed to the structure or by fibers integrated in the structure. In comparison to the self-activated structures the integrated actuators rigidify the structure already in the passive state; they take up further charges in the adaptive state, so that the structure itself will be less charged. In a third alternative the actuators can be placed at the borders of a structure.

This paper will discuss design procedures and the efficiency of strain-induced structures and of structures with integrated discrete strain actuators more in detail. An example for the third alternative, so so-called structures with changeable support reactions, is given in Weilandt [6].

2. Strain induced adaptive pane

The interferences of the homogenous stress distribution in a pane near a cutting/hole was investigated already 1898 by Kirsch [2]. The stress distribution in a pane of infinite size charged at its edges with uniformly distributed loads p_x and p_y , which vary between $p_x/\sigma_\infty = -1$ and $p_x/\sigma_\infty = 1$ respectively $p_y/\sigma_\infty = -1$ and $p_y/\sigma_\infty = 1$, can so be described by the stress function F which contains a symmetric and an asymmetric part.

$$F = \underbrace{\frac{p_x + p_y}{2 \cdot h} \cdot [r^2 - 2 \cdot a^2 \cdot \ln r]}_{\text{symmetric part}} + \underbrace{\cos 2\varphi \cdot \frac{p_x - p_y}{2 \cdot h} \cdot \left[r^2 - 2 \cdot a^2 + \frac{a^4}{r^2} \right]}_{\text{asymmetric part}}$$

$$\sigma_r = \frac{1}{r} \cdot \frac{\partial F}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial^2 F}{\partial \varphi^2} \tag{1}$$

$$\sigma_\varphi = \frac{\partial^2 F}{\partial r^2}$$

$$\tau_{r\varphi} = -\frac{\partial}{\partial r} \left(\frac{1}{r} \cdot \frac{\partial F}{\partial \varphi} \right)$$

Depending on the ratio p_x/p_y the stresses increase near the edge of the hole by a concentration factor up to $2 \div 4$ in relation to the stresses σ_∞ in the homogenous stress field.

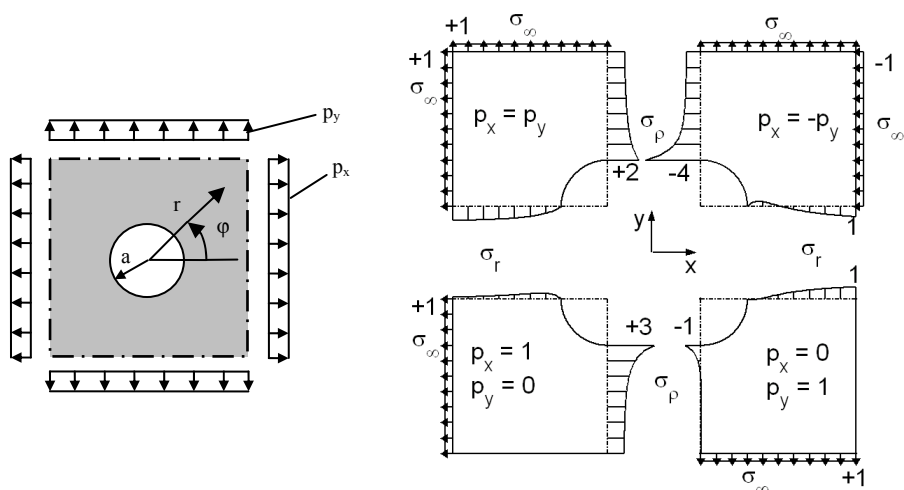


Figure 1: Pane with circular hole and its stress distribution due uniformly distributed loads of different ratios p_x/p_y

For a self-activated structure with an induced strain distribution ($\delta_{aktiv}(r, \varphi)$) the stress distribution in the activated state can be determined analogous to the stress distribution in pane charged with variable temperature. Therefore Föppl [1] gives the determining equation:

$$\Delta\Delta F + E \cdot \alpha_t \cdot \Delta t = \Delta\Delta F + E \cdot \Delta\delta_{aktiv} = 0 \quad (2)$$

The aim of the self-activation of the structure shall be defined with the homogenization of the stress field, so with a reduction of the stress concentration at the edge of the hole. Therefore the optimal distribution of the induced strains can be found by analyzing the symmetric and the asymmetric part of the stress function F.

2.1. Symmetric load cases

Comparing the homogenous stress functions of a pane without hole (3) with the stress functions of the system with hole in the passive state (4) for the symmetric load case

$$\begin{aligned} \sigma_{r \text{ passiv}, \infty, \text{sym}} &= \frac{p_x + p_y}{2 \cdot h} \\ \sigma_{\varphi \text{ passiv}, \infty, \text{sym}} &= \frac{p_x + p_y}{2 \cdot h} \end{aligned} \quad (3)$$

$$\begin{aligned} \sigma_{r \text{ passiv}, \text{with hole}, \text{sym}} &= \frac{p_x + p_y}{2 \cdot h} \cdot \left[1 - \frac{a^2}{r^2} \right] \\ \sigma_{\varphi \text{ passiv}, \text{with hole}, \text{sym}} &= \frac{p_x + p_y}{2 \cdot h} \cdot \left[1 + \frac{a^2}{r^2} \right] \end{aligned} \quad (4)$$

show that the stresses in the activated state have to fulfill the following boundary conditions (5):

$$\begin{aligned} \sigma_{r \text{ aktiv}}(a) &= 0 \\ \sigma_{r \text{ aktiv}}(r \rightarrow \infty) &= 0 \quad \text{and} \quad \sigma_{\varphi \text{ aktiv}}(a) = -\frac{p_x + p_y}{2 \cdot h} \\ \sigma_{\varphi \text{ aktiv}}(r \rightarrow \infty) &= 0 \end{aligned} \quad (5)$$

Considering the relation between strains and stresses and the compatibility conditions (6)

$$\begin{aligned} \sigma_{r,aktiv} &= \frac{E}{1-\mu^2} \left[\frac{\partial u}{\partial r} + \mu \cdot \frac{u}{r} - (1+\mu) \cdot \delta_{aktiv} \right] \\ \sigma_{\varphi,aktiv} &= \frac{E}{1-\mu^2} \left[\frac{u}{r} + \mu \cdot \frac{\partial u}{\partial r} - (1+\mu) \cdot \delta_{aktiv} \right] \\ \frac{\partial \sigma_r}{\partial r} &= \frac{\sigma_\rho - \sigma_r}{r} \end{aligned} \quad (6)$$

the optimal distribution of the induced strains and the resulting stresses in the adaptive state can be found with:

$$\begin{aligned} \delta_{aktiv,sym} &= \frac{a}{E \cdot r} \cdot \frac{p_x + p_y}{2 \cdot h} \\ \sigma_{r\,adapt,sym} &= \frac{p_x + p_y}{2 \cdot h} \cdot \left[1 - \frac{a}{r} \right] \\ \sigma_{\varphi\,adapt,sym} &= \frac{p_x + p_y}{2 \cdot h} \end{aligned} \quad (7)$$

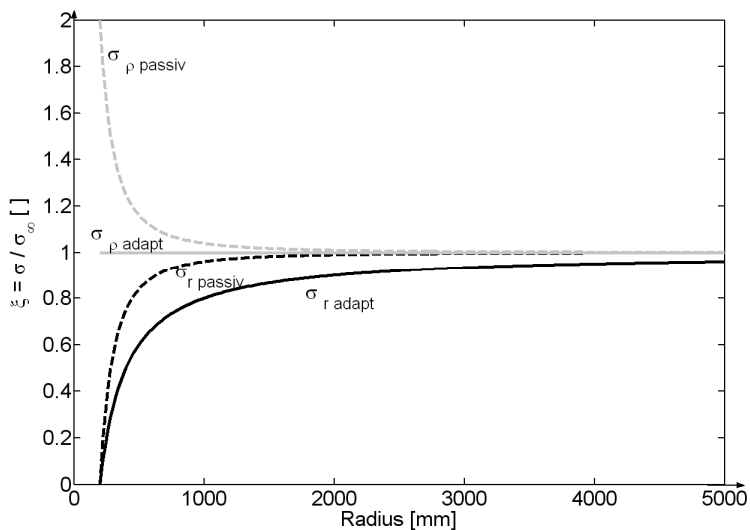


Figure 2: stress concentration factor in the adaptive and the passive state in the symmetric load case; $a=200$ mm

The resulting plot of the stress concentration factor in the adaptive state (Figure 2) shows that the increased tangential stresses can be neutralized completely by the activation of the structure.

1.2. Asymmetric load cases

The optimal induced strain distribution for asymmetric load cases can be determined in a similar manner. But in this case it is necessary to analyze the stress function F and the determining equation (2), instead of regarding the stress-strain relation and the compatibility conditions. With the boundary conditions for the stresses in the activated state (8)

$$\begin{aligned}
 \sigma_{r \text{ aktiv}}(a) &= 0 \\
 \sigma_{r \text{ aktiv}}(r \rightarrow \infty) &= 0 \\
 \sigma_{\varphi \text{ aktiv}}(r \rightarrow \infty) &= 0 \\
 \sigma_{\varphi \text{ aktiv}}(a) &= \frac{p_x - p_y}{2 \cdot h} \cdot 3 \\
 \tau_{r\varphi \text{ aktiv}}(a) &= 0 \\
 \tau_{r\varphi \text{ aktiv}}(r \rightarrow \infty) &= 0
 \end{aligned} \tag{8}$$

and the approach (9) for the stress function in the activated state

$$F_{\text{aktiv}} = \underbrace{\cos 2\varphi \cdot \left[c_1 + \frac{c_2}{r^2} \right]}_{\text{Solutions for } \Delta\Delta F=0} + \cos 2\varphi \cdot c_3 \cdot r \tag{9}$$

the optimal induced strain distribution for asymmetric load cases can be found with:

$$\delta_{\text{aktiv,asym}} = \cos 2\varphi \cdot \frac{p_x - p_y}{2 \cdot h} \cdot \frac{1}{E} \left[\frac{3 \cdot a}{r} \right] \tag{10}$$

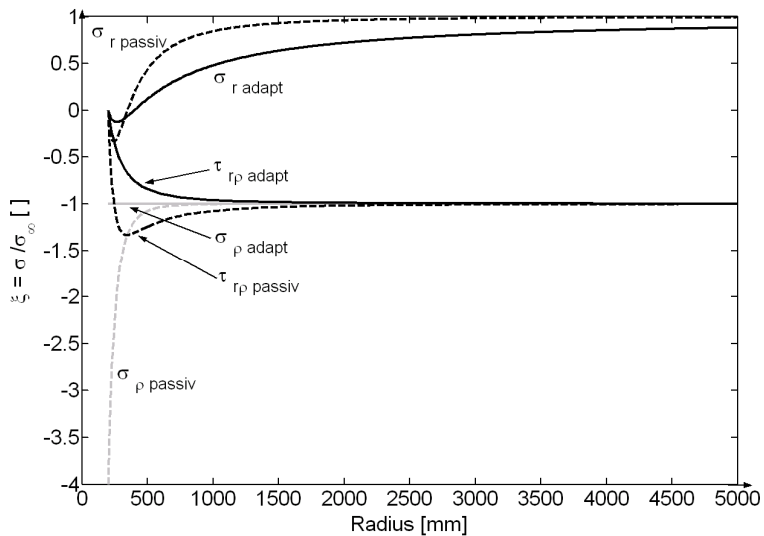


Figure 3: stress concentration factor in the adaptive and the passive state in the asymmetric load case; $a=200$ mm

The stress concentration factor for the resulting adaptive state in the asymmetric load case is shown in Figure 3. As for the symmetric load case the stresses in tangential direction can be reduced to the value of the homogenous stress field without hole. The shear stresses and the stresses in radial direction are on the other hand influenced in a larger area.

In a similar way the optimal induced strain distribution can be determined for strain-induced panes with a limited activated area. In this case the stresses in tangential direction can't be reduced to the values of the homogenous stress field anymore, but the stress concentration factor is already reduced significantly for small activated areas (Figure 4).

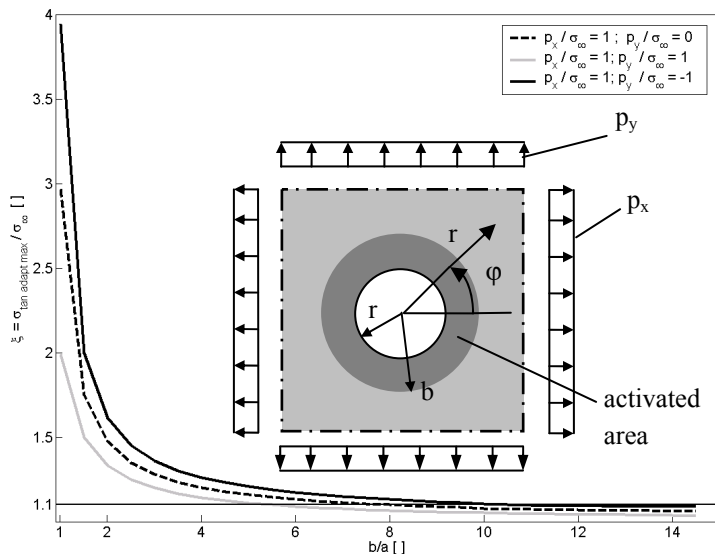


Figure 4: Maximum stress concentrations around the hole in the adaptive state depending on the size of limited activated areas for different ratios p_x/p_y

For more complicated systems the author proposes to determine the optimal induced strain distribution by a numerical approach. This approach was derived from the optimizations procedures which were developed for shape optimization problems. It describes the strain distribution by methods of computer aided geometric design, as for example linear interpolation, Bézier-splines or B-splines, which allow a minimization of the total amount of optimization variables by simultaneous compliance with continuousness of the strain distribution.

The shown example of the pane with a circular hole already demonstrates the high potential for adaptive panes. Strain induced structures are not the only way to integrate the adaptive approach to planar structures. Another approach is to activate the structure by integrated discrete induced strain or stiffness actuators. These systems will be presented in the following example.

3. Pane with integrated discrete strain actuators

Integrated discrete strain actuators rigidify already in the passive state the structural system. In the adaptive state they attract further stresses, so that the base structure itself is discharged. Adaptive structures with integrated discrete strain actuators have so a different load bearing behavior than structures with induced strain actuators as these elude high stress concentrations by yielding.

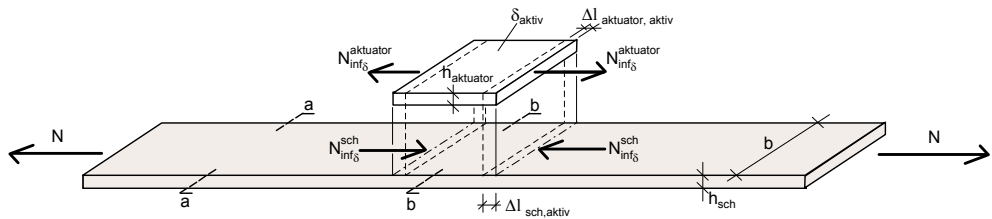


Figure 5: adaptive structure with integrated strain actuator

Adaptive structures with integrated strain actuators (Figure 5) can be compared in their load bearing behavior to non-activated structures with local reinforcement (Figure 6). The structures with integrated strain actuators have however due to its controllable strains δ_{aktiv} a variable stiffness, which corresponds to a non-realizable variable thickness of a passive local reinforcement. As the required stiffness for maximum load situations can be introduced in the system by energy, less material has to be mobilized.

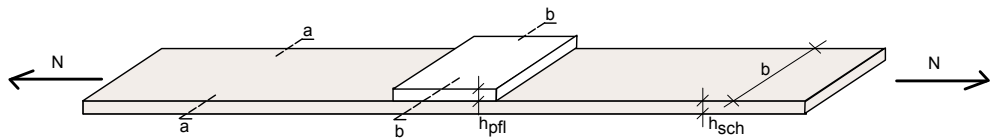


Figure 6: non-activated structure with local reinforcement

For the design of adaptive structures with integrated strain actuators it is necessary to determine the optimal position and geometry of the actuators. Aim of the design is, as for the strain induced pains, to reduce the inhomogeneities in the stress field of the investigated structure. The integrated strain actuators are therefore defined as optimal, if a defined reduction of inhomogeneities in the stress field can be achieved with a minimum actuator volume. The actuators are hereby defined as elements of constant thickness and of mono-axial material. Several actuators can be integrated in one structure. These actuators with different orientation are activated with one constant induced strain.

One approach to determine the optimal position of the integrated actuators is to determine in a first approach the optimal position of local reinforcements in a non-activated structure for each different load case. Afterwards the position, orientation, but also the required induced strains/ thickness, of the integrated strain actuators can be deviated by superposition of the non-active local reinforcements and by consideration of the following relation between the actuator and the non-active reinforcement thickness

$$h_{aktuator} = \frac{\delta_{aktiv} \cdot E_{sch} \cdot E_{aktuator} \cdot h_{pfl} + \sigma_{aktuator, b-b} \cdot E_{sch} \cdot h_{pfl}}{E_{aktuator} \cdot \sigma_{aktuator, b-b}} \quad (11)$$

Herein is E_{sch} the elastic modulus of the structure and the non-active reinforcement, $E_{aktuator}$ the elastic modulus of the integrated strain actuator and $\sigma_{aktuator}$ the maximum sustainable stress of the actuators.

For this approach the author proposes to determine the position of the non-active reinforcements by a discretized optimality criteria method presented by Zhou [10], which was adapted to the given optimization problem.

For the above mentioned example of a pane with a circular hole (see Figure 1 with $a = 10 \text{ mm}$, $h_{sch} = 10 \text{ mm}$) the resulting locations of non-active reinforcements are given in Figure 7 for different load cases, a maximum reinforcement thickness $h_{pfl} = 10 \text{ mm}$ and a reduction of the maximum stress concentration factor to $\sigma/\sigma_{\infty} = 1.70$.

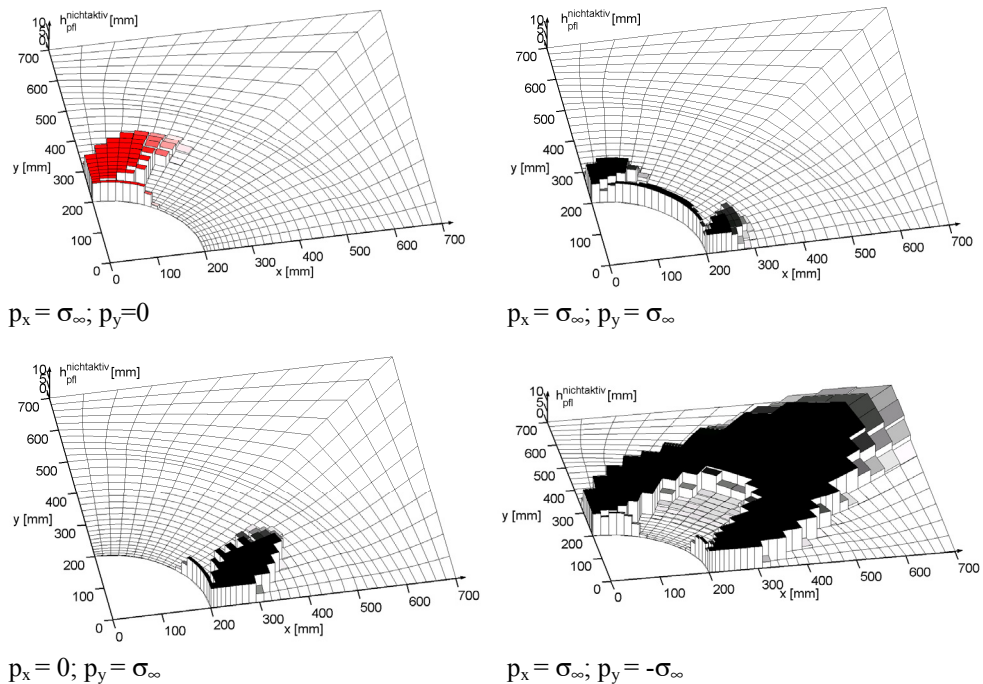


Figure 7: non-active local reinforcement with $h_{pfl} \leq 10 \text{ mm}$, $\sigma/\sigma_{\infty} = 1,70$ and different relations p_x/p_y

Superposing of the different non-active reinforcements gives the location of the integrated strain actuators, which have to be divided in the next step in actuator zones of different orientation and different levels of induced strains. The following criteria have to be considered in this process:

- Orientation of the decisive principal stress
- Ratio between the two principal stresses
- Shifting of the orientation of the principal stresses between different load cases

Two different proposals for the orientation and number of actuator zones are given in Figure 8. Depending on the numbers of the zones the efficiency factor of the adaptive system increases. The maximum achievable reduction of the stress concentration factor is hereby defined by the results of the positioning of the non-active local reinforcements.

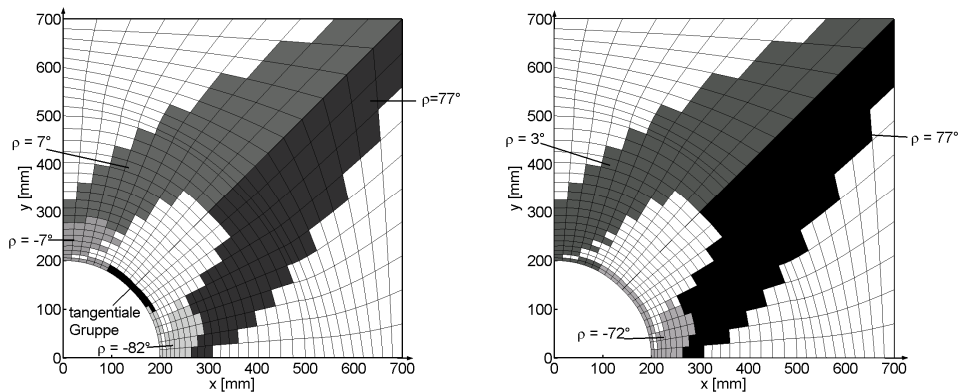


Figure 8: location of the integrated strain actuators by dividing in $n=5$ (left side) and $n=3$ (right side) zones of different orientation and induced strain

It is shown in Figure 9 that already with a small amount of actuator zones ($n=3$) the stress concentration factor is reduced significantly compared to a passive non-reinforced structure. With an increasing amount of actuator zones the stress concentration factor approaches the achievable value.

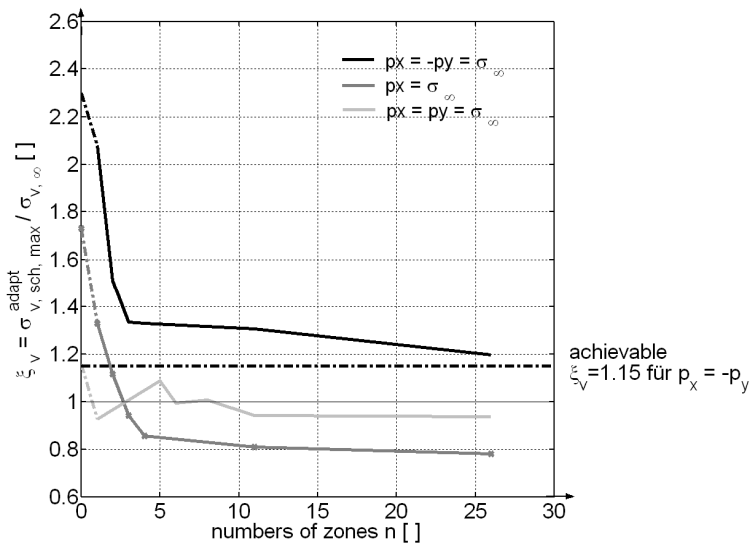


Figure 9: stress concentration factor of the maximum von-Mises-stresses in the adaptive state compared to the maximum stress in the homogenous stress field $\sigma_{v,\infty}$ for different numbers of actuator zones

4. Conclusion

The high potential of adaptive systems for light weight structures is shown by the given examples. The aim of the activation was hereby the reduction of stress concentrations, which yields according to the principals of lightweight structures to homogenous stress fields and so to a homogenous exploitation of the mobilized material under changing loading conditions.

Decisive for the further development of adaptive structures is an integral approach to the design of these structures. Hereby the actuators may not be designed as an additional element of a passive structure, but they have to be integrated to the design by the beginning.

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